THERMAL EFFECTS ON THE STRAIN ENERGY RELEASE RATE FOR EDGE DELAMINATION IN CRACKED LAMINATED COMPOSITES

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SUMMARY: In this paper, edge delaminations in cracked composite plates are analytically investigated. A theoretical model based upon a sub-laminate approach is used to determine the strain energy release rate, G^{ed}, in $[\pm\theta_m/90_n]_s$ carbon/epoxy laminates loaded in tension. The analysis provides closed-form expressions for the reduced stiffness due to edge delamination and matrix cracking and the total energy release rate. The parameters controlling the laminate behaviour are identified. It is shown that the available energy for edge delamination is increased notably due to transverse ply cracking. Also thermal stresses increase substantially the strain energy release rate and this effect is magnified by the presence of matrix cracking. Prediction for the edge delamination onset strain is presented and compared with experimental data. The analysis could be applied to ceramic matrix composite laminates where similar mechanisms develop, but further experimental evidence is required.

KEYWORDS: edge delamination, matrix cracking, composite laminates, strain energy release rate, thermal stresses, delamination onset strain

INTRODUCTION

The fracture process of composite laminates subjected to quasi-static of fatigue tensile loading involves a sequential accumulation of damage in the form of matrix cracking, splitting and delaminations before the catastrophic failure. Matrix cracking in the transverse 90° plies occurs first, followed by edge or/and local delamination as the applied load increases. Edge delaminations develop due to interlaminar stresses near the free edges of the laminate and in the vicinity of matrix crack tips. They result in loss of stiffness and strength of the structure and are, along with impact damage, one of the most serious failure modes in fibre-reinforced composite structures.

Strain energy release rate approach is becoming increasingly accepted for predicting the onset and growth of delaminations. O'Brien [1, 2] proposed a simple method to model edge delaminations based on the simple rule of mixture analysis and the classical laminate theory and derived a closed form expression for the energy release rate associated with the edge delamination. The effect on thermal stresses on the edge delamination behaviour in epoxy-based composite laminates was examined in [3, 4]. Thermal stresses were found to reduce significantly the delamination onset strain. More recently, a finite element model [5], which includes the effect of thermal stresses in the calculation of the strain energy release rate, was used to study edge delaminations in fibre-reinforced thermoplastics. However, in none of these studies the coupling effect of matrix cracking that substantially influences the development of the edge delamination was taken into account. In this paper, the effect of thermal stresses on the strain energy release rate associated with the edge delamination coupling with transverse ply cracking is investigated for balance symmetric composite laminates under tensile loading.

THEORETICAL MODELLING

The damage observed in $[\pm\theta_m/90_n]_s$ CFRP laminates under monotonic tensile loading can be represented by the idealised configuration shown in Fig. 1. It consists of matrix cracks in the 90° ply and delaminations developing on both edges of the laminate. Ought to matrix cracking, delamination shifts from one $(-\theta/90)$ interface to another through the cracks in the 90° plies [7, 8], thus forming a single delamination surface at the edge.

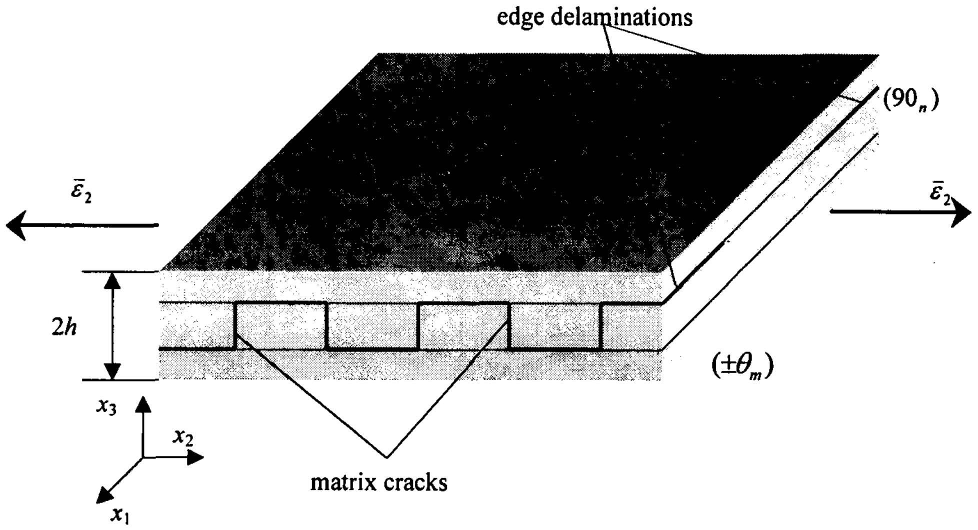

Fig. 1: Idealised damage configuration of the edge delamination in a transversally cracked composite laminate

Idealised laminate element (Fig. 1) can be schematically segregated into the laminated portion (I), which contains matrix cracks only, and two delaminated portions (II), where matrix cracks and delaminations are present (Fig. 2). Following Reference [3], the energy release rate G^{ed} associated with the edge delamination under a constant remote strain can be written as a

$$G^{ed} = -\frac{\partial U}{\partial A^{ed}} = -\frac{1}{2lw}\frac{\partial U}{\partial D^{ed}}, \quad A^{ed} = 2lwD^{ed}, \quad D^{ed} = \frac{w_{\mathrm{II}}}{w} \tag{1}$$

where U denotes the strain energy stored in the laminate element, A^{ed} is the delamination area, w_{II} denotes the delamination depth, and l and $2w$ are the laminate element dimensions (Fig. 2). The strain energy U may be written as

$$U = U(C_{\mathrm{I}}, C_{\mathrm{II}}, D^{ed}) = U_{\mathrm{I}}(C_{\mathrm{I}}, D^{ed}) + U_{\mathrm{II}}(C_{\mathrm{II}}, D^{ed}) \tag{2}$$

where U_{I} and U_{II} are strain energies stored in the laminated (I) and delaminated (II) regions of the laminate element, and C_{I} and C_{II} are crack densities in the regions I and II, respectively ($0 < C_{\mathrm{I}} \le C_{\mathrm{II}}$, [6]).

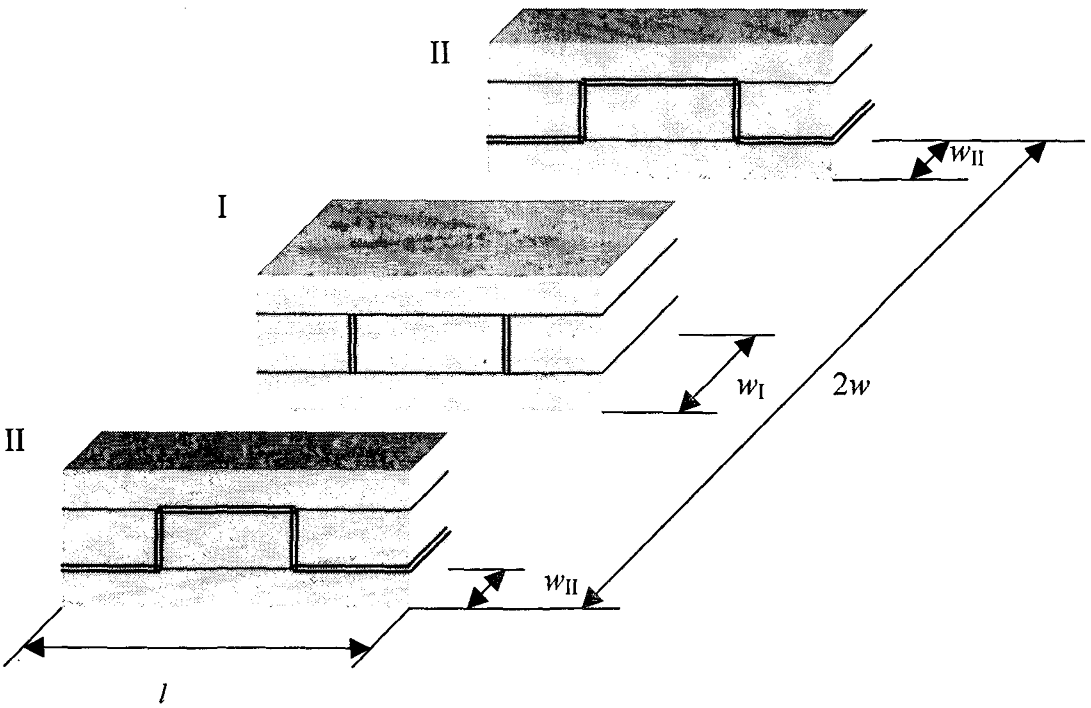

Fig. 2: Schematic of the cracked laminate element with edge delaminations: (I) laminated portion; (II) delaminated portion

In order to capture the effect of thermal stresses, a ply-by-ply calculation of the strain energy in the laminated (I) and delaminated (II) portions of the laminate is required. Application of the classical lamination theory to calculate the strain energy then yields

$$U_{\mathrm{I}}(C_{\mathrm{I}}, D^{ed}) = 4hlw(1-D^{ed})\sum_{k=1}^{N}\frac{u_{\mathrm{I}}^{(k)}(C_{\mathrm{I}})}{N}, \quad U_{\mathrm{II}}(C_{\mathrm{II}}, D^{ed}) = 4hlwD^{ed}\sum_{k=1}^{N}\frac{u_{\mathrm{II}}^{(k)}(C_{\mathrm{II}})}{N} \tag{3}$$

where N is the number of ply groups. Strain energy densities $u_{\mathrm{I}}^{(k)}, u_{\mathrm{II}}^{(k)}$ of the k^{th} ply group in the laminated (I) and delaminated (II) portions of the laminate element are

$$u_i^{(k)} = \frac{1}{2}(\{\bar{\varepsilon}_i\}'_k + \{\bar{\varepsilon}_i^T\}'_k)[\bar{Q}]_k(\{\bar{\varepsilon}_i\}_k + \{\bar{\varepsilon}_i^T\}_k), \quad i = \mathrm{I}, \mathrm{II} \tag{4}$$

where $\{\bar{\varepsilon}_i\}_k$ and $\{\bar{\varepsilon}_i^T\}_k$ are the strain and thermal strain vectors of the k^{th} ply group in the i^{th} portion of the laminate element, and $[\bar{Q}]_k$ is the reduced in-plane stiffness matrix of the k^{th} ply group. Superscript (′) denotes transpose of a vector. Using the lamination theory, the strains of k^{th} ply group in the laminated (I) and delaminated (II) portions can be related to the macrostrains, which are evaluated from the applied strain, assuming that the two portions are loaded in parallel in the x_2-direction and in series in the x_1-direction. In the delaminated portion, the two sublaminates, $(\pm\theta_m)$ and $(\pm\theta_m/90_{2n})$ for the $[\pm\theta_m/90_n]_s$ lay-up, carry load in parallel in the x_1- and x_2-directions.

Substitution of Eqns. (2)—(4) into Eqn. (1) leads to the following simple expression for the strain energy release rate associated with edge delamination

$$G^{ed}(\bar{\varepsilon}, C_{\mathrm{I}}, C_{\mathrm{II}}) = 2h\sum_{k=1}^{N}\frac{(u_{\mathrm{I}}^{(k)}(C_{\mathrm{I}}) - u_{\mathrm{II}}^{(k)}(C_{\mathrm{II}})}{N} \tag{5}$$

The effect of matrix cracking in the 90° ply group is taken into account through the reduced in-plane stiffness matrix $[\bar{Q}]_k$. It is related to the in-plane stiffness matrix $[\hat{Q}]_k$ of the undamaged lamina via the In-situ Damage Effective Functions (IDEFs) $\Lambda_{22}, \Lambda_{66}$ [6-8]

$$\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix}_k = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} & 0 \\ \hat{Q}_{21} & \hat{Q}_{22} & 0 \\ 0 & 0 & \hat{Q}_{66} \end{bmatrix}_k - \begin{bmatrix} \frac{(\hat{Q}_{12})^2}{\hat{Q}_{22}}\Lambda_{22} & \hat{Q}_{12}\Lambda_{22} & 0 \\ \hat{Q}_{21}\Lambda_{22} & \hat{Q}_{22}\Lambda & 0 \\ 0 & 0 & \hat{Q}_{66}\Lambda_{66} \end{bmatrix}_k \qquad (6)$$

The IDEFs $\Lambda_{22}, \Lambda_{66}$ for a specific matrix crack density can be determined from the micro/macromechanical analysis of a representative sandwich element defined by a pair of cracks. An improved 2-D shear lag approach is used to obtain the in-plane microstresses in the cracked 90° lamina. Consequently, the closed form expressions for the IDEFs are derived as

$$\Lambda_{jj} = 1 - \left(1 - \frac{C_{\mathrm{I}}}{\beta_j}\tanh\frac{\beta_j}{C_{\mathrm{I}}}\right)\left[1 + \gamma_j \frac{C_{\mathrm{I}}}{\beta_j}\tanh\frac{\beta_j}{C_{\mathrm{I}}}\right]^{-1}, \quad j = 2,6 \qquad (7)$$

Constants β_j and γ_j depend solely on the elastic properties of the $(\pm\theta_m)$ and (90_n) sublaminates and their thickness ratio.

The strain energy release rate given by Eqn. (5) can be used to predict the edge delamination onset strain. According to Griffith's energy principle, edge delamination initiates when

$$G^{ed}(\bar{\varepsilon}, C_{\mathrm{I}}, C_{\mathrm{II}}) = G_c^{ed} \qquad (8)$$

where G_c^{ed} is the critical strain energy release rate associated with edge delamination, determined experimentally (material property). Matrix crack densities, at which delamination initiates, are lay-up dependent values and are also taken from the experimental data.

NUMERICAL RESULTS

The influence of the thermal stresses on the strain energy release rate associated with edge delamination is examined for T300/934 carbon fibre/epoxy laminates. Lamina properties were taken from [3] and are as follows: E_{11}=144.8Gpa, E_{22}=11.38Gpa, G_{12}=6.48Gpa, G_{23}=3.45Gpa, $\nu_{12} = 0.3$, $\alpha_1 = 0.36 \cdot 10^{-6}{}^{\circ}\mathrm{C}^{-1}$, $\alpha_2 = 28.8 \cdot 10^{-6}{}^{\circ}\mathrm{C}^{-1}$, ply thickness t=0.132mm. The applied strain is $\bar{\varepsilon}_2 = 1\%$.

Figure 3 shows the strain energy release rate G^{ed} associated with edge delamination as a function of the matrix crack density in the delaminated region C_{II} for a $[\pm 25/90_3]_s$ T300/934 laminate. Crack density in the laminated region was assumed to be $C_{\mathrm{I}} = 0$. When there is no matrix cracking in the delamination region, the effect of the thermal stresses on the energy release rate is small. As the matrix crack density increases, thermal stresses increase

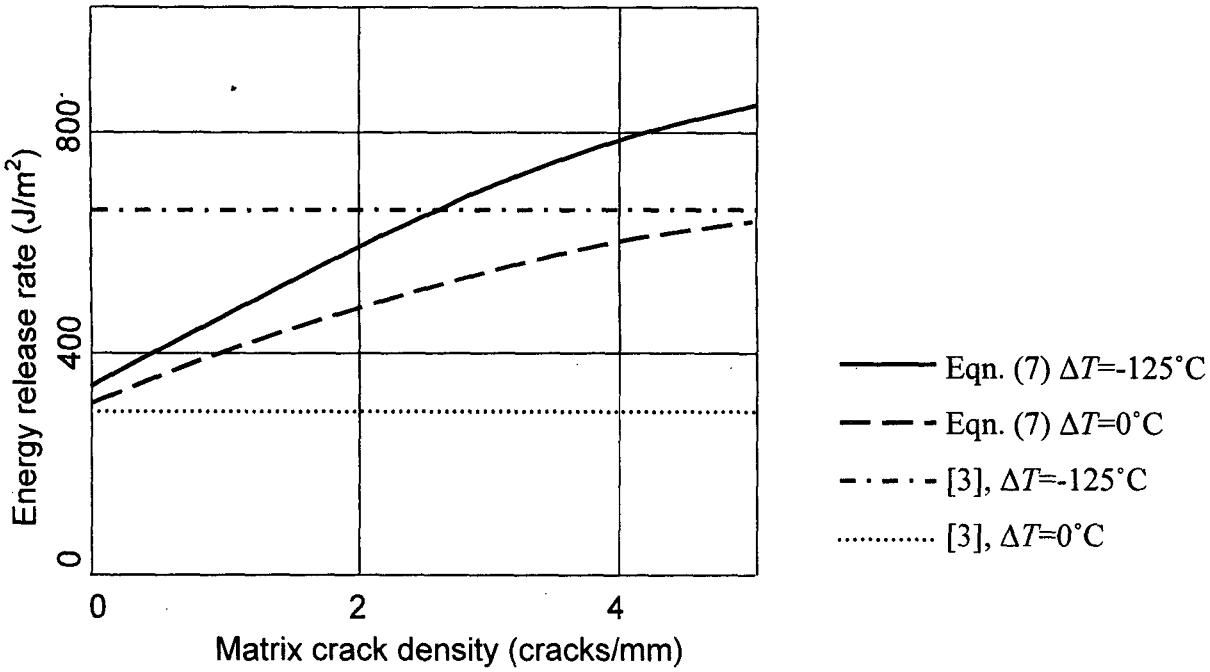

Fig 3: Energy release rate associated with the edge delamination as a function of matrix crack density for T300/934 $[\pm 25/90_3]_s$ laminate with and without thermal stresses

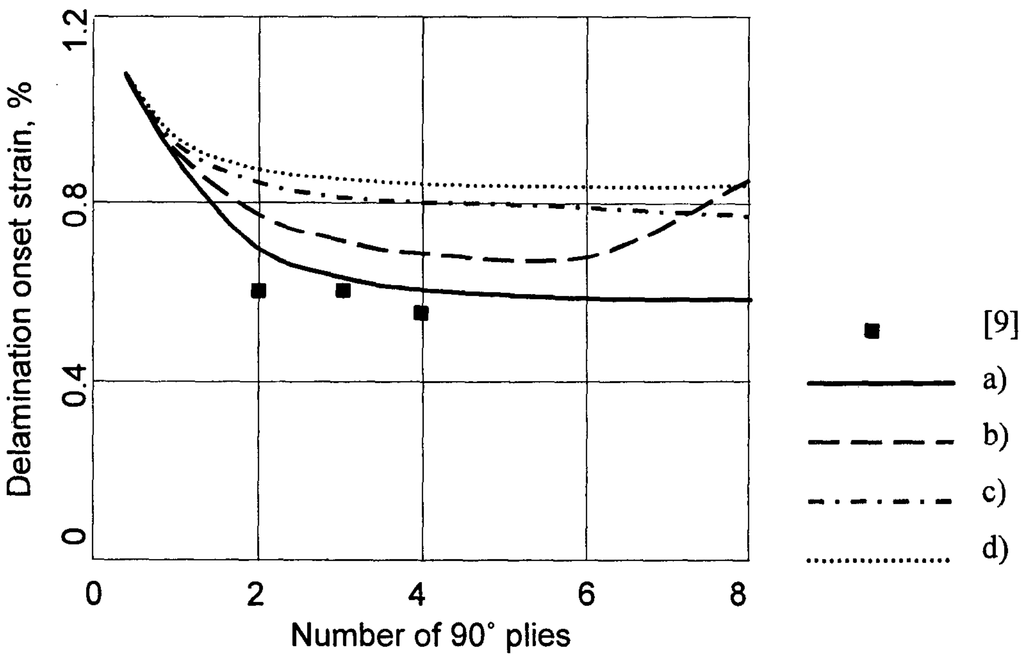

Fig. 4: Delamination onset strain as a function of the number of 90° plies for T300/934 $[\pm 25/90_3]_s$ laminate: a) including the effect of thermal stresses and matrix cracking; b) including the effect of thermal stresses only; c) including the effect of matrix cracking only; d) not including the effect of thermal stresses and matrix cracking

the energy available for edge delamination. This means that the influence of the thermal stresses is magnified significantly by matrix cracking. Predictions based on Eqn. (5) are compared with the results obtained in Reference [3], where the effect of matrix cracking was neglected. Figure 4 shows the edge delamination onset strain for T300/934 $[\pm 25/90_n]_s$ carbon fibre/epoxy laminates as a function of the number of 90° plies. Matrix crack densities, required to solve Eqn. (8), were taken from Reference [9], while the critical value of the strain energy release rate was assumed to be $G_c^{ed} = 216\text{J/m}^2$. Matrix cracks in T300/934 $[\pm 25/90_n]_s$ plates were observed to span the whole width of the laminate, so that $C_\text{I} = C_\text{II}$. Besides that, $[\pm 25/90/\mu 25]$ and $[\pm 25/90]_s$ lay-ups exhibited no matrix cracking prior to edge delamination, i.e. $C_\text{I} = C_\text{II} = 0$ for $n = 1/2, 1$. It can be seen that matrix cracking and thermal stresses significantly reduce the edge delamination onset strain. Theoretical predictions taking into account the effect of matrix cracking and thermal stresses correlate with the experimental data much better than those that neglect both or either of these factors.

CONCLUSIONS

The effect of thermal stresses on the strain energy release rate and onset strain associated with the edge delamination in balance symmetric composite laminates has been examined using an analytical model that takes into account coupling between edge delamination and matrix cracking. To incorporate thermal stresses, strain energy release rate was calculated on ply-by ply basis, while matrix cracking was taken into account via the reduced in-plane stiffness properties of the 90° ply group. Thermal stresses were found to increase the strain energy release rate and decrease substantially delamination onset strain. The effect of thermal stresses is magnified significantly by the presence of matrix cracks. The analytical model described in this work could be applied to laminated ceramic matrix composites where similar damage mechanisms are observed.

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