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분포 마찰력을 받는 드럼 브레이크-슈의 동적안정성과 진동

O류봉조*, 류시웅**, 杉山 吉彦**, 오부진***

Dynamic Stability and Vibration of a Drum Brake Shoe under a Distributed Frictional Force

Bong-Jo Ryu, Si-Ung Ryu, Yoshihiko Sugiyama and Boo-Jin Oh

ABSTRACT

In this paper, dynamic stability and vibration characteristics of a flexible shoe in drum brake systems are investigated. The frictional force between the drum and the shoe is assumed as a distributed frictional force, while the shoe is modeled as an elastic beam supported by two translational springs at both ends and elastic foundations. Governing equations of motion are derived by energy expressions, and numerical results are calculated by finite element method. Through the numerical simulation, critical distributed frictional forces are calculated by changing the stiffness of two translational springs and elastic foundation parameters. It is also shown that the beam loses its stability by flutter and divergence depending on the stiffness of elastic supports and elastic foundation parameters. Finally, the time responses of the beam corresponding to their instability types are demonstrated.

1. INTRODUCTION

Drum brakes are adopted by many kinds of heavy vehicles such as trucks and busses, because the brake systems are simple and reliable. However braking operation may generate uncomfortable noises and vibrations in the braking systems. These noises and vibrations may be referred to as moan, groan, squeal, judder and so on, depending on the frequency ranges.

These different terminologies for the brake noises imply that there can be many different mechanisms for the noises and vibrations. As to the mechanism of brake noises, the earlier theory suggested that the noise could be caused by negative friction-induced vibration. It is noted that the theory based upon one-degree of freedom model[1~4].

However the recent theories of brake noises have based on the multi-degree of freedom model subjected to nonconservative loads induced by constant frictional forces between the drum and the shoe[1~4]. The theory states that the drum brake squeal is attributable to dynamic instability of elastic brake systems under a constant nonconservative loads[1~4]. The intended aim of the present paper is to show the possibility of dynamic instability of a brake shoe when it is assumed to be flexible and subjected to a distributed frictional force, while the dynamic instability of drums has been studied well so far[4]

* Member, Dept. of Mechanical Design Engng., Hanbat National University

** Dept. of Aerospace Engng., Univ. of Osaka Prefecture

*** Member, Graduate School Student, Hanbat National University

2. ANALYSIS

2.1 Mathematical Model

Figure 1 depicts a conceptual sketch of a standard drum brake system[4]. Figure 2 shows an idealized model of a drum-shoe system.

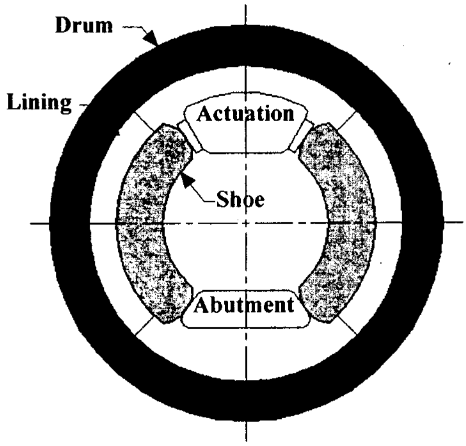

Fig. 1 Conceptual sketch of a drum brake system.

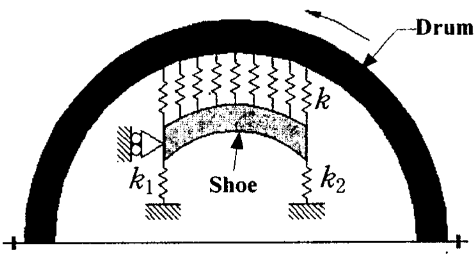

Fig. 2 Idealized drum-shoe system.

The shoe is now assumed to be elastic and thus flexible, though the real shoes are very stubby and rigid. In reality, there is a lining between the drum and the shoe. The lining is modeled for simplicity into a Winkler-type elastic foundation of the spring constant k. It is assumed that the shoe is supported at both ends elastically by two linear springs of different spring constants k_1 and k_2. The shoe is then assumed to be subjected to a distributed follower force induced by dry friction between the drum and the shoe. Distributed tangential follower force applied to a column was first considered by Pflüger in his book[5].

Fundamental aspects of the effect of distributed follower forces on the stability of columns were complied in a book by Leipholz[6]. However the existence of such kind of follower forces has been an issue of severe criticism so far[6]. Figure 2 implies one example of the existence of a distributed tangential follower force. Intended aim of the present paper is to demonstrate the possible existence of distributed tangential follower force induced by a dry friction between the drum and the shoe in drum brake systems.

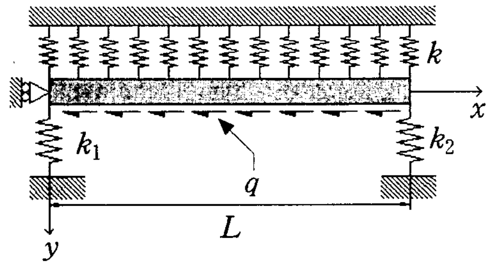

Fig. 3 Column model of a brake shoe under a distributed follower force.

Figure 3 shows a simplified column model imaged from the model in Figure 2. The column is assumed to be an elastic, uniform, and straight column of total length L, mass per unit length m, and bending stiffness EI.

2.2 Finite Element Formulation

Extended Hamiton's principle for the nonconservative system under consideration can be written in the form.

$$\delta\int_{t_1}^{t_2}(T+W_c-U-U_s)dt+\int_{t_1}^{t_2}(\delta W_{nc})dt=0 \qquad (1)$$

Energy expressions for the mathematical model in Figure 3 are written in the following forms.

The kinetic energy of the uniform column ;

$$T = \frac{1}{2}\int_0^L m(\frac{\partial y}{\partial t})^2 dx \qquad (2)$$

The work done by the conservative component of the distributed follower force ;

$$W_c = \frac{1}{2}\int_0^L q(L-x)(\frac{\partial y}{\partial x})^2 dx \qquad (3)$$

The virtual work done by the nonconservative component of the distributed follower force ;

$$\delta W_{nc} = -\frac{1}{2}\int_0^L q(\frac{\partial y}{\partial x})^2 \delta y dx \qquad (4)$$

The potential energy of the column due to bending ;

$$U = \frac{1}{2}\int_0^L EI(\frac{\partial^2 y}{\partial x^2})^2 dx \qquad (5)$$

The strain energy stored in the elastic foundation and the two spring supports ;

$$U_s = \frac{1}{2}\int_0^L ky^2 dx + \frac{1}{2}k_1 y^2(0,t) + \frac{1}{2}k_2 y^2(L,t) \qquad (6)$$

After substitution of equations (2)-(6) into equation (1), transformation leads to

$$\int_{t_1}^{t_2}\int_0^L [m(\frac{\partial y}{\partial t})\delta(\frac{\partial y}{\partial t}) + q(L-x)(\frac{\partial y}{\partial x})\delta(\frac{\partial y}{\partial x})$$

$$+ EI(\frac{\partial^2 y}{\partial x^2})\delta(\frac{\partial^2 y}{\partial x^2}) - q(\frac{\partial y}{\partial x})\delta y - ky\delta y]dxdt$$

$$-\int_{t_1}^{t_2}[(k_1 y\delta y)_{x=0} + (k_2 y\delta y)_{x=L}]dt = 0 \qquad (7)$$

For simplicity the following dimensionless quantities are introduced :

$$\xi = \frac{x}{L}, \quad \eta = \frac{y}{r}, \quad \tau = \frac{t}{L^2}\sqrt{\frac{EI}{m}}, \quad F = \frac{qL^3}{EI},$$

$$K = \frac{kL^4}{EI}, \quad K_1 = \frac{k_1 L^3}{EI}, \quad K_2 = \frac{k_2 L^3}{EI} \qquad (8)$$

where F is the distributed follower force parameter. K is the nondimensional elastic foundation parameter. Further, K_1 and K_2 are the nondimensional parameters describing the spring constants of the two elastic supports, respectively.

Equation (7) with equation (8) can be written in the dimensionless form.

$$\int_{t_1}^{t_2}\int_0^L [\eta_\tau \delta\eta_\tau + F(1-\xi)\eta_\xi \delta\eta_\xi + F\eta_\xi \delta\eta - \eta_{\xi\xi}\delta\eta_{\xi\xi} - K\eta\delta\eta]$$

$$d\xi d\tau - \int_{t_1}^{t_2}[(K_1\eta\delta\eta)_{\xi=0} + (K_2\eta\delta\eta)_{\xi=1}]d\tau = 0 \qquad (9)$$

In order to obtain a characteristic equation of small motion of the column, the column is divided into N equal elements to be compatible with finite element method. Then equation (9) can be written as

$$\int_{t_1}^{t_2}[\sum_{i=1}^{N}\int_{\frac{1}{N}(i-1)}^{\frac{1}{N}i}[\eta_\tau\delta\eta_\tau + F(1-\xi)\eta_\xi\delta\eta_\xi + F\eta_\xi\delta\eta - \eta_{\xi\xi}\delta\eta_{\xi\xi}$$

$$-K\eta\delta\eta]d\xi - \{[(K_1\eta\delta\eta)_{\xi=0} + (K_2\eta\delta\eta)_{\xi=1}]\}d\tau = 0 \qquad (10)$$

Substitution of local coordinates ($\zeta = N\xi - i + 1$) into equation (10) yields the following discretized equation ;

$$\int_{t_1}^{t_2}[\sum_{i=1}^{N}\{\eta_\tau^{(i)}\delta\eta_\tau^{(i)} + FN(N-i+1-\xi)\eta_\xi^{(i)}\delta\eta_\xi^{(i)}$$

$$-FN\eta_\xi^{(i)}\delta\eta^{(i)} - N^4\eta_{\xi\xi}^{(i)}\delta\eta_{\xi\xi}^{(i)} - K\eta^{(i)}\delta\eta^{(i)}\}d\xi$$

$$-\{(K_1\eta^{(1)}\delta\eta^{(1)})_{\xi=0} + (K_2\eta^{(N)}\delta\eta^{(N)})_{\xi=1}\}]d\tau = 0 \qquad (11)$$

The dimensionless displacement η can be assumed to take the form.

$$\{\eta^{(i)}(\zeta,\tau)\} = \{e^{(i)}(\zeta)\}\cdot\{v^{(i)}(\tau)\} \qquad (12)$$

By substitution of equation (12) into (11), finally the characteristic equation is obtained in the matrix form.

$$[M]\{v\}_{\tau\tau} + [K]\{v\} = 0 \qquad (13)$$

The displacement field varies with time according to an exponential law.

$$\{v(\tau)\} = \{X\} e^{(\lambda\tau)} \tag{14}$$

Finally, the global characteristic equation can be obtained in the form.

$$|[M]^{-1}[K] + \lambda^2 [E]| = 0 \tag{15}$$

where, $[E]$ is the unit matrix.

The stability of the system under consideration is determined by the sign of real part σ of complex eigenvalue $\lambda = \sigma \pm i\omega (i = \sqrt{-1})$. If $\sigma < 0$, the system is stable ; if $\sigma > 0$ and $\omega = 0$, the system is statically unstable, i.e., divergence type instability ; if $\sigma > 0$ and $\omega \neq 0$, the system is dynamically unstable, i.e., flutter type instability ; if $\sigma = 0$, the critical distributed follower force(F_{cr}) arises.

3. NUMERICAL RESULTS

Figure 4 shows the relation between the critical distributed follower force and the stiffness of the two equal supports $K_1 = K_2$, for the different values of elastic foundation modulus K=0.0, 10, 100. Flutter type instability can occur in the range $K_1 = K_2 \leq$ 39.976.

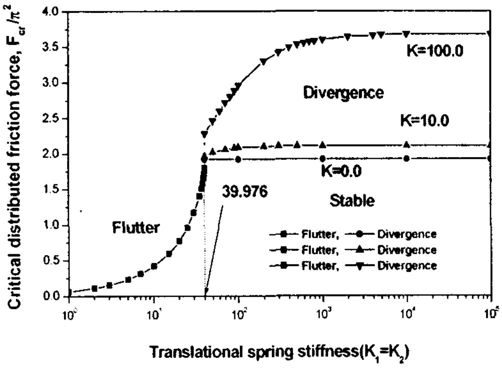

Fig. 4 Critical distributed friction forces depending on elastic foundations and translational springs.

It is noted that the critical distributed follower force does not change for the value of K in this range. This intuitively unexpected behavior was first reported by Smith and Herrmann[8]. At the critical point $(K_1 = K_2)_{cr}$=39.976, the critical distributed follower force jumps, and the instability type changes from flutter to divergence. Only the divergence type of instability occurs in the range $K_1 = K_2 \geq 39.977$.

When K=0.0, the critical distributed follower force, F_{cr}/π^2=1.921, is constant for sufficiently large values of $K_1 = K_2$. The critical flutter value F_{cr}/π^2=1.921 for the beam simply supported at both ends agrees with the earlier results in the references(6, 9). When K=10.0, 100.0, the critical distributed follower force increases with the increasing $K_1 = K_2$.

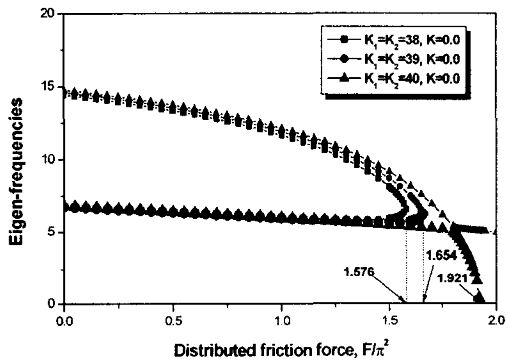

Fig. 5a First and second eigen-frequencies for distributed friction forces(K=0.0).

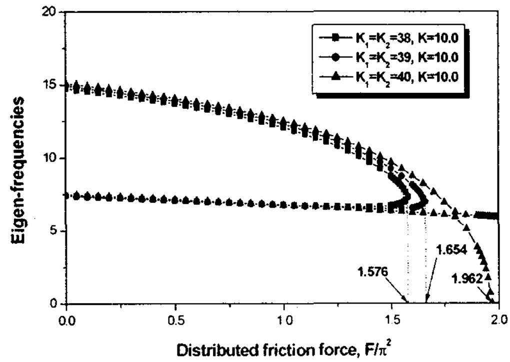

Fig. 5b First and second eigen-frequencies for distributed friction forces(K=10.0).

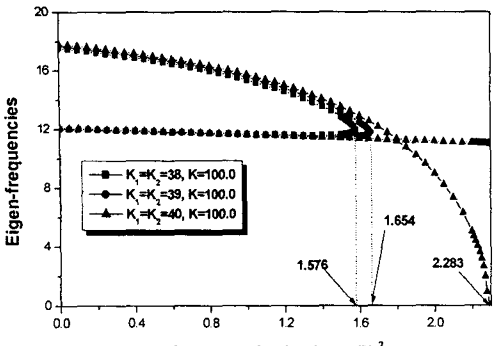

Fig. 5c First and second eigen-frequencies for distributed friction forces(K=100.0).

It is now interesting to observe eigen-value curves to understand how the instability changes from flutter to divergence can take place at the critical stiffness of $K_1 = K_2$=39.976.

Figure 5(a)~(c) show the eigen-frequencies curves of the first and second mode for various values of the support stiffness $K_1 = K_2$ =38, 39, 40. Figure 5(a) shows the eigen-frequencies for K=0.0.

It is observed in Figure 5(a) that the first and second eigen-frequencies coincide each other, and thus the flutter type instability can occurs for $K_1 = K_2$ =38 and 39. The flutter values of the distributed follower forces are F_{cr}/π^2=1.576 and 1.654. Divergence occurs as the first eigen-frequency becomes zero, at F_{cr}/π^2 =1.921 for $K_1 = K_2$=40.0.

It is seen in Figure 5(b) and (c) that the first and second eigen-frequencies become larger as the elastic embedding is stiffer. It is confirmed that the flutter value of distributed follower force is constant regardless its elastic foundation, while the eigen-frequencies by themselves depend on the stiffness of the elastic foundation as seen in Figure 5(a). (b) and (c) [8].

Figure 6 shows the relation between the critical distributed follower force and the stiffness the lower support K_1, when K=0.0 and K_2=10.0, 100.0, 1000.0. There are three regions of instability, A, B, and C. The critical distributed follower force for divergence instability increases monotonically as the value of K_1 is small enough and increasing.

Then flutter type instability takes place for the stiffness K_1 of about the order of 10.0. After a sharp climb of flutter value at K_1=40.5, divergence occurs again for K_1>40.5. The critical distributed follower force value F_{cr}/π^2=1.921, remains constant for the considerably large value K_1.

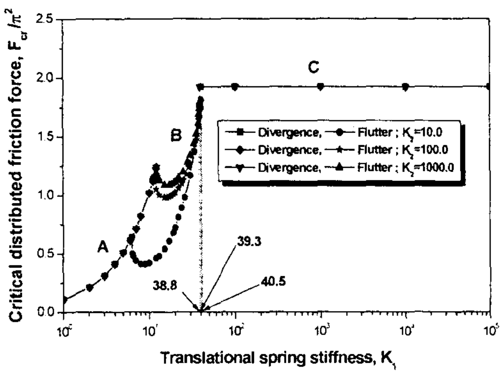

Fig. 6 Effect of the right translational spring on stability of beams, when K=0.0.

Figure 7 depicts typical unstable configurations at the B region of instability in Figure 6.

In this figure, we can see the flutter response with increasing time.

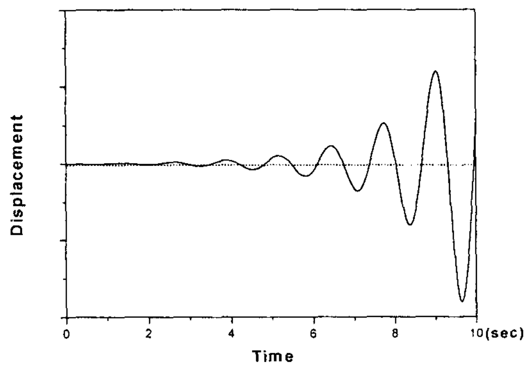

Fig. 7 Time response of the region B in Figure 6 (K = 0.0, K_1 = 20.0, K_2 = 10.0).

4. CONCLUDING REMARKS

The brake shoe is simplified into an elastically supported columns subjected to a distributed follower due to a dry friction. The evidence of the existence a distributed follower force applied to a flexible column has been demonstrated by the present paper. The paper has suggested that the instability of shoe can be considered as one of the many possible mechanisms for drum brake squeal, if the shoe is weak and thus flexible.

Though Hulten[4] concluded that the instabilities due to follower forces are negligible in real drum brake systems applied to automobiles, there are however some flexible shoes which are made from rubber and applied to bicycles.

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