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On the Linear Harmonic Analysis of Engine Exhaust and Intake Systems

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ABSTRACT

Linear harmonic analysis is a convenient and generally accurate method to use for the acoustic analysis of intake and exhaust silencers for IC engines. The major uncertainty in this form of modelling is the characterisation of the source, which is inherently nonlinear and time-variant. Experimental methods are generally used to determine the source characteristics, and in particular the indirect method is most suitable for an IC-engine source. With reference to an idealised linear time-variant source, it is found that the characteristics of a time-variant source as determined by the indirect method have no physical relevance. The direct method of experimental measurement appears to have some advantage over the indirect method, although in practice it is difficult to apply to an IC engine source. Again, an idealised linear time-variant source can be used to indicate that the characteristics of a time-variant source as determined by the direct method also have no physical relevance. Strangely, these meaningless measured source properties can nevertheless be used to accurately predict the radiated noise from an IC engine and silencer system.

1. Introduction

There are various techniques available by which to model the characteristics of the exhaust or intake system of an internal combustion (IC) engine. Each technique has its associated merits and demerits, and the answer to the question of 'which is best?' hinges upon the purpose for which the modelling is being done. If the goal is to model engine performance, for which the intake and exhaust systems influence the cylinder charging and discharging, then the most appropriate technique is undoubtedly a time-variant, non-linear analysis [1]. Generally a one-dimensional analysis, by either the method of characteristics or finite differences, will give sufficient accuracy for this purpose. However, the internals of silencer (or muffler) boxes tend to be very complicated and give rise to three-dimensional flow patterns, which thus require a three-dimensional model for really accurate analysis. Even a one-dimensional non-linear analysis is, computationally, very time-consuming and hence expensive. Thus although the output from such a model, in particular the time-varying flow in the intake/exhaust orifice, yields information about the noise output from the intake/exhaust system, generally a different modelling technique is preferred when the goal is the acoustic characterisation of the system.

Despite the generally high noise levels in an exhaust downpipe, a linear, frequency-domain analysis is capable of giving very accurate predictions of the acoustic performance of both exhaust and intake silencer systems [2]. Relative to a non-linear analysis, it is very quick and enables one to make realistic representations of the complex internals which are typical of commercial silencers. It is quite feasible to use a three-dimensional finite element or boundary element analysis of the silencer box. However at low frequencies, typically below 1 kHz on the intake side and 2 kHz on the exhaust side, a one-dimensional analysis with appropriate end corrections to model the effects of evanescent three-dimensional waves at geometrical discontinuities is just as accurate. Since an engine's primary noise source is of low frequency, and the greatest difficulty in silencer design is to reduce low frequency noise, it is conventional to use a one-dimensional linear model in an iterative manner for silencer design work. Such a model can be PC-based, will take only a few seconds to analyse a complete frequency

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spectrum for a given design, and will enable equally rapid editing of a given design.

2. Linear Frequency-Domain Analysis

A schematic of the linear frequency domain analysis of the exhaust side of an engine is shown in Figure 1(a), where everything downstream of the exhaust valves is regarded as an acoustic load, Z_l on the engine. The equivalent electrical network for the linear acoustic representation of the engine-exhaust model is shown in Figure 1(b).

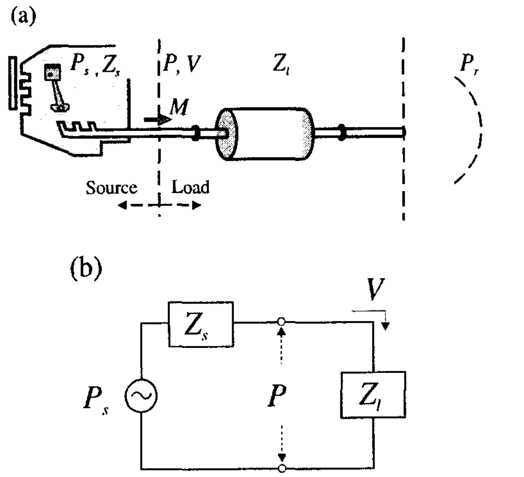

Figure 1. Source-load model. (a) Acoustic source-load system, (b) electro-acoustic circuit.

There are three basic elements to the network: the noise (engine) source; the exhaust system from downpipe to tailpipe; and the radiation characteristics of the tailpipe orifice. The latter, even considering convective effects of the mean exhaust flow from the engine, is well-known in the form of a radiation impedance. The major modelling effort goes into the determination of the transfer matrix of the exhaust system. The complete system is sub-divided into a sequence of basic acoustic elements, for each of which the relationship between the acoustic pressure and velocity at the inputs of the element to those at the outputs is known, as an element transfer matrix. Typically the coefficients of the matrix depend upon geometrical data, temperature, frequency and flow data. The overall transfer matrix then follows from matrix multiplication of the individual element matrices.

The transmission loss of the silencer can be deduced from the overall transfer matrix alone. Comparison of theoretical and experimental values of transmission loss, using some simple source for the latter, can be a useful means to determine the accuracy of the modelling of the individual acoustic elements under ideal conditions. The theoretical transmission loss can be used as a guide in the overall acoustic design of a silencer, but it is of limited usefulness because the effects of the downpipe and tailpipe are not evident with this noise measure. Furthermore it is difficult to measure the transmission loss of an operating exhaust system because to do so requires in-duct pressure measurements, and hence exposing delicate microphones to the hot and corrosive exhaust gas.

Ideally, one would like to predict the absolute levels of radiated noise from the outlet orifice of a given silencer system. In order to achieve this ideal, it is necessary to know both the strength and the impedance of the engine source. A simpler goal is to predict insertion loss, the difference in radiated sound levels between two different silencer systems for the same source, measured at some fixed free-field location relative to the tailpipe orifice. It is inherently assumed that the source is exactly the same for both systems. Prediction of insertion loss requires knowledge of the source impedance, but not its strength. Both downpipe and tailpipe effects are evident in this noise measure. Thus both radiated noise level and insertion loss are more useful than transmission loss, and are much simpler to measure, but are harder to predict in that they demand complete or partial knowledge of the noise source. Source characterisation represents the single major uncertainty in the entire process of linear frequency domain modelling of IC engine silencer systems. This stems to a large degree from the fact that an engine source is in reality non-linear and time-variant.

From Figure 1(b), it follows that the circuit equations are

$$P_{sn}(\omega_n) - P_n(\omega_n) = Z_{sn}(\omega_n)V(\omega_n) \tag{1a}$$

$$P_n(\omega_n) = Z_{l_n}(\omega_n)V(\omega_n) \tag{1b}$$

where the source impedance Z_s and, for radiated noise, the source strength P_s are required for each frequency component ω_n. The impedance is complex, consisting of a resistance and a reactance. The source data is generally obtained by experimentation, using either a direct or an indirect method.

3. Indirect Method of Source Measurement

Indirect methods make use of measurements relating to two or more varying loads on the same source. Again it is implicitly assumed that the source does not change as the load is varied. If phase information is retained then at least two loads are required [3], else at least four loads [4] are normally used. In order to reduce the effect of measurement errors a larger number of loads [5,6] are often used in order to perform some sort of averaging on the over-determined system. Even multi-load methods, with minimal experimental error, give predominantly negative source resistance values for IC-engine sources, see Figure 2.

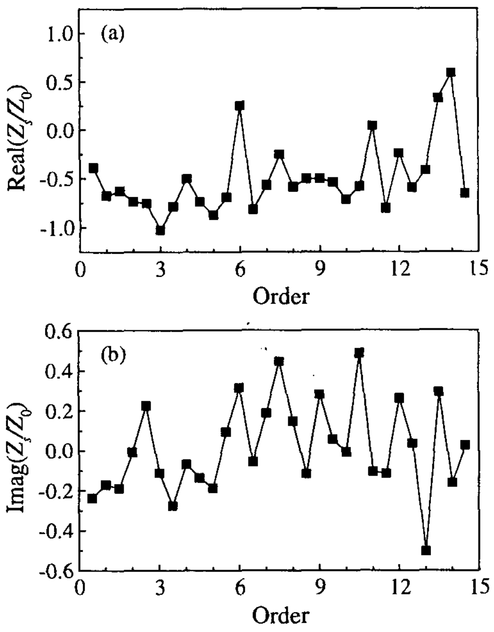

Figure 2. The measured source characteristics for the exhaust of an internal combustion engine [6]: (a) source resistance; (b) source reactance.

Negative source resistance is physically implausible, since it implies that the energy of an acoustic wave reflected by the source is greater than the energy of the incident wave. Despite this, if the measured source strength and impedance are used to compute the radiated sound level from a different load, the predictions are often in excellent agreement with experimental results, see Figure 3.

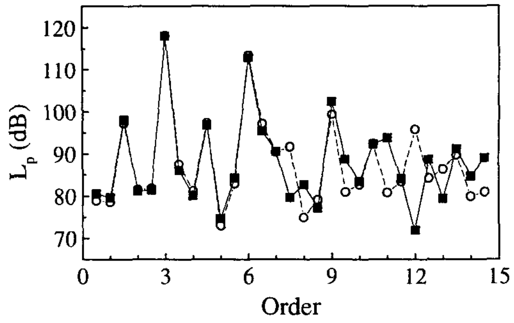

Figure 3. The sound pressure level at 50 cm from the output end of the exhaust system of an IC engine [6]: (a) -■-, predicted; (b)-o-, measured.

A previous paper [7] discussed many possible reasons for the negative source resistance as frequently measured by the indirect method. With regard to an IC-engine source, the two most likely causes would appear to be non-linearity and time-variance.

4. Analysis of Indirect Method

Since the valve motion makes an IC-engine inherently time-variant, it is worthwhile to determine whether time-variance alone can cause the negative resistance as measured by the indirect method. For this purpose, an idealised constant pressure source was considered [8], with flow velocity $u(t)$ through the open valve governed by the linear equation

$$\overline{\rho}\overline{c}u(t)=C_D\left[P_s(t)-P(t)\right], \qquad (2)$$

where C_D is a constant non-dimensional discharge coefficient and $\overline{\rho}$, $\overline{c}$ are the mean density and speed of sound of the gas flow through the valve, respectively. The time-variant nature of the source is introduced by letting the valve area $\tilde{A}(t)$ vary periodically with time, with period T. In particular the valve is assumed to be closed over some portion of the cycle, at which time of course the velocity of discharge will be zero. Thus, from equation (2), the equation of discharge throughout a cycle can be written as

$$\overline{\rho}\overline{c}V(t)=C_D\tilde{A}(t)\left[P_s(t)-P(t)\right], \quad \tilde{A}(t)\begin{cases}\neq 0, & 0\le t\le \tau,\\ =0, & \tau\le t\le T,\end{cases} \qquad (3)$$

where $V(t)$ is the volume velocity of the flow through the valve. The variables can each be expanded as complex Fourier series, and if only the first $N+1$ harmonics are non-negligible, then equation (3) results in the matrix system

$$\{V\}=[A]\{P_s - P\} \tag{4}$$

where $[A]$ is a non-dimensional admittance matrix. Its inverse is the non-dimensional impedance matrix of the source.

The conventional time-invariant source model as represented by Figure 1(b) is characterised by equation (1a). Comparison of equations (1a) and (4) indicates that they can be equivalent only if the admittance matrix is diagonal, which implies that the valve area $A(t) = A_0$, a constant. The admittance matrix is then not only diagonal, but has constant coefficients, as of course will its inverse the impedance matrix.

Results have been obtained for a particular form of valve motion using only the first 10 acoustic harmonics, i.e. N=10, and a fundamental frequency of 25 Hz. Figure 2 shows the precise time history of the valve open area together with the calculated values from the Fourier expansion followed by its inverse, using N = 10.

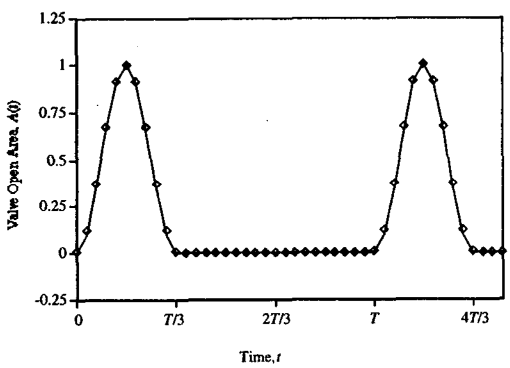

Figure 4. Time history of the non-dimensional valve open area. ——— , exact; ◊ , calculated for N=10.

The corresponding coefficients of the admittance matrix are all real and it is found that many of the off-diagonal terms are of similar magnitude to the diagonal terms, and are certainly not negligible. The coefficients of the impedance matrix $[A]^{-1}$ are all extremely large and, if sufficient harmonics were used, they would all become infinite, since the valve closed condition corresponds to $Z_s(t) \to \infty$ over part of each cycle. In other words only the admittance matrix can be used since for sufficient harmonics it is singular and its inverse does not exist. Thus in all cases where the valve is completely closed over part of the cycle, impedance is not a useful concept and one needs to use the admittance instead.

Two loads, pipes of length 1.1 m and 1.7m respectively, were used in order to evaluate an effective source impedance from the two-load method. The results are shown in Figure 5.

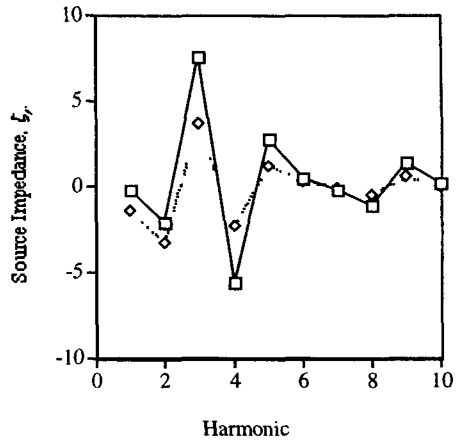

Figure 5. The effective impedance of the source, if it were time-invariant. —□— , resistance; --◊-- , reactance.

The values are complex and their magnitude bears no relation to the terms in the impedance matrix. In particular, note that the real part of many of the modal values is negative. The corresponding values of admittance, the more rational measure to use, are similarly complex and bear no relation to the real values in the

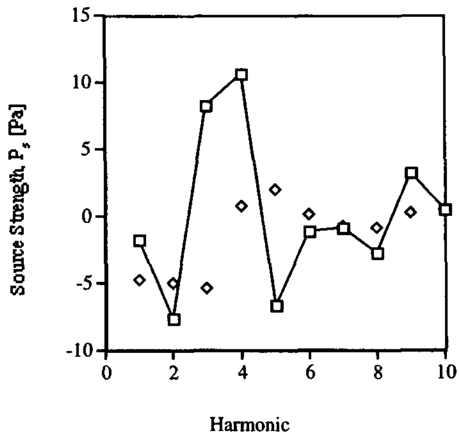

Figure 6. The effective strength of the source, if it were time-invariant. —□— , real part; --◊--, imaginary part.

admittance matrix. The two-load method yields an effective source strength as well as impedance, and these values are shown in Figure 6. The exact value is 10 Pa, real and constant. In summary, the effective source properties as measured by the two-load method, which would have been precise for a time-invariant source, bear no relationship whatsoever to the actual properties of a linear time-variant source.

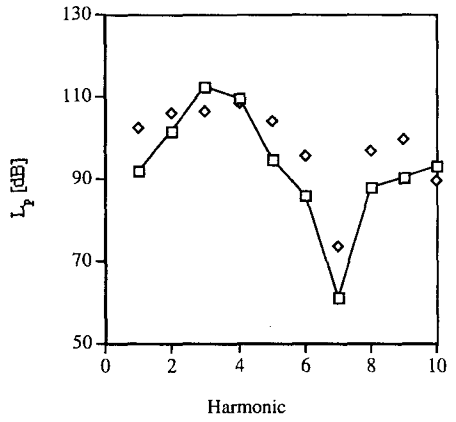

Figure 7. The sound level at the inlet to a pipe of length 1.4 m. —□— , exact; --◇--, calculated from effective time-invariant source characteristics.

Figure 7 shows the sound pressure level in the exhaust duct just downstream of the valve, as calculated for a third load, namely a pipe of length 1.4 m. A comparison is shown between the exact values and the values obtained when one assumes a linear time-invariant source with the source properties as determined from the above two-load analysis. The agreement is seen to be quite acceptable, despite the observations above.

The effective source properties as measured by the two-load method will actually be different for different pairs of loads, even when the actual ideal time-variant source is unaltered by the load. Thus the multi-load methods are effectively performing an averaging process, even in the absence of any experimental error.

5. Direct Method of Source Measurement

The direct method requires the use of a second source, placed somewhere in the load section, of much greater magnitude than the primary source such that sound output from the latter is negligible, see Figure 8. The impedance of the effectively passive source termination is then measured by conventional techniques [9-11]. The problem for an IC-engine source is to provide a dominant second source and, even if this is possible, the resultant sound level is likely to be so high as to make nonlinear effects significant. Furthermore pressure measurements are required within the flow duct which is a hostile environment, as noted before. Thus indirect methods are generally used for IC- engine sources, and have the additional benefit of yielding the source strength as well as its impedance. However, the source resistance of loudspeakers, fans and blowers as measured by the direct method has always been found to be positive. Furthermore, some nonlinear numerical simulations of the direct method for an IC-engine source also result in positive resistance [7]. Hence, in view of the preceding analysis which indicates that the properties of a time-varying source as given by the indirect method have no meaning, it is worthwhile to enquire whether, in principle, the direct method is any better.

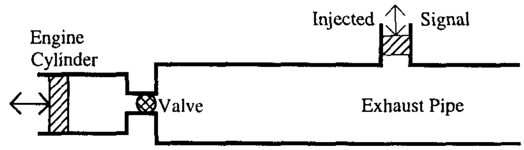

Figure 8. Conventional direct method system.

6. Analysis of Direct Method

Consider [12] an idealised time-varying linear source which discharges into a system through a valve according to equation(3). The velocity amplitude of motion of the piston source is considered to be negligibly small in comparison to the velocity field induced by the injected signal source as is required by the direct method. Thus the source is effectively a constant volume region.

The conventional four-pole matrix for a uniform pipe can be used to relate the acoustic properties at the source outlet to those of the injected signal, and similarly the acoustic properties at the piston end of the source region can be related to those at the inlet to the valve. The solution for the acoustic pressure and velocity at the outlet of the valve then follows, and their ratio gives the 'measured' source impedance, the reflectivity of the source end of the system.

Once again, constant pressure source and constant area valve conditions represent the only case in which the 'measured' source impedance corresponds to the exact one. However constant pressure source conditions now require a source of very large volume. Furthermore, as before, for a time-varying valve an admittance matrix is required, which cannot be inverted to an impedance matrix when the valve motion is such that it is fully closed over part of the cycle. Once again the 'measured' source impedance will be dependent upon the coefficients of the four-pole matrix of the discharge pipe, that is upon the characteristics of the 'load'.

Figure 9 shows the effectively 'measured' source impedance for a particular case. The valve opening frequency has been taken as 50Hz, and N = 10 as before. The duct and cylinder lengths are 1.7m and 0.2m respectively. The 'resonance peaks' in Figure 9 are then seen to correspond closely to frequencies of the injected signal which match harmonics of the valve opening frequency, except when such a frequency also corresponds to a multiple of half-wave resonance in the duct. More

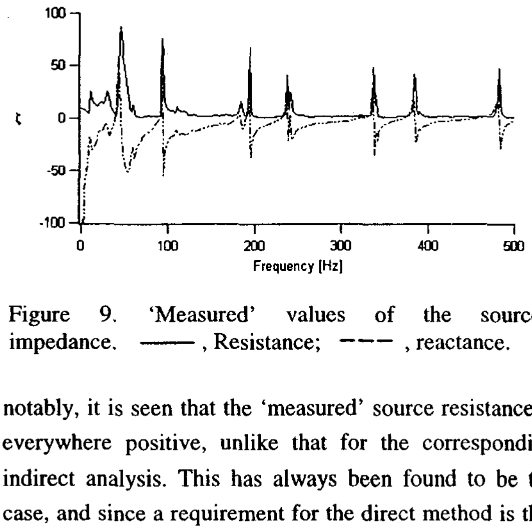

Figure 9. 'Measured' values of the source impedance. ——— , Resistance; – – – , reactance.

notably, it is seen that the 'measured' source resistance is everywhere positive, unlike that for the corresponding indirect analysis. This has always been found to be the case, and since a requirement for the direct method is that the source must be essentially passive, it is to be expected. However it is equally clear that the 'measured' impedance values bear no relation whatsoever to even the diagonal coefficients of the impedance matrix at the corresponding frequencies.

7. Conclusions

The greatest uncertainty in the linear acoustic analysis of an IC engine silencer is in the specification of the strength and impedance of the source. The source parameters as conventionally measured by the indirect method have no physical significance when the actual source is time variant. The direct method of measurement is difficult to apply to an IC engine and in principle leads to a similar outcome, although at least the resistance remains positive.

Acknowledgements

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