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# Exponential 스텝사이즈를 이용한 스마트안테나용 블라인드 LMS 알고리즘

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## A novel class of LMS Algorithms with exponential step size for Smart Antenna Applications

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### 요 약

블라인드 형태의 새로운 LMS 알고리즘을 제안한다. 기존의 LMS 알고리즘의 스텝사이즈를 지수를 이용하여 가변시켜서 수렴속도가 향상되도록 한 두가지 방식을 보여준다. 본 제안방식은 디지털 신호의 Finite Constellation 특성을 이용하여 Projection 방식을 이용했다. 제안알고리즘의 성능을 증명하기 위해서, AWGN 채널과 다중경로 Rayleigh Fading 채널상황에서 시뮬레이션을 수행하였다.

### ABSTRACT

In this paper, we propose two novel blind LMS algorithms, called exponential step size LMS algorithms (ES-LMS), for adaptive array antennas whose convergence speed is increased, hence they are much more capable of tracking the desired signal than the conventional LMS algorithms. Both of the algorithms require neither spatial knowledge nor reference signals since they use the finite symbol property of digital signal. Computer simulations were carried out in CDMA environment affected by multi-path Rayleigh fading to verify the performance of the two proposed algorithms.

### 키워드

Smart antenna, exponential step size, blind algorithms, adaptive beamforming.

## I. Introduction

LMS algorithm has been widely used in various applications, namely, adaptive filter, system identification and antenna beamforming because of its low computational complexity. However, since LMS algorithm is a member of the family of stochastic gradient algorithms, an appropriate choice of step size is very important for the algorithm to converge. A small step size will ensure small misadjustments in steady state, but the algorithm will converge slowly. On the other hand, a large step size will provide faster convergence and better tracking capabilities, but will result in higher misadjustments. In order to increase convergence speed, a variety of solutions have been proposed in the literature, for example the transform-domain LMS

(TR-LMS) algorithms, [1]-[3], the exponentially weighted step size NLMS, [4], some gradient adaptive step size LMS algorithms [5]-[6], or the coherent LMS algorithms [7]. Based on this background, we consider the application of LMS algorithm for smart antenna beamforming. Smart antennas are a promising approach for increasing capacity in wireless communications. Consequently, various blind and non-blind beamforming algorithms for smart antennas have been proposed in the literature. Nevertheless, blind algorithms are of more interest because they require no training signals, thus resulting in bandwidth efficiency. Such algorithm employed the advantage of signal properties such as constant modulus (CM) [8], decision-directed (DD) [9], or finite alphabet (FA) [10]. In this letter we propose two novel blind LMS algorithms, called

Exponential step size LMS (ES-LMS), for smart antenna applications. Convergence speeds of the two proposed algorithms are faster than their conventional counterparts. Simulation results showed that ES-LMS1 and ES-LMS2 are much more capable of tracking signal sources than the conventional LMS algorithms.

## II. The proposed algorithms

First, consider M signals impinging at an array of N sensors, the received signal vector or the data vector  $\vec{x}(t)$  is represented by

$$\vec{x}(t) = \sum_{k=1}^M s_k(t) \vec{a}(\theta_k) + \vec{n}(t) \quad (1)$$

where,  $\vec{x}(t) = [x_0(t), x_1(t), \dots, x_{N-1}(t)]^T$  is the received signal vector.  $s_k(t)$  is the  $i^{th}$  impinging signal.  $\vec{n}(t)$  is the additive white gaussian noise (AWGN) at the array. And  $\vec{a}(\theta_k) = [a_1(\theta_k), a_2(\theta_k), \dots, a_N(\theta_k)]^T$  is the steering vector.

The array output is given by

$$y = \vec{w}^H \vec{x} \quad (2)$$

Second, let us consider the conventional LMS algorithm in which the beamforming weights are updated as follows [11]:

$$\vec{w}(n+1) = \vec{w}(n) - \mu \vec{x}(n) e^*(n) \quad (3)$$

Where  $\mu$  is the step size parameter, which controls the convergence speed of the algorithm.  $\vec{x}(n)$  is the received signal vector at the  $n^{th}$  snapshot.  $e(n)$  is the error between the desired signal and the reference signal or training signal defined by [11]

$$e(n) = d(n) - y(n) \quad (4)$$

Where  $d(n)$  is the training sequence.

In both proposed algorithms (ES-LMS1 and ES-LMS2), based on the fact that  $y(n)$  is confined in a finite set of symbols, the use of training sequence is unnecessary. After finding  $y(n)$  by equation (2), we can perform as follows:

-Project  $y(n)$  onto discrete constellations, denoted as  $\text{Pr}[y(n)]$

-Find the error by  $e(n) = \text{Pr}[y(n)] - y(n)$ .

In order to improve the convergence speed of the conventional LMS algorithms, the step sizes used for the two proposed algorithms are not constant but are varied as follows

-For ES-LMS1:

$$\mu = \mu_0 \exp(b \|\vec{x}(n) e^*(n)\|_2) \quad (5)$$

-For ES-LMS2:

$$\mu = \mu_0 \exp\left(b \frac{\|\vec{x}(n) e^*(n)\|_2}{\|\vec{x}(n)\|_2}\right) \quad (6)$$

Where,

$\|\vec{a}\|_2$  denotes the induced norm of the vector  $\vec{a}$ .  $\mu_0$  and  $b$  ( $b \geq 0$ ) are constants.

The two proposed LMS algorithms can be summarized as follows:

### a) ES-LMS1 algorithm

1. Initialize

$$\mu_0, b, \vec{w}(0), n = 0$$

2. Update weight vector,  $n = n + 1$

Receive a new snapshot.

$$y(n) = \vec{w}^H(n-1) \vec{x}(n)$$

Project  $y(n)$  onto discrete constellations,  $\text{Pr}[y(n)]$ .

$$e(n) = \text{Pr}[y(n)] - y(n)$$

Calculate the step size

$\mu = \mu_0 \exp(b \|\vec{x}(n) e^*(n)\|_2)$ , as in equation (5)

$$\vec{w}(n) = \vec{w}(n-1) - \mu \vec{x}(n) e^*(n)$$

3. Repeat until the weight vector converges.

### b) ES-LMS2 algorithm

Perform the same as ES-LMS1 algorithm, except that the step size is calculated by equation (6)

$$\mu = \mu_0 \exp\left(b \frac{\|\vec{x}(n) e^*(n)\|_2}{\|\vec{x}(n)\|_2}\right)$$

## III. Convergence condition for the two proposed algorithms

In order for the conventional LMS algorithm to converge, the step size must satisfy the following condition [11]

$$0 < \mu < 1 / \text{Tr}(R) \quad (7)$$

Where  $R = E[\vec{x}(n) \vec{x}^H(n)]$  is the correlation matrix of the vector signal in equation (1). Therefore, in the two proposed algorithms,  $\mu_0$  and  $b$  are chosen such that the step sizes in equation (5) and (6) must satisfy the condition (7). Theoretically, if the weight

vectors of the proposed algorithms converge to the optimum solution, the step sizes in equations (5) and (6) will converge to the constant  $\mu_0$ . In practice, however, rather than terminating on the optimum solution, the weight vector  $\vec{w}(n)$  compute by the LMS algorithm executes a random motion around the minimum point of the error performance surface [12]. Consequently, the step size  $\mu$  given by equations (5) and (6) is always larger than  $\mu_0$ . Therefore, in order to maintain the same steady-state excess MSE as in the conventional LMS algorithm  $\mu_0$  should be chosen to be smaller than the constant step size in the conventional LMS algorithm.

IV. Simulation results

Fig. 1 illustrates the learning curves of the two proposed algorithms and the conventional LMS-CM algorithm. The simulation was carried out in the CDMA environment using BPSK modulation. SNR was kept at 0 dB. The channel is assumed to be multi-path Rayleigh fading channel with AWGN. The processing gain is 64. The number of users is 3. Besides, the velocity of the mobile user is 80 km/h. The number of multi-path is 30. The initial direction of arrival (DOA) of the desired user is fixed. And the carrier frequency is 900 MHz.

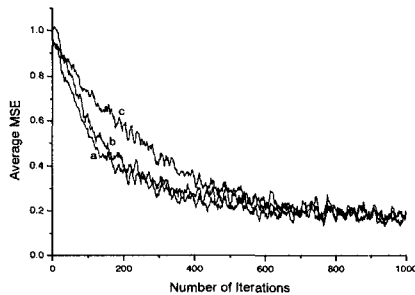


Fig.1 Learning curves of ES-LMS1 algorithm (a), ES-LMS2 algorithm (b), and LMS-CM algorithm (c) in Rayleigh fading channel with AWGN

As shown in Fig. 1, even though  $\mu_0$  is chosen to be much smaller than the constant step size  $\mu$  of the LMS-CM algorithm (0.00005 for ES-LMS1 and 0.00001 for

ES-LMS2 compared with 0.0005 for LMS-CM), the two proposed algorithms still converge to a steady-state more rapidly than LMS-CM algorithm.

Fig. 2 illustrates the BER performance versus the SNR in the Rayleigh fading channel with AWGN. The conditions for the simulation are almost the same as those used in Fig. 1, except that SNR is changed from 0 to 14 dB. As can be seen from Fig. 2, BER of the two proposed algorithms is nearly the same as that of LMS-CM algorithm.

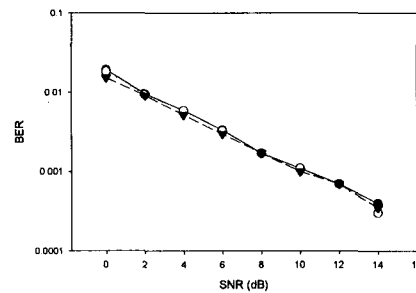


Fig.2 BER performance versus SNR for ES-LMS1, ES-LMS2, and LMS-CM algorithms in the Rayleigh fading channel with AWGN, number of antenna elements are 6, number of users are 3, processing gain is 64).

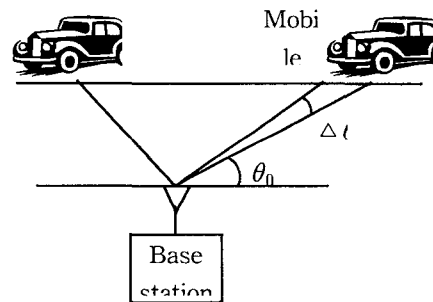
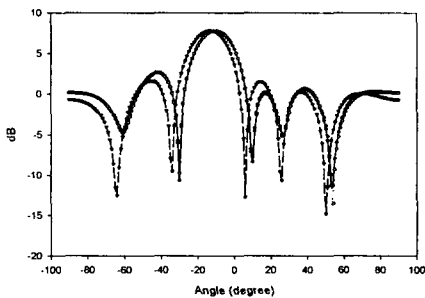


Fig. 3 Model for verifying tracking capability of ES-LMS1 and ES-LMS2 algorithms

The model shown in Fig. 3 verifies the tracking ability of ES-LMS1 and ES-LMS2 algorithm. In this simulation, as the mobile user was in motion, the DOA of the desired user was changed. The initial DOA of the desired subscriber is  $\theta_0$  and is supposed to change by an amount of  $\Delta\theta$  degree at every snapshot.

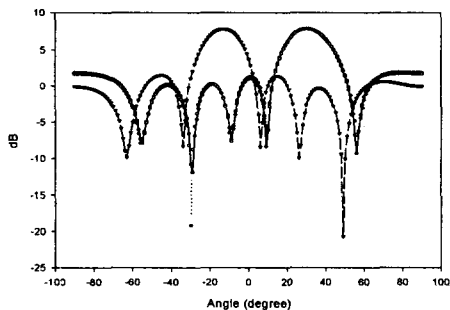
**Fig. 4** and **Fig. 5** show the beam patterns of the three algorithms after 2000 snapshots. The conditions used for the simulation are almost the same as those used in Fig. 1, except that the SNR is 3 dB, the initial DOA of the desired signal is  $-50^\circ$ , the angle change  $\Delta\theta$  is  $0.02^\circ$  and  $0.04^\circ$  for Fig. 4 and Fig. 5, respectively.

From Fig. 4 we see that the two proposed algorithms are able to track and find the exact DOA of the desired user after 2000 snapshots, the main lobes are at  $-10^\circ$ , while the LMS-CM shows an angle that is slightly different from the actual DOA.



**Fig. 4.** Beam patterns of ES-LMS1, ES-LMS2, and LMS-CM algorithms in Rayleigh fading channel with AWGN.

**Fig. 5** gives us a clearer proof to the tracking capability of ES-LMS1 and ES-LMS2. After 2000 snapshots, ES-LMS1 and ES-LMS2 still show the exact DOA of the desired user ( $+30^\circ$ ) while LMS-CM shows a very different angle from the real DOA ( $-12^\circ$ ).



**Fig.5** Beam patterns of ES-LMS1, ES-LMS2, and LMS-CM algorithms in Rayleigh fading channel with AWGN.

## V. Conclusions

In this paper we propose two novel blind LMS algorithm (ES-LMS1 and ES-LMS2) whose convergence rates are more dramatic than that of the conventional LMS-CM algorithm, thus resulting in a very good capability of tracking the desired signal. Furthermore, because of exploiting the finite constellation property of digital signal, the two proposed algorithms are applicable not only for BPSK modulation but also for other modulation techniques such as QPSK or QAM. Thus, ES-LMS1 and ES-LMS2 are good choices for real time smart antenna applications.

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