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유한요소법을 이용한 전자기력 계산방법의 비교

Yan Xiuke*, 류재섭, 고창섭
*Shenyang University of Technology, 충북대학교 전기공학과

Comparison of Force Calculation Methods in Finite Element Method

Yan Xiuke*, Jae-seop Ryu, Chang Seop Koh
*School of Electrical Engineering, Shenyang University of Technology
Department of Electrical Engineering, Chungbuk National University,

Abstract - The magnetic force calculation methods, the Maxwell's stress tensor method, virtual work method, and nodal force method, are reviewed. The methods are applied to the magnetic force calculation of 2D linear and nonlinear problems. As the results, the convergence of the methods as the number of elements increases, accuracy of the methods, and integral path dependence of the methods are discussed. Finally some recommendations on the usage of the methods, including the determination of the integral path, are given.

1. Introduction

In the design of electromagnetic devices the accurate magnetic field analysis and the precise force calculation are very important. The finite element method, nowadays, is being considered as the most accurate for the precise electro-magnetic field analysis in both the linear and nonlinear problems. However, as far as the force computation is concerned, there is no definite conclusion as to which method is the most accurate although several algorithms such as Amperes force law, the Maxwell's stress tensor method(MSTM), the virtual work method(VWM), and the equivalent source method are developed and being used[1].

Among the methods, the Amperes force law and the equivalent source method are considered less accurate than the other methods and seldom used except some special purposes[2]. MSTM is one of the most widely used methods. The method, however, has been proved to give different results for different integration paths, and converge very slowly as the number of elements increases[2,3].

VWM using local Jacobian derivative, on the other hand, is developed very recently and is known to be more compatible with the finite element method because the elements used in the force calculation are also used in the field calculation[4]. In the method, however, the derivation of the formula is quite difficult and the application to the complex real engineering problems is also not easy. It is because distinguishing the deformed elements due to the virtual displacement of the object is not easy. This problem becomes more severe for the 3D problems.

The nodal force method(NFM), which is derived from the Maxwell stress tensor with the virtual displacement interpolated by shape function, is very recently developed[5]. According to the reference [5], this method is reported as accurate as VWM, and easily applicable to the 2D and 3D problems. Furthermore, in this method, the distribution of the local force can be easily computed. In these reasons, the method seems very attractive.

In this paper, MSTM, VWM, and NFM are reviewed, and the three methods are proved numerically to give essentially same results. The influence of the integration path on the computed result is also investigated for MSTM and VWM. Finally, through the numerical examples, some recommendations are given on the usage of the force calculation methods.

2. Force Calculation Methods

2.1 The Maxwell stress tensor method

According to the electromagnetic field theory, the force densities in conducting material and the magnetized material are given ($\vec{J}\times\vec{B}$) and ($-H^2\nabla\mu$), respectively [6]. Hence, neglecting the magneto-restriction force, the magnetic force density, generally, can be represented as follows:

$$\vec{f}=\vec{J}\times\vec{B}-\frac{1}{2}H^2\nabla\mu \qquad (1)$$

where the symbols have their usual meanings. Using the Amperes law and vector identities, (1) can be manipulated as follows:

$$\begin{aligned}\vec{f} &=\mu(\nabla\times\vec{H})\times\vec{H}-\frac{1}{2}H^2\nabla\mu\\ &=\nabla\cdot(\vec{B}\ \vec{H})-\frac{1}{2}\nabla(\mu H^2)\\ &=\nabla\cdot\overline{T}\end{aligned} \qquad (2)$$

where $\overline{T}$ is the Maxwell stress tensor matrix of which the elements are defined as follows:

$$T_{ij}=H_i B_j-\delta_{ij}\,\omega_m \qquad (3)$$

where δ_{ij} is the Kronecer's delta function, and ω_m is stored magnetic co-energy density.

The global force, hence, exerting to the object that occupies the volume(v) can be calculated as follows

$$\vec{F} = \int_v \nabla \cdot \overline{T} dv = \oint_s \overline{T} \cdot ds \tag{4}$$

where s is an arbitrary closed surface enclosing the object. In the numerical implementation with the finite element method, generally, the integration surface is set in air region. In 2D application especially, it can be expressed by[1,3,6]:

$$\vec{F} = \oint_l \left\{ (B_n H_t) \hat{t} + \frac{1}{2}\left(\frac{1}{\mu_0} B_n^2 - \mu_0 H_t^2\right) \hat{n} \right\} dl \tag{5}$$

where $\hat{t}$ and $\hat{n}$ are the tangential and normal unit vectors at the integration path, respectively.

2.2 The virtual work method

According to the electromagnetic energy conversion theory, the force exerted on the object can be calculated from the variation of the stored energy with respect to the displacement of the object. The force along q direction, hence, is represented as follows [4]:

$$F = -\frac{dW'_m}{dq}\bigg|_{A=constant} \tag{6}$$

where the stored magnetic energy is defined as,

$$W'_m = \int_\Omega \int^B H \cdot dB d\Omega \tag{7}$$

For the simplicity of the mathematical manipulation, the 2D isotropic magnetic material surrounded by air will be considered.

Using the first order triangle element in the finite element analysis, vector potential A can be interpolated as follows in the element (e):

$$A^{(e)} = \sum_{m=i,j,k} N_m A_m^{(e)} \tag{8}$$

where N_m is the shape function defined as

$$N_m = \frac{1}{2\triangle}(a_m + b_m x + c_m y) \quad (m=i,j,k), \tag{9-a}$$

$$a_i = x_j y_k - x_k y_j,\ b_i = y_j - y_k,\ c_i = x_k - x_j \tag{9-b}$$

With the help of the above equations, the magnetic energy in an element (e) per unit length can be expressed by

$$\vec{F}_q = -\sum_{e=1}^{M} \frac{1}{\partial q}\left[\frac{1}{4\mu^{(e)}}\{A^{(e)}\}^T [K^{(e)}]\{A^{(e)}\}/|S^{(e)}|\right] \tag{10-a}$$

$$K_{ij}^{e} = b_i b_j + c_i c_j, \quad i,j = 1,2,3. \tag{10-b}$$

where $|S^{(e)}|$ is twice of the element area.

The global force exerting on the body along q direction, using (6) and (10-a), can be written as

$$\vec{F}_q = -\sum_{e=1}^{M} \frac{1}{4\mu^{(e)}}\left[\{A^{(e)}\}^T \frac{1}{|S^{(e)}|}\frac{\partial [K^{(e)}]}{\partial q}\{A^{(e)}\} + A^{(e)T}[K^{(e)}]\{A^{(e)}\}\frac{\partial |S^{(e)}|^{-1}}{\partial q}\right] \tag{11}$$

where M denotes the number of elements, and μ is kept constant in an element in the derivation of (11).

In this method, the virtual displacement is occurred to all the nodal points on the object, and the corresponding energy variation in each element is computed and summed up to give the global force. If the object is rigid body, however, the energy variation due to the virtual displacement of the object occurs at only the elements that surround the object. In real implementation, hence, only the nodal points on the surface of the object are virtually moved and the corresponding energy variations are computed for the deformed elements only and summed up for global force.

2.3 Nodal force method

The Maxwell's magnetic stress tensor $\overline{T}$ can be represented as following:

$$\overline{T} = [\,\overline{T_x^{\rightarrow}},\ \overline{T_y^{\rightarrow}},\ \overline{T_z^{\rightarrow}}\,]^T \tag{12}$$

The magnetic volume force density, $\vec{f}$ and surface force density, $\vec{g}$ can be driven from the Maxwell's stress tensor are defined as follows[5]:

$$\vec{f} = \sum_{i=x,y,z} \nabla \cdot \overline{T_i}\ \vec{i} \tag{13-a}$$

$$\vec{g} = \sum_{i=x,y,z} (\overline{T_i}|_2 - \overline{T_i}|_1) \cdot \vec{n} \tag{13-b}$$

where $\vec{n}$ is the unit normal vector from region 1 to region 2.

For the virual displacement $\delta\vec{l}$ of the material, the virtual work done by the forces is expressed as

$$\delta W = \int_\Omega \vec{f} \cdot \delta\vec{l}\, \Omega + \int_\Gamma \vec{g} \cdot \delta\vec{l}\, d\Gamma = -\int_\Omega [\overline{T_x} \cdot (\nabla \delta l_x) + \overline{T_y} \cdot (\nabla \delta l_y) + \overline{T_z}(\nabla \delta l_z)] d\Omega \tag{14}$$

The virtual displacement of the rigid body can be interpolated using the nodal shape function, as shown in Fig.1, and can be written as

$$\delta\vec{l} = \sum_n N_n \vec{l}^n \tag{15}$$

where N_n and $\delta\vec{l}^n$ are the nodal shape function and the virtual displacement of the n-th nodal point. Substituting (15) into (14), the virtual work can be expressed as follows:

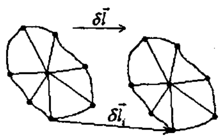


Fig.1 The displacement of a rigid body and nodal points.

$$\delta W = -\sum_n \int_\Omega (\vec{T}_x \cdot \nabla N_n \hat{x} + \vec{T}_y \cdot \nabla N_n \hat{y} + \vec{T}_z \cdot \nabla N_n \hat{z}) d\Omega \cdot \delta \vec{l}^n \quad (16)$$

Finally the force acting on the n-th nodal point is given as:

$$\vec{F}_n = -\int_\Omega (\vec{T}_x \cdot \nabla N_n \hat{x} + \vec{T}_y \cdot \nabla N_n \hat{y} + \vec{T}_z \cdot \nabla N_n \hat{z}) d\Omega \quad (17)$$

where, the volume integration will be performed only for the elements containing the n-th nodal point. The global force on the rigid body can be easily found by simple summation of the nodal forces at nodes on the body.

3. Numerical Examples

A simple magnetic lifter, shown in Fig. 2, is taken as a model, and the developed force calculation methods are applied to compute the force between the iron core and iron bar. For the magnetic field analysis, the adaptive mesh refinement technique, with the local error estimation using the field continuity condition at the interface of the elements, is used to refine the mesh. In order to see the relation between the integration path and computed force, the several integration paths are defined:

Path 1: Lines connecting the mid-points of the triangles surrounding the iron bar.
Path 2: A rectangular box that is 0.001[mm] apart from the iron bar.
Path 3: A rectangular box that is 0.9[mm] apart from the iron bar.
Path 4: A rectangular box that is 2.0[mm] apart from the iron bar.
Path 5: A rectangular box that the upper edge is 5.0[mm] and the other three edges are 10.0[mm] apart from the iron bar.

At first, the non-linear case is studied, where the applied current is 34200[AT]. In the magnetic field analysis the Newton-Raphson

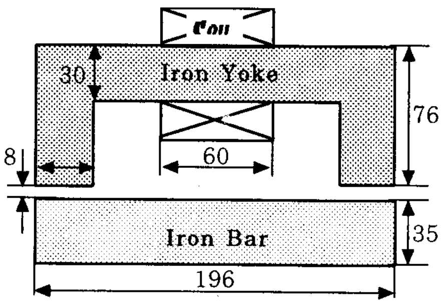


Fig.2 2D magnetic lifter.

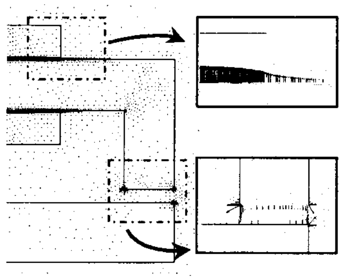
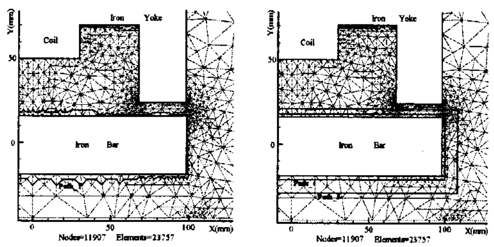


Fig.3 Distribution of the force densities in non-linear case.

method is incorporated. Fig.3 shows the distribution of the force densities on the magnetic material when the number of elements is 23,757. It is found that the forces are concentrated on the lower part of the iron yoke, and the force between the exciting current and iron yoke is quite big. The relative positions of the integration paths for the Maxwells stress tensor method are shown in Fig. 4. The computed forces using MSTM with different integration path, VWM, and NFM are compared in Fig.5. It can be seen, from this figure, that the MSTM gives the same result with the VWM and NFM so long as the integration path is taken as *Path 1* as in Fig.4-(a). It is also found that, for the accurate computation and fast convergence of the force, the VWM and NFM are more recommendable than MSTM and *Path 1* should be taken if the MSTM is used.

When computing the force with VWM, the deformed elements by the virtual displacement of the body should be taken, at least theoretically, as the elements that surround the body, as shown in Fig. 6-(a). In the computation, however, the deformed elements are chosen differently for the Path 2~5 as shown in Fig. 6-(b), and the corresponding forces are computed using VWM.

The computed forces with different deformed elements are compared in Fig.7. From the results, it is found that the different deformed elements give almost same result so long as the deformed elements are not too far from the surface of the object. It is very important advantage of VWM because the method can be easily applied to the real complex engineering model.



(a) Path 1 (b) Path 4 and 5

Fig.4. The relative positions of the integration paths for MSTM.

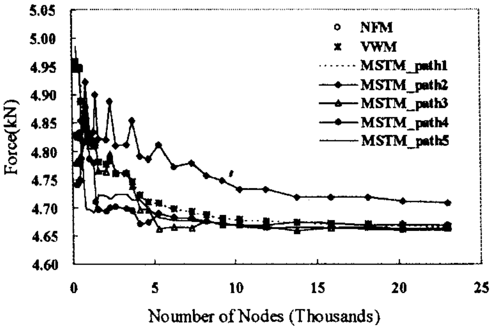


Fig.5. Convergence of the computed forces as the number of elements increases(non-linear).

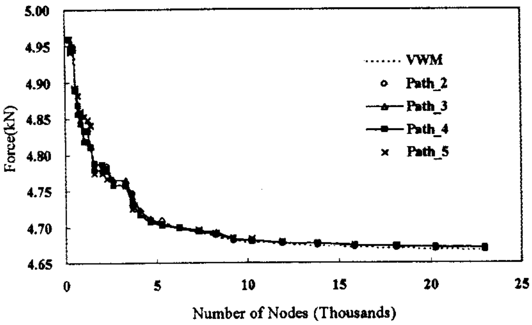


Fig.7 Dependence of the force on the moving surface in VWM.

Secondly, the linear case, where the applied current is 1140[AT] and the relative magnetic permeability of the iron yoke and bar is 3,000, is studied. The distribution of the force densities, which are computed using the NFM, are shown in Fig.8. It is found that the distribution of the force densities is quite different from that in non-linear case, and the force is highly concentrated at the corner of the iron yoke where the magnetic flux density is very high. The convergence of the computed forces is compared in Fig.9. In the nonlinear case, the three methods give the same result so long as the *Path 1* is used for the MSTM.

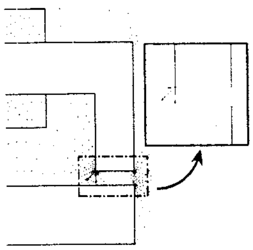


Fig.8 Distribution of the force densities(linear).

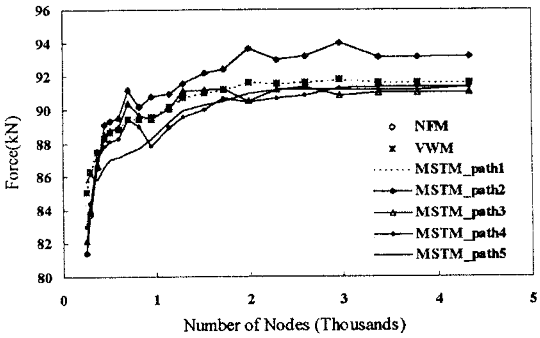


Fig.9 Convergence of the forces as the number of elements increases for linear case

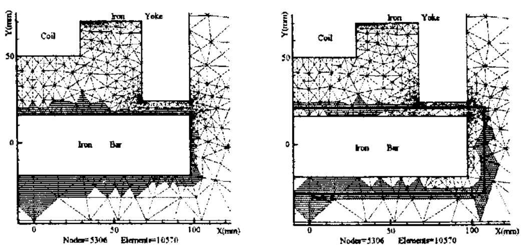
4. Conclusins

The equivalence of MSTM, VWM, and NFM are proved numerically for the linear and non-linear problems. In the viewpoint of the precise force computation, the following conclusions are obtained for 2D problems through some numerical examples.

(1) The three methods give the exactly same force so long as the integral path is taken as lines connecting the mid-points of the triangles surrounding the object.

(2) The NFM is recommendable for the precise force computation with finite element field analysis in linear and non-linear cases because the method gives as fast convergence as the other methods and the distribution of the local force, and is very simple to be implemented.

(3) When VWM is used, the moving surface can be taken arbitrary without the loss of accuracy so long as the moving surface is not too far from the surface of the object.



(a) (b)

Fig.6 The distorted elements for the VWM

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