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# 스마트안테나용 블라인드 LMS 및 MMSE 알고리즘

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## New Blind LMS and MMSE Algorithms for Smart Antenna Applications

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### 요 약

기준신호를 필요로 하지않는 블라인드 방식의 새로운 LMS 및 MMSE 알고리즘을 제안한다. 디지털 신호의 Finite Constellation 특성을 이용하여 Projection 방식으로 본 알고리즘을 구현했다. 제안 알고리즘의 성능을 증명하기 위해서, AWGN 채널과 다중경로 Rayleigh Fading 채널상황에서 시뮬레이션을 수행하였다.

### ABSTRACT

We propose two new blind LMS and MMSE algorithms called projection-based least mean square (PB-LMS) and projection-based minimum mean square error (PB-MMSE) for smart antennas. Both algorithms employ the finite constellation property of digital signal to transform the conventional LMS and MMSE algorithms into blind algorithms. Computer simulations were carried out in the AWGN channel and Rayleigh fading channel with AWGN in CDMA environment to verify the performance of the two proposed algorithms.

### 키워드

LMS, MMSE, Blind algorithms, CDMA

### 1. Introduction

Wireless communication is now playing an important part in supporting various voice and data services. However, the limitation of available radio frequency resource poses a major challenge to these systems. One promising solution to this problem is to use smart antennas (or adaptive antennas). Research on adaptive beamforming algorithms for smart antennas has been carried out and various blind as well as non-blind algorithms have been achieved [1]-[5]. Most conventional beamforming algorithms are based on prior knowledge of the array manifold. Therefore, their performance strongly depends on reliable knowledge of the manifold. On the other hand, in many applications such as digital

communications the array manifold is poorly defined because of a highly variable propagation environment. In order to overcome this weakness, training sequences are used. Nevertheless, this approach results in bandwidth inefficiency. Recently, various blind algorithms, which exploit the advantage of signal properties such as constant modulus (CM) [2], decision-directed (DD) [3-4], or finite alphabet (FA) [5], have been developed. Blind methods require no training signals for the demodulation process. Thus, bandwidth efficiency can be improved. Moreover, in cellular applications, blind algorithms can be used to reject interference from adjacent cells [5].

In this paper, we propose two new blind algorithms, called PB-LMS and PB-MMSE that

are based on the finite alphabet property of digital signals to simultaneously estimate the weight vector and the desired signal. Simulation results show that the performance of these two proposed algorithms is comparable to that of LMS-CM and steepest-descent decision-directed LMS (SD-DD-LMS) algorithms.

## II. Proposed Algorithms

Consider M signals impinging at an array of N elements. A schematic diagram of the basic receiver system is illustrated in Fig. 1.

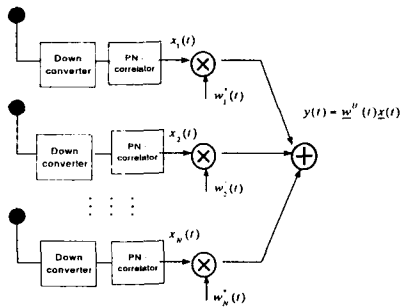


Fig. 1: Schematic diagram of the receiver.

The received signal vector or data vector at the  $k^{th}$  snapshot is represented by:

$$\vec{x}(k) = \sum_{i=0}^M s_i(k) \vec{a}(\theta_i) + \vec{n}(k) \quad (1)$$

Where,

$\vec{x}(k) = [x_0(k), x_1(k), \dots, x_{N-1}(k)]^T$  is the received signal vector.

$s_i(k)$  is the  $i^{th}$  impinging signal.

$\vec{n}(k)$  is additive white gaussian noise (AWGN) at the array.

$\vec{a}(\theta_i) = [a_1(\theta_k), a_2(\theta_k), \dots, a_N(\theta_k)]$  is the steering vector.

The array output or the desired signal can be expressed as follows:

$$y = \vec{w}^H \vec{x} \quad (2)$$

The problem addressed in this paper is to estimate the weight vector  $\vec{w}$  so that the desired signal can be recovered, given the received signal vector  $\vec{x}(n)$ .

### A. LMS algorithm with projection

Let us consider the conventional LMS algorithm in which the beamforming weights are updated as follows [6]:

$$\vec{w}(k+1) = \vec{w}(k) - \mu \vec{x}(k+1) e^*(k+1) \quad (3)$$

where,  $\mu$  is the step size parameter, which controls the convergence speed of the algorithm.  $\vec{x}(k)$  is the received signal vector at the  $k^{th}$  snapshot.  $e(k)$  is the error between the desired signal  $y(k)$  and the reference signal  $d(k)$  defined by:

$$e(k) = d(k) - y(k) \quad (4)$$

In the proposed algorithms, based on the fact that  $y(k)$  is confined in a finite set of symbols, the use of training or reference signal is unnecessary. After finding the desired signal as in equation (4), we can perform as follows:

- Project  $y(k)$  onto discrete constellations, denoted as  $\text{Pr}[y(k)]$ .

- Calculate the error by

$$e(k) = \text{Pr}[y(k)] - y(k).$$

The LMS algorithm with projection (PB-LMS) can be summarized as follows:

1. Initialize

$$\vec{w}(0), k=0$$

2. Update weight vector,  $k = k + 1$ .

Receive a new snapshot.

$$y(k) = \vec{w}^H(k-1) \vec{x}(k)$$

Project  $y(k)$  onto discrete constellations,  $\text{Pr}[y(k)]$ .

$$e(k) = \text{Pr}[y(k)] - y(k)$$

$$\vec{w}(k) = \vec{w}(k-1) - \mu \vec{x}(k) e^*(k)$$

3. Iterative until the weight vector converges.

### B. MMSE algorithm with projection

In the MMSE approach, the cost function to be minimized is [6]:

$$J(\vec{w}) = E[|\vec{w}^H \vec{x}(k) - d(k)|^2] \quad (5)$$

An adaptive solution that minimizes the cost function can be expressed as [6]:

$$\vec{w}(k+1) = \vec{w}(k) - \frac{1}{2} \mu \nabla J(\vec{w}) \quad (6)$$

where

$$\nabla J(\vec{w}) = 2E[\vec{x}(k) \vec{x}^H(k)] \vec{w} - 2E[\vec{x}(k) d^*]$$

$$\text{or } \nabla J(\vec{w}) = 2R\vec{w} - 2\vec{p} \quad (7)$$

$R = E[\vec{x}(k) \vec{x}^H(k)]$  is the correlation

matrix of the received signal vector.  $\vec{p} = E[\vec{x}(k)d^*(k)]$  is the cross-correlation of the received signal vector and the training sequence. In MMSE algorithm with projection, after finding  $y(k)$  as in equation (2), we follow the same steps as described in subsection A to find  $\Pr[y(k)]$ . The proposed MMSE algorithm can be expressed as follows:

1. Initialize

$$\vec{w}(0), R(0), P(0), k=0.$$

2. Update the weight vector,  $k = k + 1$

Receive a new snapshot

$$y(k) = \vec{w}^H(k-1)\vec{x}(k)$$

Project  $y(k)$  onto discrete constellations,  $\Pr[y(k)]$ .

$$R(k) = fR(k-1) + \vec{x}(k)\vec{x}^H(k)$$

$$\vec{p}(k) = f\vec{p}(k-1) + \vec{x}(k)\Pr^*[y(k)]$$

$$\nabla J(\vec{w}(k)) = R(k)\vec{w}(k-1) - \vec{p}(k)$$

$$\vec{w}(k) = \vec{w}(k-1) - \mu \nabla J(\vec{w}(k))$$

3. Iterative until the weight vector converges.

In this algorithm, parameter  $f$ ,  $0 < f \leq 1$ , is called the *forgetting factor*, and  $\mu$  is also the step size parameter.

### III. Discussions on the proposed algorithms

The proposed algorithms are similar to several other blind algorithms that have been developed recently such as Multi-target CM (MT-CM) algorithm and Steepest-Decent DD(SD-DD) algorithm. In the former and the latter algorithm, the errors  $e(k)$  are determined by:

$$e(k) = 2\left(y(k) - \frac{y(k)}{|y(k)|}\right) \quad (8)$$

and,

$$e(k) = y(k) - \text{sgn}(\text{Re}(y(k))) \quad (9)$$

respectively.

It can be inferred from equations (8) and (9) that in order to generate the error signals, the MT-CM algorithm use the constant envelop property of the signal, while the SD-DD employs the sign of the signal [6]. On the contrary, the proposed algorithms use the finite constellation of digital signal to make the error signals.

An other issue that we would like to comment here is the difference between the conventional

MMSE algorithm and the proposed MMSE counterpart. In the conventional MMSE algorithm, the autocorrelation matrix  $R$  and the cross-correlation  $\vec{p}$  are defined as [3]:

$$R = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \vec{x}(n)\vec{x}^H(n) \quad (10)$$

$$\vec{p} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \vec{x}(n)d^*(n) \quad (11)$$

where  $N$  is the number of snapshots. In this case, the calculations of  $R$  and  $\vec{p}$  depend not only on the present snapshot, but also on the past snapshots. The accurate estimates of  $R$  and  $\vec{p}$  can be achieved only if  $N$  is sufficiently large. This results in the large memory to store the snapshots and the expense of computational load. To overcome these weaknesses, we apply a factor called *forgetting factor* for computing  $R$  and  $\vec{p}$ , as in [7]. The purpose of this factor is to drive the degree of the dependence of  $R$  and  $\vec{p}$  on the past snapshots. The larger the factor  $f$  is, the more  $R$  and  $\vec{p}$  depend of the past snapshots and vice versa.

### IV. Computer simulation results

Fig. 2 illustrates the BER performance corresponding to the SNR in the AWGN channel and multi-path Rayleigh fading channel with AWGN.

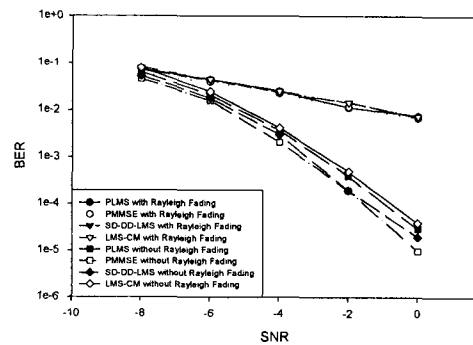


Fig. 2 BER performance versus SNR for SD-DD-LMS, LMS-CM and the two proposed algorithms in the AWGN channel and multi-path Rayleigh fading channel with AWGN.

The simulation was carried out in CDMA

environment using BPSK modulation. The processing gain is 64. The number of users is 10. The number of array elements is 10. In addition, the velocity of the mobile user is 80 km/h. The number of multi-path is 22. And the carrier frequency is 900 MHz.

As can be seen from Fig. 2, in the absence of Rayleigh fading, the BER of the PB-MMSE algorithm is slightly better than that of the conventional LMS algorithm using CM and SD-DD properties with error signals as in (12) and (13), respectively. The BER of the PB-LMS algorithm is slightly worse than that of SD-DD-LMS but better than that of LMS-CM. However, in the Rayleigh fading channel with AWGN, the BER of the four algorithms is nearly the same.

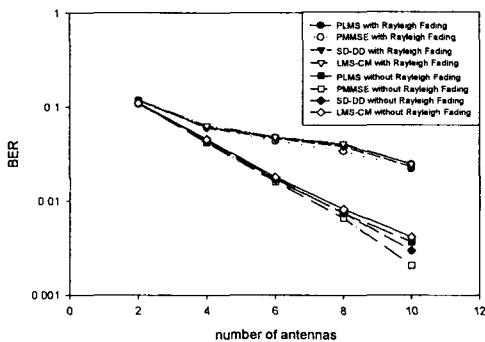


Fig. 3 BER performance versus number of antennas for SD-DD-LMS, LMS-CM and the two proposed algorithms in the AWGN channel and multi-path Rayleigh fading channel with AWGN.

Fig. 3 shows the dependence of BER on the number of array elements. The conditions for this simulation is almost the same as those used in Fig.2, except that the number of antennas is changed at SNR of -4 dB. As shown in Fig. 3, in both AWGN channel and multi-path Rayleigh fading channel, BER performance for the four illustrated algorithms is almost the same. Nonetheless, of the four algorithms, PB-MMSE still prove to have the best BER performance.

## V. Conclusions

In this paper, we propose two new blind LMS and MMSE algorithms for smart antennas. Both of the approaches have the BER performance

comparable to that of SD-DD-LMS and LMS-CM algorithms. Moreover, because of applying the finite constellation property of digital signal, the two proposed algorithms are applicable not only for BPSK, but for other modulation techniques such as QPSK and QAM as well. Besides, the PB-LMS and PB-MMSE algorithms are not complex compared with SD-DD-LMS and LMS-CM algorithms. Thus the two proposed algorithms are good choices for real time smart antenna applications.

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