Multichannel Blind Equalization using Multistep Prediction and Adaptive Implementation

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Abstract—Blind equalization of transmission channel is important in communication areas and signal processing applications because it does not need training sequence, nor does it require *a priori* channel information. Recently, Tong *et al.* proposed solutions for this problem exploit the diversity induced by antenna array or time oversampling, leading to the second order statistics techniques, for example, subspace method, prediction error method, and so on. The linear prediction error method is perhaps the most attractive in practice due to the insensitive to blind equalizer length mismatch as well as for its simple adaptive filter implementation. Unfortunately, the previous one-step prediction error method is known to be limited in arbitrary delay. In this paper, we induce the optimal delay, and propose the adaptive blind equalizer with multi-step linear prediction using RLS-type algorithm. Simulation results are presented to demonstrate the proposed algorithm and to compare it with existing algorithms.

I. Introduction

Multipath propagation appears to be a typical limitation in mobile digital communication where it leads to severe intersymbol interference (ISI). The classical techniques to overcome this problem use either periodically sent training sequences or blind techniques exploiting higher order statistics (HOS). Adaptive equalization using training sequence wastes the bandwidth efficiency but in blind equalization, no training is needed and the equalizer is obtained only with the utilization of the received signal. Since the seminal work by Tong *et al.* the problem of estimating the channel response of multiple FIR channel driven by an unknown input symbol has interested many researchers in the signal processing areas and communication fields [3].

For the most part, algebraic and second-order statistics (SOS) techniques have been proposed that exploit the structural techniques (Hankel, Toeplitz matrix, *et al.*) of the single-input multiple-output (SIMO) channel or data matrices. The information on channel parameters or transmitted data is typically recovered through subspace decomposition of the received data matrix (deterministic method) or that of the received data correlation matrix (stochastic method). Subspace-based techniques lay in the fact that they rely on the existence of numerically well-defined dimensions of the noise-free signal or noise subspaces. Since these dimensions are obviously closely related to the channel length, subspace-based techniques are extremely sensitive to channel order mismatch[6].

The prediction error methods (PEM) offer an alternative to the class of techniques above. The PEM offers great practical advantages over most other proposed techniques. First, channel estimation using the PEM remains consistent in the presence of the channel length mismatch. This property guarantees the robustness of the technique with respect to the difficult channel length estimation problem. Another significant advantage of the PEM is that it lends itself easily to a low-cost adaptive implementation such as adaptive lattice filters. But the delay cannot be controlled with existing one-step linear prediction[5][6][8]. In this paper, we proposed novel adaptive blind equalizer algorithms based on multistep prediction method. Also, we present simulation results comparing the proposed method and existing algorithm.

II. Problem Formulation

Let $x(t)$ be the signal at the output of a noisy channel

$$x(t)=\sum_{k=-\infty}^{\infty}s(k)h(t-kT)+v(t) \tag{1}$$

where $s(k)$ denotes the transmitted symbol at time kT, $h(t)$ denotes the continuous-time channel impulse response, and $v(t)$ is additive noise. The fractionally spaced discrete-time model can be obtained either by time oversampling or by the sensor array at the receiver[6]. The oversampled single-input single-output (SISO) model results SIMO model as in Fig. 1. The corresponding discrete SIMO model is described as follows

$$x_i(n)=\sum_{k=0}^{L-1}s(k)h_i(n-k)+v_i(n),\ i=0,1,\cdots,P-1 \tag{2}$$

Let

$$\begin{aligned}\mathbf{x}(n)&=[x_0(n)\cdots x_{P-1}(n)]^T\\ \mathbf{h}(n)&=[h_0(n)\cdots h_{P-1}(n)]^T\\ \mathbf{v}(n)&=[v_0(n)\cdots v_{P-1}(n)]^T\end{aligned} \tag{3}$$

We represent $x_i(n)$ in a vector form as

$$\mathbf{x}(n)=\sum_{k=0}^{L-1}s(k)\mathbf{h}(n-k)+\mathbf{v}(n) \tag{4}$$

Stacking N received vector samples into an $(NP\times 1)$-vector, we can write a matrix equation as

$$\mathbf{X}_N(n)=\mathbf{Hs}(n)+\mathbf{V}_N(n) \tag{5}$$

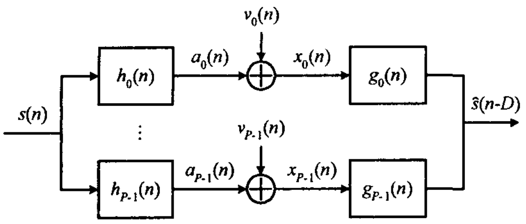

Fig. 1. SIMO channel with P subchannels and equalizer.

where $\mathbf{H}$ is a $NP\times(N+L-1)$ block Toeplitz matrix, $\mathbf{s}(n)$ is $(N+L-1)\times 1$, $\mathbf{X}_N(n)$, and $\mathbf{V}_N(n)$ are $NP\times 1$ vectors.

$$\begin{aligned}\mathbf{s}(n)&=[s(n)\cdots s(n-L-N+2)]^T\\ \mathbf{X}_N(n)&=[\mathbf{x}^T(n)\cdots\mathbf{x}^T(n-N+1)]^T\\ \mathbf{V}_N(N)&=[\mathbf{v}^T(n)\cdots\mathbf{v}^T(n-N+1)]^T\\ \mathbf{H}&=\begin{bmatrix}\mathbf{h}(0)&\cdots&\mathbf{h}(L-1)&\cdots&\mathbf{0}\\ \vdots&\ddots&\vdots&\ddots&\vdots\\ \mathbf{0}&\cdots&\mathbf{h}(0)&\cdots&\mathbf{h}(L-1)\end{bmatrix}\end{aligned}\tag{6}$$

We assume the following throughout in this paper.

A1) The input sequence $s(n)$ is zero mean and white with variance σ_s^2

A2) The noise is stationary with zero mean and white with variance σ_v^2.

A3) The sequences $s(n)$ and $v(n)$ are uncorrelated.

A4) The matrix $\mathbf{H}$ has full rank, i.e., the subchannels $h_i(n)$ have no common zeros to satisfy the Bezout equation.

A5) The dimensions of $\mathbf{H}$ obey $NP>L+N$.

The ith subchannel $h_i(n)$ is equalized by the filter $g_i(n)$, as shown in Figure 1. A ZF equalizer whose subchannels are order L_g is described by

$$\sum_{i=0}^{P-1}\sum_{k=0}^{Lg-1}h_i(n-k)g_i(k)=\delta(n-D)\tag{7}$$

Above equation can be written in matrix form as follows

$$\mathbf{g}^T\mathbf{H}=\mathbf{e}_{D+1}^T\tag{8}$$

where $\mathbf{e}_{D+1}$ is a $(N+L)\times 1$ vector with a 1 as the $(D+1)$st element and zeros elsewhere.

A ZF equalizer is proved in [3]-[5] and we consider noise-free case. The correlation matrix of received signal of (5) is

$$\mathbf{R}=E[\mathbf{X}_N(n)\mathbf{X}_N^H(n)]=\sigma_s^2\mathbf{H}\mathbf{H}^H\tag{9}$$

From (8), we induce the zero-delay equalizer

$$\mathbf{g}_0^T=\sigma_s^2\mathbf{H}(0)\mathbf{R}^+=\sigma_s^2[\mathbf{h}^H(0)\ \mathbf{0}]\mathbf{R}^+\tag{10}$$

where $\mathbf{H}(0)$ is the first column of matrix $\mathbf{H}$ and $\mathbf{R}^+$ denotes the Moore-Penrose inverse of $\mathbf{R}$.

Considering that an arbitrary-delay blind equalizer, it is proved in [4] that an equalizer $\mathbf{g}_D$ with D-delay can be obtained from $\mathbf{g}_0$ as

$$\mathbf{g}_D^T=\mathbf{g}_0^T\mathbf{R}_D\mathbf{R}^+\tag{11}$$

where $\mathbf{R}_D=E[\mathbf{X}_N(n-D)\mathbf{X}_N^H(n)]$

III. Multistep Prediction And LS Approach To Adaptive Blind Equalization

A. Multistep Prediction Based Blind Equalization

A zero-delay ZF equalizer based on linear prediction is proposed in [5] and [6]. We know that

$$\mathbf{g}_0^T=\mathbf{h}^H(0)[\mathbf{I}_P-\mathbf{P}_{N-1}]\tag{12}$$

where $-\mathbf{P}_{N-1}$ is $P\times P(N-1)$ prediction coefficient matrix. It is obtained by minimizing the prediction error variance in [6] or using least squares lattice (LSL) algorithm[5]. And a D-step forward predictor of order N produces an estimation of the received signal $\mathbf{x}(n)$ based on the N previous signal $\mathbf{x}_N(n-D)$[11].

$$\hat{\mathbf{x}}(n)=\mathbf{a}_1\mathbf{x}(n-D)+\cdots+\mathbf{a}_N\mathbf{x}(n-N-D+1)\tag{13}$$

The D-step forward prediction error is given by

$$\begin{aligned}\mathbf{f}_D(n)&=\mathbf{x}(n)-\hat{\mathbf{x}}(n)\\ &=\begin{bmatrix}\mathbf{I}_P & \mathbf{0}_{P\times P(D-1)} & -\mathbf{A}_N\end{bmatrix}\mathbf{X}_{N+D}(n)\\ &=\mathbf{K}_D\mathbf{X}_{N+D}(n)\end{aligned}\tag{14}$$

Using well-known orthogonality principle between prediction error and input sequences, it produces

$$\mathbf{f}_D(n)=\mathbf{K}_D^H\mathbf{X}_{N+D}(n)=\sum_{j=0}^{D-1}\mathbf{h}(j)s(n-j)\tag{15}$$

The D-delay blind equalization method considered here is based on the output of D- and $(D+1)$-step prediction error filters. A D-delay ZF equalizer can be obtained after acknowledging from (15)

$$\begin{aligned}&\mathbf{f}_{D+1}(n)-\mathbf{f}_D(n)=(\mathbf{K}_{D+1}-\mathbf{K}_D)^H\mathbf{X}_{N+D}(n)\\ &=\sum_{j=0}^{D}\mathbf{h}(j)s(n-j)-\sum_{j=0}^{D-1}\mathbf{h}(j)s(n-j)=\mathbf{h}(D)s(n-D)\end{aligned}\tag{16}$$

Therefore, the transmitted symbol can then be extracted as follows

$$s(n-D)=\frac{\mathbf{h}^H(D)}{\|\mathbf{h}(D)\|^2}(\mathbf{f}_{D+1}(n)-\mathbf{f}_D(n))\tag{17}$$

The predictor coefficients are selected such that the mean square value of $\mathbf{f}_D(n)$, i.e., $E[|\mathbf{f}_D(n)|^2]$, is minimized. Therefore, for any set of predictor coefficients $\mathbf{a}_k$ $(1\le k\le N)$,

$$\begin{aligned}&\frac{\partial E[\mathbf{f}_D(n)\mathbf{f}_D^H(n)]}{\partial\mathbf{a}_k^H}=0,\ \text{for } 0\le k\le N\\ &\Rightarrow\mathbf{r}(D+l-1)-\sum_{k=1}^{N}\mathbf{a}_k^N\mathbf{r}(k-l)=0, l=1,\cdots,N\end{aligned}\tag{18}$$

We can rewrite above equation using matrix form as follows

$$\begin{aligned}&\begin{bmatrix}\mathbf{r}(0)&\cdots&\mathbf{r}(N-1)\\ \vdots&\ddots&\vdots\\ \mathbf{r}^H(N-1)&\cdots&\mathbf{r}(0)\end{bmatrix}\begin{bmatrix}\mathbf{a}_1^N\\ \vdots\\ \mathbf{a}_N^N\end{bmatrix}=\begin{bmatrix}\mathbf{r}(D)\\ \vdots\\ \mathbf{r}(D+N-1)\end{bmatrix}\\ &\Rightarrow\mathbf{R}_N\mathbf{A}_N=\mathbf{r}_{N+D}\end{aligned}\tag{19}$$

To solve the prediction error filter coefficients matrix in

D-step predictor, it requires matrix inverse calculation for Yule-Walker equation.

B. LS Approach to Blind Equalization

The D-delay equalizer $\mathbf{g}_D$ in (11) can be estimated by linear prediction. Consider the following LS method

$$\begin{aligned}\mathbf{e}_D(n) &= \mathbf{x}(n-D)-\mathbf{p}_1\mathbf{x}(n)\\ \mathbf{e}_{D+1}(n) &= \mathbf{x}(n-D-1)-\mathbf{p}_2\mathbf{x}(n-1)\\ &\vdots\\ \mathbf{e}_{D+N-1}(n) &= \mathbf{x}(n-D-N+1)-\mathbf{p}_N\mathbf{x}(n-N+1)\end{aligned} \tag{20}$$

Equation (20) is rewritten with matrix form as

$$\mathbf{E}(n)=\mathbf{X}_N(n-D)-\mathbf{P}_N\mathbf{X}_N(n) \tag{21}$$

where $\mathbf{E}(n)$ is an $PN\times 1$ prediction error vector and $\mathbf{P}_N$ is a $PN\times PN$ projection matrix. The optimal $\mathbf{P}_N$ is obtained by minimizing as following cost function.

$$J=\mathrm{tr}\{E[\mathbf{E}(n)\mathbf{E}^H(n)]\} \tag{22}$$

Letting the derivative of (22) with respect to projection matrix equal to zero as following

$$\frac{\partial J}{\partial \mathbf{P}_N^H}=E[-\mathbf{X}_N(n-D)\mathbf{X}_N^H(n)+\mathbf{P}_N\mathbf{X}_N(n)\mathbf{X}_N^H(n)]=\mathbf{0} \tag{23}$$

So we get as

$$\begin{aligned}\mathbf{P}_N\mathbf{R}-\mathbf{R}_D&=\mathbf{0}\\ \mathbf{P}_N&=\mathbf{R}_D\mathbf{R}^+\end{aligned} \tag{24}$$

Comparing (11) with (24) we get as following equation

$$\mathbf{g}_D^T=\mathbf{g}_0^T\mathbf{P}_N \tag{25}$$

It should be noted that the blind equalizer is designed for transmitted signal recovery at a given delay D. Thus, different delay can result in different performance. To get best delay choice, [8] proposes the minimizing MSE given by

$$\mathrm{MSE}(D)=1-\mathbf{H}^H(D)\mathbf{R}^+\mathbf{H}(D) \tag{26}$$

where $\mathbf{H}(D)$ is the $(D+1)$th block column of the channel convolution matrix $\mathbf{H}$. Hence, the optimum delay can be found by

$$\arg\max_D \mathbf{H}^H(D)\mathbf{R}^+\mathbf{H}(D) \tag{27}$$

C. Adaptive Implementation

We propose the adaptive algorithm for updating the linear prediction error filter coefficients. To solve (25), we are required to compute the first linear prediction in (12) and to estimate the $\mathbf{h}(0)$. To estimate $\mathbf{h}(0)$, we use eigen-tracking method in [7]. A zero-delay blind equalizer is obtained in (12). The second linear prediction in (21) is computed and then D-delay blind equalizer can be computed in (25) with predetermined zero-delay blind equalizer. Based on well-known RLS algorithm shown in [1], we have derived an RLS-based adaptive algorithm for computing the zero-delay blind equalizer. Zero-delay and D-delay blind equalizer are obtained as described in Table 1 and Table 2.

TABLE 1. RLS algorithm for zero-delay blind equalizer.

Initialize the algorithm at time n=0, set

$\mathbf{Q}_1(0)=\delta^{-1}\mathbf{I}_{P(N-1)}$, δ = small positive constant

$\mathbf{P}_{N-1}(0)=\mathbf{0}$

For n=1,2,..., do the following

(1) First linear prediction computation.

$$\mathbf{X}_N(n)=[\mathbf{X}_{N,1}^H\ \ \mathbf{X}_{N,2}^H]$$

$$\mathbf{K}_1(n)=\frac{\lambda^{-1}\mathbf{Q}_1(n-1)\mathbf{X}_{N,2}(n)}{1+\lambda^{-1}\mathbf{X}_{N,2}(n)\mathbf{Q}_1(n-1)\mathbf{X}_{N,2}(n)}$$

$$\mathbf{e}(n)=\mathbf{X}_{N,1}(n)-\mathbf{P}_{N-1}(n-1)\mathbf{X}_{N,2}(n)$$

$$\mathbf{P}_{N-1}(n)=\mathbf{P}_{N-1}(n-1)+\mathbf{e}(n)\mathbf{K}_1^H(n)$$

$$\mathbf{Q}_1(n)=\lambda^{-1}\mathbf{Q}_1(n-1)-\lambda^{-1}\mathbf{K}_1(n)\mathbf{X}_{N,2}^H(n)\mathbf{Q}_1(n-1)$$

(2) Estimation of channel coefficient vector $\mathbf{h}(0)$ in [7].

(3) Computation $\mathbf{g}_0$ in (12).

TABLE 2. RLS algorithm for D-delay blind equalizer.

Initialize the algorithm at time n=0, set

$\mathbf{Q}_2(0)=\delta^{-1}\mathbf{I}_{PN}$, δ = small positive constant

$\mathbf{P}_N(0)=\mathbf{0}$

For n=1,2,..., do following

(1) Projection matrix computation.

$$\mathbf{K}_2(n)=\frac{\lambda^{-1}\mathbf{Q}_2(n-1)\mathbf{X}_N(n)}{1+\lambda^{-1}\mathbf{X}_N(n)\mathbf{Q}_2(n-1)\mathbf{X}_N(n)}$$

$$\mathbf{E}(n)=\mathbf{X}_N(n-d)-\mathbf{P}_N(n-1)\mathbf{X}_N(n)$$

$$\mathbf{P}_N(n)=\mathbf{P}_{N-1}(n-1)+\mathbf{E}(n)\mathbf{K}_2^H(n)$$

$$\mathbf{Q}_2(n)=\lambda^{-1}\mathbf{Q}_2(n-1)-\lambda^{-1}\mathbf{K}_2(n)\mathbf{X}_N^H(n)\mathbf{Q}_2(n-1)$$

(2) Compute the zero-delay blind equalizer $\mathbf{g}_0$

(3) Computation of D-delay blind equalizer in (18).

IV. SIMULATION RESULTS

In this section, we use computer simulations to examine the performance of the proposed method described in previous section. In this simulation, as an approximation of a three-ray multipath environment, the channel impulse response is given by

$$\begin{aligned}h_c(t)&=c(t,0.45)W(t)+0.8c(t-0.25T,0.45)\cdot\\ &\quad W(t-0.25T)-0.4c(t-2T,0.45)W(t-2T)\end{aligned} \tag{28}$$

where $c(t,0.45)$ is raised-cosine filter with roll-off factor 0.45, and $W(t)$ is a rectangular window of duration 6 symbol intervals spanning [-0.85T 5.14T]. There are four sub-channels. The MSE of symbol estimation is defined as in [3].

$$\mathrm{MSE}=E[|\hat{s}(n-D)-s(n)|^2] \tag{29}$$

For the simulations, the SNR is defined to be at the input to the equalizer in Fig. 1.

$$\mathrm{SNR}=\frac{E[|a(n)|^2]}{E[|v(n)|^2]} \tag{30}$$

For each simulation, we have used an i.i.d. input sequence drawn from a 16-QAM constellation. The noise is generated from a white Gaussian distribution at varying SNR's. The number of subchannels is four. Let the equalizer order be L_g=8. In Fig. 2, we show the MSE of the output for different delay D and the MSE obtained from (27) under SNR=20dB and 30dB. Fig. 3 and Fig. 4 show the MSE curves for MSE of the proposed RLS and existing algorithms under SNR=20dB and 30dB, respectively. In this simulation, we compare the proposed algorithm with FS-CMA, RLS algorithm proposed by Halford *et al.* (denotes Halford algorithm)[4], and LSL algorithm with forward prediction error filter (FPEF) proposed by Taylor *et al.* (denotes Taylor algorithm)[5]. It is shown that the proposed algorithm performs better than the others, whereas the Taylor algorithm has poor performance because equalizers of arbitrary delay cannot be controlled.

V. CONCLUSION

This paper presents adaptive blind equalizer based on multichannel linear prediction with optimum delay. We have developed RLS-type algorithm for updating prediction coefficient matrix as a projection matrix. Our proposed method ensures flexible delay control and provides flexibility for a practical implementation since various well-known adaptive algorithms. Furthermore, our algorithms are robust to channel order over-determination in nature of linear prediction characteristics, and do not need channel length estimation. Simulation results show that out algorithm have good performance in channel equalization. Compared with HOS-based algorithm such as CMA, our algorithms are based on SOS; thus faster convergence can be achieved.

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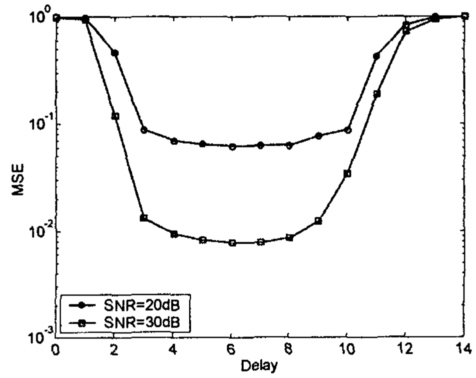

Fig. 2. MSE curves for different delay.

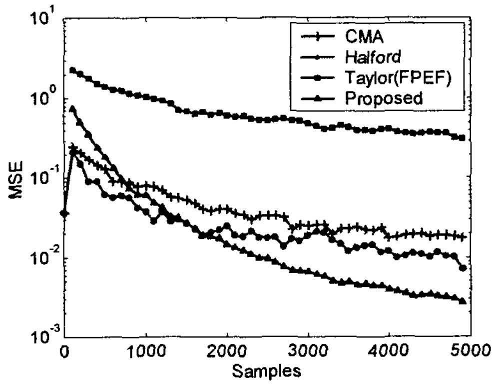

Fig. 3. MSE comparison of existing algorithms and the proposed algorithm, SNR=30dB.

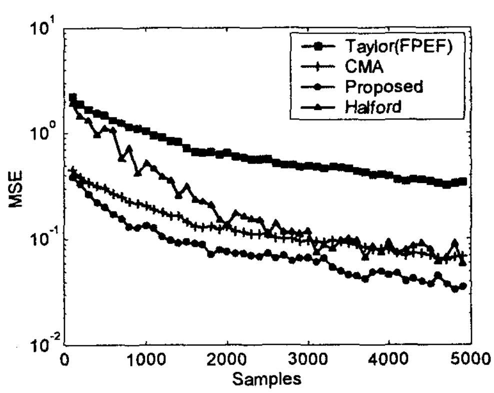

Fig. 4. MSE comparison of existing algorithms and the proposed algorithm, SNR=20dB.