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블라인드 동채널 신호 분리를 위한 순차적인 Joint Maximum Likelihood 알고리듬

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A Sequential Joint Maximum Likelihood Algorithm for Blind Co-Channel Signal Separation

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abstract

In this paper we consider a problem of blind co-channel signal separation, the goal of which is to estimate multiple co-channel digitally modulated signals using an antenna array. We employ the joint maximum likelihood estimation and present a sequential algorithm, which is referred to as *sequential joint maximum likelihood* (SJML) algorithm. It separates multiple co-channel signal on-line and converges fast in overdetermined noisy communication environment. And the computational complexity of SJML for M-QAM (M=8, 16, 64,…) signals is less expensive compared to the SLSP. Useful behavior of this algorithm are confirmed by simulations.

1. Introduction

Mobile communications are growing rapidly in the number of subscribers and in the range of services, but available radio frequency spectrum is limited. A promising solution to increase spectrum efficiency lies in exploiting spatial diversity (via antenna arrays). Array processing techniques allows multiple co-channel users per cell in order to increase the capacity.

Blind co-channel signal separation aims at estimating multiple co-channel digitally modulated signals, given only observation vector (measured at an antenna array) which consists of a superposition of signal waveforms plus additive noise. Several methods have been developed so far, among which, we pay attention to joint maximum likelihood estimation [1]. It seems that this algorithm is more of blind co-channel signal separation than conventional gradient based ICA algorithms [4, 5, 6] which did not take the effect of additive noise into account. We apply the sequential least squares method to this algorithm. Then resulting algorithm is referred to as SJML (Sequential Joint Maximum Likelihood). This algorithm converge to a solution much faster to the gradient-based ICA algorithms and shows better performance in the presence of additive white Gaussian noise. Moreover the computational complexity of SJML for M-QAM (M=8, 16, 64,…) signals is less expensive compared to the SLSP [2, 3].

2. Problem Formulation

Consider d narrowband signals entered at an array of m sensors with arbitrary characteristics. There are multiple reflected and diffracted paths from the source to the array in a wireless environment or channel. So they arrive at array sensors for different angles, and with different attenuations and time delays. Output of antenna array becomes

$$x(t) = \sum_{k=1}^{d} \sum_{l=1}^{q_k} a(\theta_{kl})\ p_k\ \alpha_{kl}\ s_k(t-\tau_{kl}) + v(t), \quad (1)$$

where $a(\theta_k)$ is the array response vector to a signal from direction θ_k, p_k is the amplitude of the k-th signal, $s_k(\cdot)$ is the k-th signal waveform which can be written as

$$s_k(t) = \sum_{n=1}^{N} b_k(n) g(t-nT), \quad (2)$$

where N is the number of symbols in a data batch, $\{b_k(\cdot)\}$ is the symbol sequence of the k-th user, T is the symbol period. And $g(\cdot)$ is the unit-energy signal waveform of duration T.
And q_k is number of subpaths for the k-th signal, a_{kl} and τ_{kl} are the attenuation and time delay corresponding to l-th subpath, $v(\cdot)$ is white complex symmetric Gaussian noise.

The antenna array output modeled as phase-shifts under the narrowband assumption. And assume that the signals are symbol-synchronous, we perform matched filtering over each symbol period T. We obtain the following equivalent discrete representation of the data

$$x(n) = \sum_{k=1}^{d} p_k \, a_k \, b_k(n) + v(n). \tag{3}$$

where a_k is the total array response vector

$$a_k = \sum_{l=1}^{D_k} a_{kl} \, e^{-j\omega_c \tau_{kl}} a(\theta_{kl}) \tag{4}$$

and ω_c is the carrier frequency.
In matrix form,

$$x(n) = As(n) + v(n). \tag{5}$$

The problem addressed in this paper is the estimation of $s(n)$, given $x(n)$, and a good estimate of A, where source signal $s(n)$ is $s(n) = [b_1(n) \cdots b_d(n)]^T$, $x(n)$ is the matched filter output for array output, array response A is a matrix which dimension is $m \times d$ and $v(n)$ is white Gaussian noise.

3. Sequential LS with Projection

We assume that the noise $v(t)$ is isotropic Gaussian with zero mean and variance σ^2. Then the log-likelihood function of matched filter output is given as

$$\log L(A, s(1), \cdots, s(N)) \propto -const - mN\log\sigma^2 - \frac{1}{\sigma^2}\sum_{n=1}^{N} \|x(n) - As(n)\|^2, \tag{6}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. We maximize log-likelihood function with respect to the unknown A and $s(n)$, $n = 1, \cdots, N$. We consider the exponentially weighted LS cost function

$$J(A(n)) = \sum_{i=1}^{n} \beta^{n-i} \, \|x(i) - A(n)\, s(i)\|^2 \tag{7}$$

where $s(n)$ belong to a certain alphabet depending on its constellation. The minimization of (6) is minimize for (8) leads to

$$\hat{s}(n) = \hat{A}^{\dagger} x(n), \tag{8}$$
$$z(n) = \mathrm{proj}(\hat{s}(n)),$$

$$\hat{A}^{(n)} = \hat{C}_{xz}(n)\, \hat{C}_{zz}^{-1}(n), \tag{9}$$

where

$$\hat{C}_{xz}(n) = \sum_{i=1}^{n} \beta^{(n-i)} x(i)\, z^T(i) = \beta\, \hat{C}_{xz}(n-1) + x(n) z^T(n), \tag{10}$$

$$\hat{C}_{zz}(n) = \sum_{i=1}^{n} \beta^{(n-i)} z(i)\, z^T(i) = \beta\, \hat{C}_{zz}(n-1) + z(n)\, z^T(n), \tag{11}$$

and the superscript $\dagger$ denotes the pseudo-inverse and $\mathrm{proj}(\cdot)$ means the projection onto its nearest alphabet. The sequential LS is applied to derive the SLSP [2, 3] that is summarized in Table 1.

$$\begin{aligned}
\hat{s}(n) &= A^{\dagger} x(n) \\
z(n) &= \mathrm{proj}(\hat{s}(n)) \\
h(n) &= P(n-1)\, z(n) \\
g(n) &= h(n)/[\beta + z(n)^T h(n)] \\
P(n) &= \frac{1}{\beta}\,\mathrm{Tri}[\,P(n-1) - g(n)\, h^T(n)] \\
e(n) &= x(n) - \hat{A}(n-1)\, \hat{z}(n) \\
\hat{A}(n) &= \hat{A}(n-1) + e(n)\, g^T(n)
\end{aligned}$$

Table 1. Algorithm outline for SLSP. The operator $\mathrm{Tri}(\cdot)$ indicates that only the upper (or lower) triangular part is calculated and its transposed version is copied to the another lower (or upper) triangular part.

Note that Pajunen and Karhunen [8] proposed a similar LS algorithm to our SLSP. Their algorithm is a nonlinear version of PAST [7], so it is referred to as the *nonlinear PAST*. The difference between the SLSP and the nonlinear PAST is that the former exploits the generative model, whereas the latter does the recognition model. As will be demonstrated in simulations, the SLSP is better in the presence of white Gaussian noise. The benefit of learning the generative model was also emphasized in [9].

4. Sequential Joint Maximum Likelihood

For the case of noise-free data, the estimate of s is obtained by a linear transform, $\hat{s}(n) = \hat{A}^{\dagger} x(n)$, given the estimate of A. It was pointed out in [1] that the reconstruction of original sources requires a nonlinear transform in the presence of noise.

As in [1], we assume that all sources have identical distributions and the noise is isotropic white Gaussian with zero mean and variance σ^2. Then the MAP cost function is given by

$$\log L(\mathbf{A}, \mathbf{s}(1), \cdots, \mathbf{s}(n))$$
$$\propto -\sum_{n=1}^{N}\left[\frac{1}{2}(\| \mathbf{A}\mathbf{s}(n)-\mathbf{x}(n)\|^2_{\Sigma^{-1}}+\sum_{i=1}^{n} f_i(s_i(n))\right]+C, \quad (12)$$

where $\|\mathbf{e}\|^2_{\Sigma^{-1}}$ is defined as $\mathbf{e}^T\Sigma^{-1}\mathbf{e}$ and $f_i(\cdot)=-\log p_i(\cdot)$ ($p_i(\cdot)$ represent the probability density function of source s_i). And C is an irrelevant constant. The independent component s_i are here constrained to have unit variance.

The optimal nonlinear function h for reconstructing independent components s_i is given by

$$\hat{\mathbf{s}}=h(\hat{\mathbf{A}}^{-1}\mathbf{x}), \quad (13)$$

where

$$h^{-1}(u)=(1-\sigma^2)u+\sigma^2 f'(u), \quad (14)$$

where $f'(u)=\frac{df(u)}{du}$.

Since we are dealing with digitally modulated communication signals, it is reasonable to assume that all sources have uniform distribution with zero mean and unit variance. Then the probability density function is given by

$$p_s(s_i)=\frac{1}{2\sqrt{3}}\{u(s+\sqrt{3})-u(s-\sqrt{3})\}, \quad (15)$$

where $u(\cdot)$ denotes the unit step function. From this, assumes the noise variance σ^2 is very small, we obtain the truncation operator from (14), (15)

$$h(u)=sign(u)\min(|u|,\sqrt{3}). \quad (17)$$

The truncation operator in (17) clips the values outside the interval $[-\sqrt{3}, \sqrt{3}]$, since the uniformly distributed random variable with unit variance cannot exceed $\pm\sqrt{3}$.

In [1], the alternating variable method was applied to find a local maximum of (12). Here we apply the sequential LS to derive our SJML algorithm that is summarized in Table 2.

The only difference between SLSP and SJML lies in the reconstruction of $\hat{s}$, given $\hat{A}$. In SLSP, we used a finite alphabet property so that the projection onto its nearest alphabet followed LS estimation. In SJML, the optimal nonlinear reconstruction was calculated under the uniform density model. With the different choice of the nonlinear reconstruction function h, the SJML is applicable to the case where sources have super-Gaussian distribution. For the case of super-Gaussian distribution, the sparse code shrinkage operator was shown to be efficient in the task of denoising [10].

5. Simulation

We demonstrate the useful behavior of SJML and compare its performance to SLSP, the nonlinear PAST and the conventional ICA algorithm.

We assume a uniform linear 3-element antenna array with each element being half wavelength spaced.

We consider two digitally modulated QPSK (Quadriphase Shift Keying) sources with angles of arrival, [10°, 30°]. Randomly-chosen initial value is assigned to $\mathbf{A}(0)$ or $\mathbf{W}(0)$. The density matrix is assigned to $\mathbf{P}(0)$. For SJML and SLSP, we used the forgetting factor β is 0.99 and for the ICA algorithm (with $\phi_i(\hat{s}_i)=|\hat{s}_i|^2\hat{s}_i$), we used the learning rate $\eta=0.001$. At the SNR, we carried out 5 independent runs and calculated averaged BER (see Fig. 1). As shown in Fig. 1, our algorithm, SJML outperforms the SLSP, the nonlinear PAST and the ICA algorithm in the presence of white Gaussian noise.

$\mathbf{u}=\mathbf{A}^{\dagger}(n-1)\mathbf{x}(n)$
$\hat{\mathbf{s}}(n)=\text{sign}(\mathbf{u})\min(\lvert\mathbf{u}\rvert,\sqrt{3})$
if $n=1$, $\hat{\mathbf{z}}(n)=\frac{\hat{\mathbf{s}}(n)}{\lVert\hat{\mathbf{s}}(n)\rVert}$
else $\mathbf{c}(n)=[1\ \hat{\mathbf{s}}(n)]$
$\hat{\mathbf{z}}(n)=\frac{\hat{\mathbf{s}}(n)}{\lVert\mathbf{c}(n)\rVert}$
$\mathbf{h}(n)=\mathbf{P}(n-1)\mathbf{z}(n)$
$\mathbf{g}(n)=\mathbf{h}(n)/[\beta+\mathbf{z}(n)^T\mathbf{h}(n)]$
$\mathbf{P}(n)=\frac{1}{\beta}\text{Tri}[\mathbf{P}(n-1)-\mathbf{g}(n)\mathbf{h}^T(n)]$
$\mathbf{e}(n)=\mathbf{x}(n)-\hat{\mathbf{A}}(n-1)\hat{\mathbf{z}}(n)$
$\hat{\mathbf{A}}(n)=\hat{\mathbf{A}}(n-1)+\mathbf{e}(n)\mathbf{g}^T(n)$

Table 2. Algorithm outline for SJML

Besides the BER performance, we also evaluated the performance of algorithm in terms of the performance index (PI) that is defined by

$$\text{PI}=\frac{1}{n(n-1)}\sum_{i=1}^{n}\left\{\left(\sum_{k=1}^{n}\frac{|g_{ik}|}{\max_j|g_{ij}|}\right)+\left(\sum_{k=1}^{n}\frac{|g_{ki}|}{\max_j|g_{ji}|}-1\right)\right\}, \quad (18)$$

where g_{ij} is the (i, j)-element of the global system matrix $\mathbf{G}=\hat{\mathbf{A}}^{\dagger}\mathbf{A}=\mathbf{W}\mathbf{A}$. The smaller value of PI, the better performance. The convergence of all these algorithms are shown in Fig. 2. Since SJML and SLSP employ the sequential least squares, they converge to a solution much faster than the gradient based ICA algorithm. The SLSP shown the fastest convergence because it exploits the finite alphabet property. However the computational complexity of SLSP for M-QAM (M=8, 16, 64, …) will be increased compared to the SJML, because the SLSP needs more search to project the data into its nearest constellation.

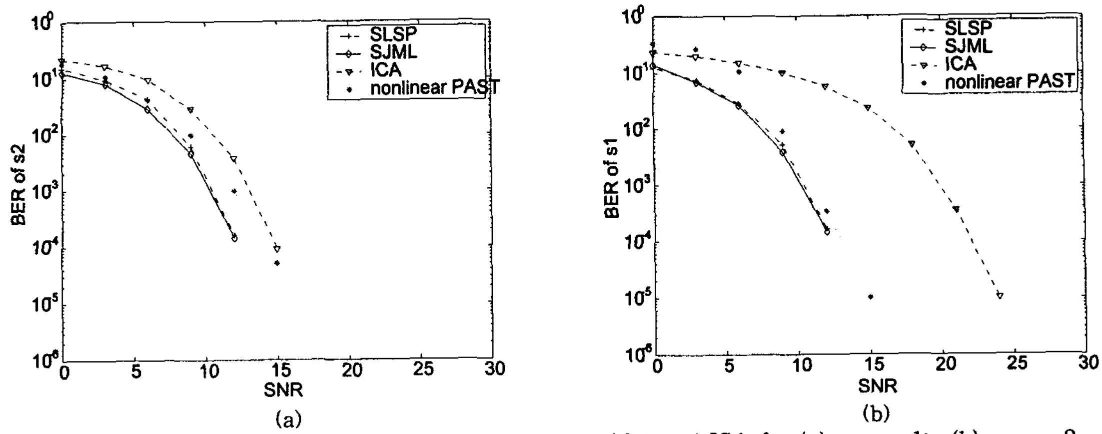

Fig. 1. BER performance of SJML, SLSP, the nonlinear PAST and ICA for (a) source 1; (b) source 2.

6. Conclusion

In this paper we proposed the sequential algorithm, SJML for blind co-channel signal separation. The key ingredient in the derivation of this algorithm was the sequential LS method. The algorithm is much faster than the gradient based source separation algorithms and are free of learning rate. The SJML differs from the SLSP only in the part of reconstruction of sources, given the estimate of $\boldsymbol{A}$. But the computational complexity of SJML for M-QAM (M=8, 16, 64, ⋯) signals is less expensive compared to the SLSP, because it employed the optimal nonlinear reconstruction function whereas the SLSP exploited the finite property. Simulations verified the high performance of our algorithm.

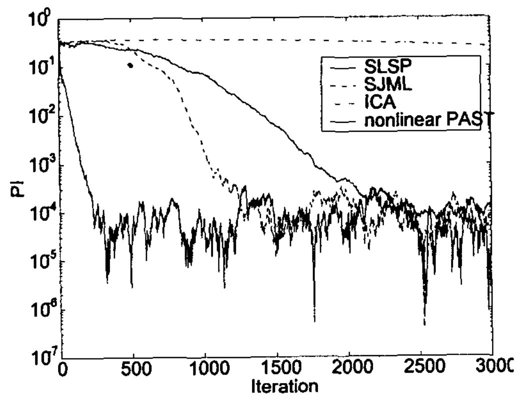

Fig. 2. The convergence comparison for SJML, SLSP, the nonlinear PAST and ICA.

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7. Reference

[1] A. Hyvärinen, "Independent component analysis in the presence of Gaussian noise by maximizing joint likelihood," *Neurocomputing*, vol. 22, pp. 49-67, 1998.

[2] S. Tawlar, M. Viberg and A. Paularj, "Blind estimation of multiple co-channel digital signals using an antenna array," *IEEE Signal Processing Letters*, vol. 1, Feb. 1994.

[3] I. Jang, J. Kim, S. Choi, "A sequential LS approach to blind co-channel signal separation," in *Proc. KICS 2000* Fall, 2000.

[4] S. Amari and A. Cichocki, "Adaptive blind signal processing neural network approaches," in *Proc. of the IEEE, Special Issue on Blind Identification and Estimation*, vol. 86, no. 10, pp. 2026-2048, Oct. 1998.

[5] J. F. Cardoso and B. H. Laheld, "Equivariant adaptive source separation," *IEEE Trans, Signal Processing*, vol 44, no. 12, pp. 3017-3030, Dec. 1996.

[6] S. Choi, A. Cichocki and S. Amari, "Flexible independent component analysis," *Journal of VLSI Signal Processing*, vol. 26, pp. 25-38, Aug. 2000.

[7] B. Yang, "Projection approximation subspace tracking," *IEEE Trans. on Signal Processing*, vol. 43, no. 1, pp. 95~107, January 1995.

[8] P. Pajunen and J. Karhunen, "Least-squares methods for blind source separation based on nonlinear PCA," *International Journal of Neural Systems*, vol. 8, pp. 601-612, 1998.

[9] S. Choi, A. Cichocki, L. Zhang and S. Amari, "Approximate maximum likelihood source separat -ion using the natural gradient," in *Proc.IEEE Workshop in Signal Processing Advances in Wire -less Communications*, (Taoyuan, Taiwan), pp. 235-238, 2001.

[10] A. Hyvärinen, "Separse code shrinkage: Denoising of nongaussian data by maximum likeli -hood estimation," *Neural Computation*, vol. 11, no. 7, pp. 1739-1768, 1999.