Iterative Decoding for a Turbo-Coded OFDM/CDMA System

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ABSTRACT

In this paper, the performance of an OFDM/ CDMA system with turbo coding is analyzed and simulated in a multipath fading channel. Bit error probability is derived for a Rayleigh fading channel. In an OFDM/CDMA system, an OFDM and a CDMA techniques are combined to exploit the advantages of both techniques. For decoding of turbo code, MAP decoding algorithm is employed. From the simulation results, it is demonstrated that turbo coding offers considerable coding gain with reasonable encoding/decoding complexity. Also, it is shown that the BER performance is substantially improved by increasing the interleaver length for a fixed code rate and the increasing number of iterations used in the decoding process. The results in the paper can be applied to the design of OFDM-based CDMA system.

I. INTRODUCTION

So far, spread-spectrum (SS) technique has mainly been employed for military applications. Since late 1980's, the SS technique has been focused on the commercial technology, for example, mobile cellular communications employing CDMA (code division multiple access). The advantages of CDMA technique include: 1) high immunity against multipath distortion, 2) no need for frequency planning, 3) high flexibility in system design, 4) easier variable rate transmission, 5) soft capacity, and 6) soft handoff, etc.

The advantages of an OFDM technique and a CDMA system have motivated many researchers to investigate the feasibility of the combination between them. By combining two techniques, it has been confirmed through extensive simulations that the OFDM/CDMA typically has following advantages over the single-carrier CDMA system [1]: 1) OFDM/CDMA system offers more flexibility, for example, in terms of processing gain control per subcarrier or guard time control for optimization of spectral efficiency, 2) OFDM/CDMA system achieves higher system capacity by higher order of frequency diversity, 3) it is easy for OFDM/CDMA system to obtain the frequency characteristics of interference for interference suppression since all the processings are performed in the frequency-domain, 4) 'null symbol' can be used for time synchronization of each frame, 5) there is no need for equalization because all the echoes are absorbed within the guard interval, and 6) BER (bit error rate) performance is more gracefully degraded as the number of users increases.

There have been many channel coding schemes (such as RS code, BCH code, convolutional code, etc.) to improve performance of many systems by compensating channel distortion in wireless, or optical, or magnetic channels [2,3]. Recently, in the channel coding community, there has been much focus on turbo code (also termed *parallel concatenated convolutional code*) introduced by Berrou *et al* in 1993 [4]. The substantial coding gain through turbo coding has been confirmed for a CDMA system in a wireless channel as well as in an AWGN channel. However, to the best of my knowledge, there has not been an attempt to obtain coding gain for an OFDM/CDMA system yet.

In this paper, the performance of OFDM/CDMA system with turbo coding is analyzed and simulated in a multipath fading channel. Bit error probability is derived for a frequency-selective Rayleigh fading channel. In OFDM/CDMA system, the advantages of an OFDM and a CDMA techniques are combined to achieve more enhanced performance. In the decoding procedure, MAP (maximum a posteriori) decoding algorithm is adopted because it achieves better performance than the other suboptimal algorithms. And, implementation issues on nonlinear distortion, FFT, CDMA and turbo decoder are discussed.

The paper is organized as follows: In Section II, the OFDM/CDMA system is described. In Section III, bit error probability for a turbo coded OFDM/CDMA system is derived for a Raleigh fading channel. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

II. SYSTEM MODEL

The overall block diagram of a multicarrier DS/CDMA system is shown in Fig. 1. The transmitter shown in Fig. 1. (a) consists of spreaders, turbo encoder, serial-to-parallel converters, and OFDM modulator. The *ith* processor of Fig. 1. (a) is shown in Fig. 1. (b). The receiver shown in Fig. 1. (c) is composed of OFDM demodulator, parallel-to-serial converter, and turbo decoder. The data sequence of each user is spread by unique spreading sequence. After spreading, the signals of all active users are added and mapped onto the N_c subcarriers. In order to reduce the receiver complexity, the transmitted signals of the N_u users are grouped into K blocks. Then, the data sequence is spread by the spreading sequence of period *L/K*.

III. PERFORMANCE ANALYSIS

In the performance analysis, the followings are assumed: 1) chip synchronization is perfect, 2) BPSK modulation is used for data and spreading sequences, and 3) power control is perfect.

At the *kth* block, the transmitted signal is given by

$$S_k = \sum_{i=(N_u/K)k+1}^{(N_u/K)(k+1)} d_i c_i , \tag{1a}$$

$$= [s_{1,k}, s_{2,k}, \cdots, s_{L/K,k}]^T , \tag{1b}$$

where d_i is data sequence with bit duration T_b, c_i is spreading sequence with chip duration T_c, and X^T denotes the transpose of X. The S_k carries the N_u/K information bits of the *kth* block users. The total number of OFDM subcarriers is $N_c = ML$. The output of each serial-to-parallel converter is mapped onto *ML/K* subcarriers. One OFDM symbol has a duration of $T_s = MT_b$, and proceeded by a guard interval Δ to prevent ISI. The energy loss due to guard interval typically decreases as *M* increases. The output of the OFDM modulator is given by

$$x(t) = \sum_{m=0}^{M-1}\sum_{k=0}^{K-1}\sum_{l=0}^{L/K-1} s_{l,k,m} e^{j2\pi f_{mL+kL/K+l} t} , \tag{2}$$

where $s_{l,k,m}$ is the *lth* component of S_k of the *mth* data. The assigned subcarrier frequency is given by

$$f_{mL+kL/K+l} = f_{c1} + \frac{mL + kL/K + l}{T_s} , \tag{3}$$

where f_{c1} is the lowest subcarrier frequency and the subcarrier spacing is $1/T_s$. It is assumed that the guard interval is greater than the delay spread, and the channel varies slowly compared with the symbol duration. The received signal sampled at the instant N_c/T_s is given by

$$r(nT_s/N_c) = (-1)^n \sum_{m=0}^{M-1}\sum_{k=0}^{K-1}\sum_{l=0}^{L/K-1} a_{l,k,m} e^{j\phi_{l,k,m}} s_{l,k,,m} e^{j2\pi(n(mL+kL/M+l))/N_c} + \eta \tag{4}$$

where $a_{l,k,m}$ is the amplitude of fading for the subcarrier and $\phi_{l,k,m}$ is the phase of the fading component of subcarrier. The signal set $[a_{l,k,m} e^{j\phi_{l,k,m}} s_{l,k,m}]$ is the DFT (discrete Fourier transform) signals. Thus, the OFDM signal can be generated by IFFT (inverse FFT). The received signal vector R_k after FFT is given by

$$R_k = C_k S_k + Z , \tag{5}$$

where C_k is a diagonal matrix of dimension *L/K*, and *Z* is the AWGN component.

So far, the turbo decoding has primarily been performed in an AWGN channel. To apply for the multipath Rayleigh fading channel, the turbo decoder should be modified to incorporate the channel characteristics of Rayleigh fading channel. Without loss of generality, it is assumed that all-zeros codeword is transmitted. Then, the word error probability is given by

$$P_w \le \sum_{d=1}^{N} A(d)P(d) , \tag{6}$$

where *N* is block length of turbo codeword, *A(d)* is the number of codewords with Hamming distance *d*, and *P(d)* is decoding error probability of a codeword with weight *d*. To get the *A(d)*, we have to perform exhaustive search for a turbo code with fixed interleaver. So, by averaging over all possible interleavers, average weight distribution is obtained by

$$A_a(d) = \sum_{i=1}^{Q} \binom{Q}{i} p(d \mid i) , \tag{7}$$

where $p(d \mid i)$ is the probability that an input codeword with Hamming weight *i* produces a codeword with Hamming weight *d*. The average upper bounds for word error and bit error probabilities are, respectively, given by [5]

$$P_{w,a} \le \sum_{d=d_{\min}}^{N} A_a(d)P(d) , \tag{8}$$

$$P_{b,a} \le \sum_{d=d_{\min}}^{N} \sum_{i=1}^{Q} \frac{i}{Q} \binom{Q}{i} p(d \mid i)P(d) , \tag{9}$$

The probability of incorrectly decoding a codeword to another codeword which differs from each other in d bit positions $\{i_1, i_2, \cdots, i_d\}$ is given by

$$P_1 = Q\left(\sqrt{\frac{2E_s}{N_0}\sum_{k=1}^{d} a_{ik}^2}\right), \tag{10}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$ is tail integral of standard Gaussian density.

If the fading amplitudes are independent, the indexes of the different bit positions are no of importance. What only matters is the weight of incorrect codewords. Then, the decoding error probability is given by

$$P(d) = \int_{a_1} \cdots \int_{a_d} p_{\mathrm{A}}(a_1, a_2, \cdots, a_d) \cdot Q\left(\sqrt{\frac{2E_s}{N_0}\sum_{k=1}^{d} a_{ik}^2}\right) da_1 \cdots da_d \,, \tag{11}$$

where $p_{\mathrm{A}}(a_1, a_2, \cdots, a_d) = \prod_{i=1}^{d} p_A(a_i)$.

It is well known that the following bound of Q-function is useful and given by

$$Q(\sqrt{x+y}) \le \frac{1}{2} Q(\sqrt{x}) e^{-y}, \quad x, y \ge 0, \tag{12}$$

From this bound, the following relation is obtained.

$$Q\left(\sqrt{2\gamma[a_1^2 + \sum_{k=2}^{d} a_k^2]}\right) \le \frac{1}{2} Q(\sqrt{2\gamma a_1^2}) e^{-(\gamma \sum_{k=2}^{d} a_k^2)}, \tag{13}$$

where $\gamma = E_s / N_0$.

Then, from (11) and (13), the decoding error probability is upper-bounded by

$$P(d) \le \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\left(\frac{1}{1+\gamma}\right)^{d-1}. \tag{14}$$

IV. SIMULATION RESULTS

For the simulation examples, $M = T_s / T_b$ = 8, the number of subcarriers N_c = 512, transmission bandwidth B = 1.25MHz, processing gain = 128, carrier spacing B / N_c = 2.441kHz (=$1/T_s$), guard interval $\Delta = 16\mu s$, and multipath diversity order L_1 = 2. As a turbo scheme, the rate 1/3 turbo code is used. And, as a constituent code, 16-state recursive systematic convolutional codes with $d_{\min}$ = 6 are employed with code generator polynomials $(21,37)_8$ of octal representation.

In Fig. 2, bit error probability vs. SNR is shown for a different number of iterations. Simulation examples are given for the number of users N_u = 10 and interleaver length N_i = 1000. It is confirmed that the BER performance is significantly improved by turbo coding, and the performance is more improved as the number of iterations increases. Note that if the number of iterations is larger than some limit (in this result, say, 8), the performance improvement is marginal because the refinement of soft decoder output achieves diminishing returns after this limit.

In Fig. 3, bit error probability vs. SNR is shown for a different number of users. Simulation examples are given for the number of iterations = 2 and interleaver length N_i = 1000. It is shown that multiple access interference due to interfering users has a great impact on BER performance even when the turbo coding is employed.

V. CONCLUSIONS

The BER performance of the OFDM/CDMA system with turbo coding was analyzed and simulated in a multipath Rayleigh fading channel. For turbo decoding, the MAP decoding algorithm was employed. It is confirmed through simulations that turbo coding offers considerable coding gain with reasonable encoding/decoding complexity. Also, it is demonstrated that the BER performance is substantially improved by increasing the interleaver length and the number of iterations used in the decoding process. The results in the paper can be applied to the design of OFDM-based CDMA system.

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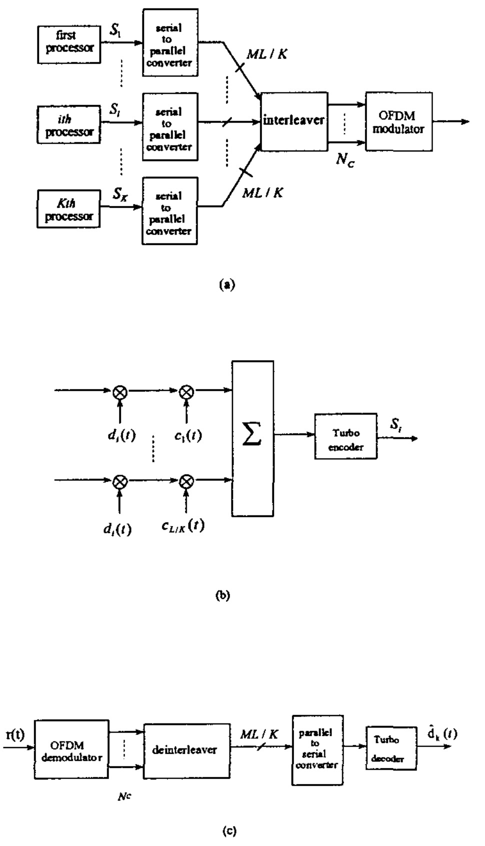

Fig. 1. Block diagram of OFDM/CDMA system.
(a) Transmitter model.
(b) The *ith* processor.
(c) Receiver model

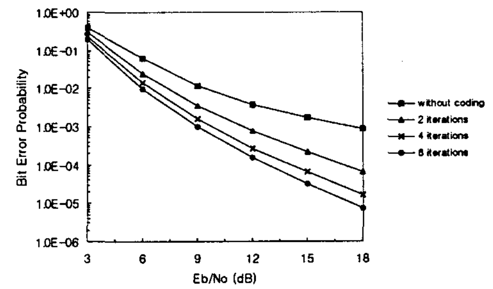

Fig. 2. Bit error probability vs. SNR for a different number of iterations.

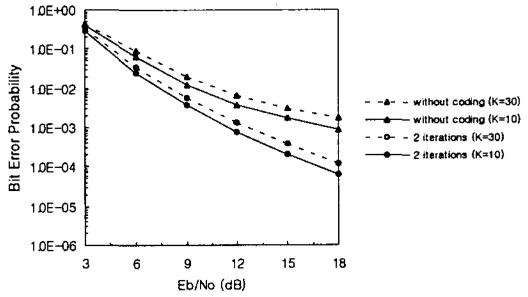

Fig. 3. Bit error probability vs. SNR for a different number of users.