

Unscented Kalman Filter를 이용한 비선형 동적 구조계의 시간영역 규명기법
Time Domain Identification of Nonlinear Structural Dynamic Systems
Using Unscented Kalman Filter

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ABSTRACT

In this study, the recently developed unscented Kalman filter (UKF) technique is studied for identification of nonlinear structural dynamic systems as an alternative to the extended Kalman filter (EKF). The EKF, which was originally developed as a state estimator for nonlinear systems, has been frequently employed for parameter identification by introducing the state vector augmented with the unknown parameters to be identified. However, the EKF has several drawbacks such as biased estimations and erroneous estimations especially for highly nonlinear dynamic systems due to its crude linearization scheme. To overcome the weak points of the EKF, the UKF was recently developed as a state estimator. Numerical simulation studies have been carried out on nonlinear SDOF system and nonlinear MDOF system. The results from a series of numerical simulations indicate that the UKF is superior to the EKF in the system identification of nonlinear dynamic systems especially highly nonlinear systems.

1. INTRODUCTION

For the purpose of damage assessment it is desirable to identify severity as well as location of the damage based on the input-output measurements. Numerous techniques are available for the recursive estimation to time-varying parameters. Many of these methods fall in the category of Least-Square Method. Unlike those methods, the equation of motion can also be represented in the states-space formulation, where the stiffness and damping matrices of the dynamic system become the unknown system parameters. For this identification problem, the extended Kalman filter technique (Yun and Shinozuka, 1980 and Hoshiya and Saito 1984) has been frequently employed to estimate the system parameters. Although the EKF inherits the fancy feature of optimal estimation of linear Kalman filter under the existence of process and measurement noises, owing to its crude linearization scheme the EKF has several drawbacks as following

1. Linearization can produce highly unstable filter performance if the time step intervals are not sufficiently small.
2. The derivations of the Jacobian matrices are nontrivial in most applications and often lead to significant implementation difficulties.

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3. Sufficiently small time step intervals usually imply high computational overhead as the number of calculations demanded for the generation of the Jacobian and the predictions of state estimate and covariance are large

Furthermore, due to the complex nature of civil infrastructures and noise-polluted measurement, there are many difficulties to apply the Kalman filtering technique and researches on improving the performance of Kalman filter have been carried out for the last two decades.

To overcome the weak points of the EKF, the UKF was recently developed as state estimator. The UKF generates a set of points, which captures the mean and covariance information, and accomplishes prediction process by using mapping those points through the nonlinear dynamic and observation equations under consideration, hence it does not require linearization process.

The purpose of this study is to investigate the performance of the UKF, which is originally developed as state estimator, for identification of nonlinear structural dynamics in comparison with the EKF based on a series of numerical simulation studies.

2. NONLINEAR SYSTEM IDENTIFICATION USING KALMAN FILTER

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete-data linear filtering problem. Since that time, due in large part to advances in digital computing, the Kalman filter has been the subject of extensive research and application. The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) *state estimation* solution.

Kalman filter possesses useful feature for *parameter estimation*. In the absence of system parameters information, Kalman filter can estimate system states and system unknown parameters such as damping coefficient, natural frequency by introducing the state vector augmented with the unknown parameters to be identified based on the input-output measurements. System identification using Kalman filter has become a branch of application of Kalman filter.

For system identification problem the dynamic and/or measurement equations are always nonlinear, the extended Kalman filter (EKF), which is nonlinear version of linear Kalman, has been frequently used for the purpose of system identification. But The EKF employs crude linearization scheme, which make use of 1st order term of Taylor series expansion of dynamic and measurement equations, several drawbacks occur as described already.

To avoid drawbacks of the EKF, the Unscented Kalman filter was recently developed by Juiler at al for *states estimation* problems. In this section, theoretical backgrounds will be described in more details.

2.1 LINEAR KALMAN FILTER

The Kalman filter addresses the general problem of trying to estimate the states $x \in \mathbb{R}^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x(k+1 | k) = Ax(k | k) + Bu(k) + w(k) \quad (1)$$

with a measurement $z \in \mathbb{R}^m$ that is

$$z(k) = Hx(k | k) + v(k) \quad (2)$$

The random variables $w(k)$ and $v(k)$ represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions

$$p(w) \sim N(0, Q) \quad (3)$$

$$p(v) \sim N(0, R) \quad (4)$$

The $n \times n$ matrix A in the difference equation (1) relates the states at time step k to $k + 1$, in the absence of either a driving function or process noise. The $n \times l$ matrix B relates the control input $u \in \mathbb{R}^l$ to the states x . The $m \times n$ matrix H in the measurement equation (2) relates the states to the measurement $z(k)$.

The Kalman filter gives optimal recursive solution in the sense of minimum variance of estimation and consists of two step structure, i.e. time updates (“prediction”) which predicts the states based on system dynamic equation and measurement update (“filtering”) which updates predicted values based on measurement information. The schematic diagram of recursive structure of Kalman filter is shown in Fig.1 and the Kalman filtering equation is shown Fig.2.

The Kalman filter, which is originally a state estimator, can be employed for parameter identification by introducing the state vector augmented with unknown parameters to be identified. These unknown parameters converges to true value of system parameters while Kalman filtering proceeds based on input-output measurements with initial guesses of $\hat{x}(0|0)$ and $P(0|0)$.

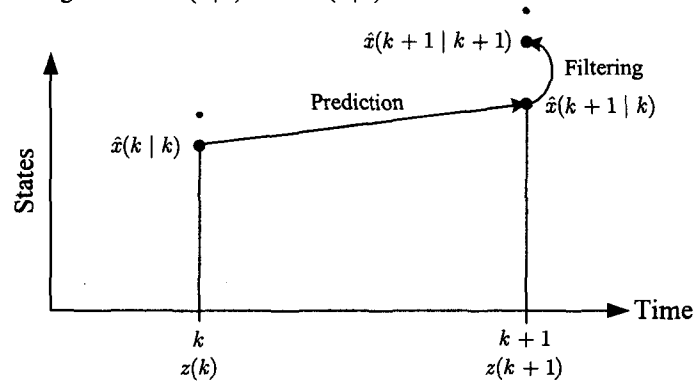


Fig.1 Recursive structure of Kalman filter

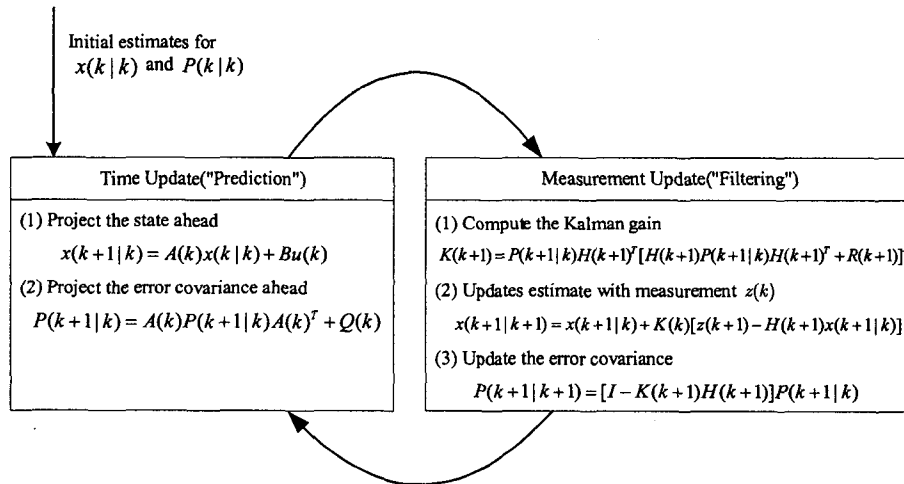


Fig.2 A complete picture of the operation of the Kalman filter

2.2 THE EXTENDED KALMAN FILTER

Let us assume that our process has a state vector $x \in \mathbb{R}^n$, but that the process is now governed by the nonlinear stochastic difference equation.

$$x(k+1) = f(x(k), u(k), w(k)) \quad (5)$$

with a measurement $z \in \mathbb{R}^m$ that is

$$z(k) = h(x(k), v(k)) \quad (6)$$

where the random variables $w(k)$ and $v(k)$ again represent the process and measurement noise as in (3) and (4). The nonlinear function $f(\bullet)$ in the difference equation (5) relates the state at time step k to the state at step $k+1$. It includes as parameters any driving function $u(k)$ and the zero-mean process noise $w(k)$. The nonlinear function $h(\bullet)$ in the measurement equation (6) relates the state $x(k)$ to the measurement $z(k)$.

In something akin to a Taylor series, one can linearize the estimation around the current estimate using the partial derivatives of the process and measurement functions to compute estimate even in the face of nonlinear relationships as following

$$x(k+1) \cong \hat{x}(k+1|k) + \mathbf{A}(x(k) - \hat{x}(k|k)) + \mathbf{W}w(k) \quad (7)$$

$$z(k) \cong \bar{z}(k) + \mathbf{H}(x(k) - \hat{x}(k|k-1)) + \mathbf{V}v(k) \quad (8)$$

where

$$\hat{x}(k+1|k) = f(\hat{x}(k|k), u(k), 0), \quad \bar{z}(k) = h(\hat{x}(k|k-1), 0)$$

$$\mathbf{A} = \left[\frac{df_{[i]}}{dx_{[j]}}(\hat{x}(k|k), u(k), 0) \right], \quad \mathbf{W} = \left[\frac{df_{[i]}}{dw_{[j]}}(\hat{x}(k|k), u(k), 0) \right]$$

$$\mathbf{H} = \left[\frac{dh_{[i]}}{dx_{[j]}}(\hat{x}(k|k-1), 0) \right], \quad \mathbf{V} = \left[\frac{dh_{[i]}}{dv_{[j]}}(\hat{x}(k|k-1), 0) \right]$$

When linearized system matrix \mathbf{A} , \mathbf{W} , \mathbf{H} and \mathbf{V} is calculated based on above equations, one can proceed the Kalman filtering process with same formulas as shown in Fig.2 for linear Kalman filter. It's worthwhile to note that the EKF employs only 1st order term in the Taylor series expansion. Hence it may cause erroneous estimations especially for highly nonlinear dynamic systems due to its crude linearization scheme.

2.3 THE UNSCENTED KALMAN FILTER

The UKF is originated from the following intuition: "With a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function". Following this intuition the UKF uses a parameterization, which captures the mean and covariance information while at the same time permitting the direct propagation of the information through an arbitrary set of nonlinear equations. It will be shown that this can be accomplished by generating a discrete distribution composed of the minimum number of points which have the same first and second (and possibly higher) moments, where each point in the discrete approximation can be directly transformed. The mean and covariance of the transformed ensemble can then be computed as the estimate of the nonlinear transformation of the original distribution while it is computed based on linearized system dynamic equation in the EKF. The UKF process doesn't employ the crude linearized scheme in prediction procedure but employs the equivalent formulation for filtering procedure with the EKF. The general procedure is illustrated in Fig.3.

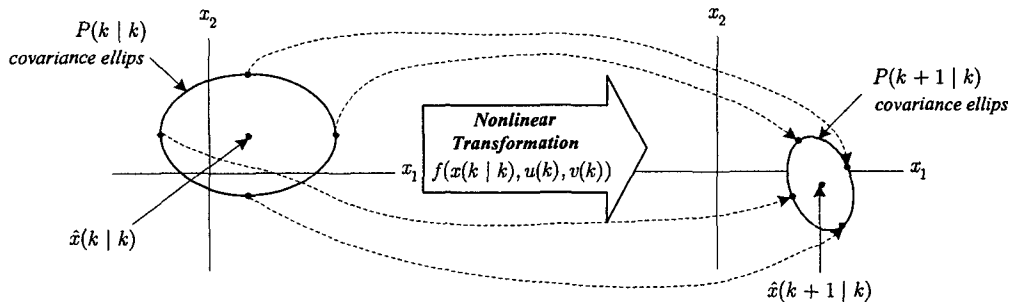


Fig.3 Basic idea of the unscented Kalman filter

Given an n -dimensional Gaussian distribution having covariance P , one generate a set of $O(n)$ points having the same covariance from the columns (or rows) of the matrices $\pm\sqrt{nP}$ (the positive and negative roots). This set of points is zero mean, but if the original distribution has mean \bar{x} , then simply adding \bar{x} to each of the points yields a symmetric set of $2n$ points having the desired mean and covariance. Because the set is symmetric its odd central moments are zero, so its first three moments are the same as the original Gaussian distribution. This is the *minimal* number of points capable of encoding this information. On the other hand, a random sampling of points from the distribution will generally introduce spurious modes in the transformed distribution even if the set of sample points has the correct mean and covariance. In a filtering application these modes will take the form of high frequency noise that may completely obscure the signal.

To restate the general problem, we have the mean $\hat{x}(k|k)$ and covariance $P(k|k)$ of the state at time k and would like to predict $\hat{x}(k+1|k)$ and $P(k+1|k)$ through the nonlinear function $f(\bullet)$. The basic method are summarize as follows:

1. Compute the set $\sigma_i(k|k)$ of $2n$ points from the columns of the matrices $\pm\sqrt{nP(k|k)}$. This set is zero mean with covariance $P(k|k)$. Compute a set of points with the same covariance, but with mean $\hat{x}(k|k)$, by translating each of the points as $\chi_i(k|k) = \sigma_i(k|k) + \hat{x}(k|k)$.
2. Transform each point through the state dynamics equations as $\chi_i(k+1|k) = f(\chi_i(k|k), u(k))$.
3. Compute $\hat{x}(k+1|k)$ and $P(k+1|k)$ by computing the mean and covariance of the $2n$ points in the set $\chi_i(k+1|k)$.

The process noise is injected into the state transition model by adding a dynamic noise covariance matrix $Q(k)$ to $P(k|k)$ before the sigma points are calculated. To predict $\hat{z}(k+1|k)$ and $P_{zz}(k+1|k)$ we apply the same intuition to $h(\bullet)$ using the set of projected sigma points $\chi_i(k+1|k)$

The basic method is generalized in two ways. First, any of the infinite number of (not necessarily square) matrix square roots can be chosen. If the orthogonal matrix square root is chosen, then the sigma points lie along the eigenvectors of the covariance matrix. Second, if copies of the prior mean $\hat{x}(k|k)$ can be included in the set of sigma points. Although the mean of the sigma points is unaffected, the distribution of the points is scaled (since they are now found from $\pm\sqrt{(n+\kappa)P(k|k)}$). In certain circumstances this scaling leads to improvements in performance.

The general formulation of the new filter is summarized in Table 1.

Table 1 General formulation of the unscented Kalman filter

1. The set of translated sigma points is computed from the $n \times n$ matrix $P(k | k)$ as

$$\begin{aligned}\sigma(k | k) &\leftarrow 2n \text{ columns from } \pm \sqrt{(n + \kappa)P(k | k)} \\ \chi_0(k | k) &= \hat{x}(k | k), \\ \chi_i(k | k) &= \sigma_i(k | k) + \hat{x}(k | k)\end{aligned}$$

2. The predicted means is computed as

$$\hat{x}(k + 1 | k) = \frac{1}{n + \kappa} \left\{ \kappa \chi_0(k + 1 | k) + \frac{1}{2} \sum_{i=1}^{2n} \chi_i(k + 1 | k) \right\}$$

3. And the predicted covariance is computed as

$$\begin{aligned}P(k + 1 | k) &= \frac{1}{n + \kappa} \left\{ \kappa [\chi_0(k + 1 | k) - \hat{x}(k + 1 | k)] [\chi_0(k + 1 | k) - \hat{x}(k + 1 | k)]^T \right. \\ &\quad \left. + \frac{1}{2} \sum_{i=1}^{2n} [\chi_i(k + 1 | k) - \hat{x}(k + 1 | k)] [\chi_i(k + 1 | k) - \hat{x}(k + 1 | k)]^T \right\}\end{aligned}$$

4. The predicted observation is calculated by

$$\hat{z}(k + 1 | k) = \frac{1}{n + \kappa} \left\{ \kappa \mathcal{Z}_0(k + 1 | k) + \frac{1}{2} \sum_{i=1}^{2n} \mathcal{Z}_i(k + 1 | k) \right\}$$

5. And the covariance is determined by

$$\begin{aligned}P_{zz}(k + 1 | k) &= \frac{1}{n + \kappa} \left\{ \kappa [\mathcal{Z}_0(k + 1 | k) - \hat{z}(k + 1 | k)] [\mathcal{Z}_0(k + 1 | k) - \hat{z}(k + 1 | k)]^T \right. \\ &\quad \left. + \frac{1}{2} \sum_{i=1}^{2n} [\mathcal{Z}_i(k + 1 | k) - \hat{z}(k + 1 | k)] [\mathcal{Z}_i(k + 1 | k) - \hat{z}(k + 1 | k)]^T \right\}\end{aligned}$$

where $P_{vv}(k + 1 | k) = P_{zz}(k + 1 | k) + R(k + 1)$

6. Finally the cross correlation matrix is determined by

$$\begin{aligned}P_{zx}(k + 1 | k) &= \frac{1}{n + \kappa} \left\{ \kappa [\chi_0(k + 1 | k) - \hat{x}(k + 1 | k)] [\mathcal{Z}_0(k + 1 | k) - \hat{z}(k + 1 | k)]^T \right. \\ &\quad \left. + \frac{1}{2} \sum_{i=1}^{2n} [\chi_i(k + 1 | k) - \hat{x}(k + 1 | k)] [\mathcal{Z}_i(k + 1 | k) - \hat{z}(k + 1 | k)]^T \right\}\end{aligned}$$

3. NUMERICAL SIMULATION

3.1 NUMERICAL SIMULATION FOR NONLINEAR SDOF SYSTEM

The governing equation of a nonlinear single-degree-of-freedom system subjected to ground acceleration may be represented by

$$\ddot{U}(t) + H(U(t), \dot{U}(t)) = -\ddot{u}_g(t) \quad (9)$$

where U, \dot{U}, \ddot{U} are the horizontal displacement, velocity and acceleration vectors of the structure all relative to the ground, $\ddot{u}_g(t)$ is the ground acceleration and $H(U(t), \dot{U}(t))$ is the normalized nonlinear restoring force.

There are many kinds of nonlinear restoring force equations, for example nonparametric model with polynomial function of the structural response, bilinear hysteretic model and hysteretic model proposed by Bouc and Wen. In this study Bouc and Wen hysteretic model is used to compare performance between UKF and EKF. The governing equations of Bouc and Wen's model are as following

$$\ddot{U}(t) + 2h\omega\dot{U}(t) + \omega^2\phi(U(t), \dot{U}(t)) = -\ddot{u}_g \quad (10)$$

$$\dot{\phi}(U(t), \dot{U}(t)) = \dot{U}(t) - \beta|\dot{U}(t)| + \gamma\dot{U}(t)|\phi(U(t))| \quad (11)$$

Equation (11) represents Bouc-Wen's hysteretic restoring force model, which was first proposed by Bouc and later generalized by Wen. In equation (11) the parameters β , γ control the hysteretic shape and degree of system degradation.

Numerical simulations are carried out for 13 by 13 cases of initial guesses of $\hat{x}(0|0)$ and $P(0|0)$. The results are shown in Fig. 4 and Fig.5 in the sense of Root Mean Square Error (RMSE) of estimated parameters. From the results the performance between the EKF and the UKF is strongly distinct. The EKF diverged almost region of simulation but the UKF shows robust estimation capability about the initial guesses of $\hat{x}(0|0)$ and $P(0|0)$.

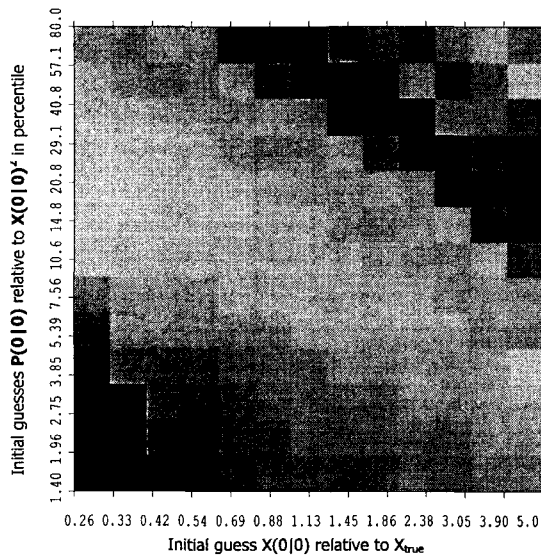


Fig.4 RMSE of the UKF for 1% noise

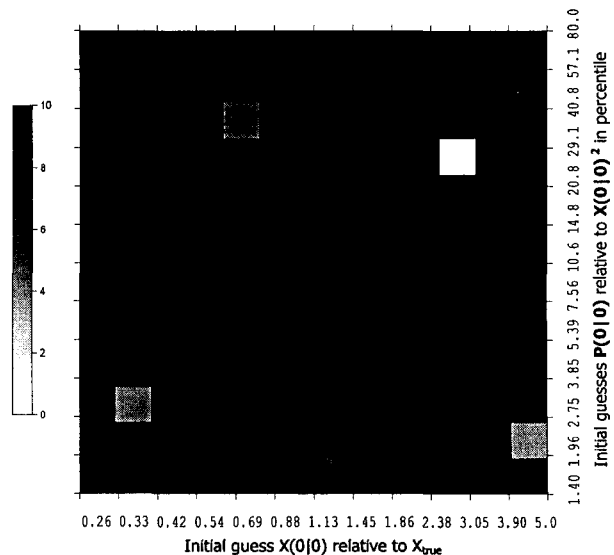


Fig.5 RMSE of the EKF for 1% noise

3.2 APPLICATION TO FIVE-STORY BUILDING STRUCTURE

Numerical simulations are carried out to show the capability of parameter identification using unscented Kalman filter. Identification is performed for the five-degree-of-Freedom shear-building model with Bouc-Wen's hysteretic nonlinear spring model between base and 1st floor using the artificial response data with 1% noise in RMS level. In this example, the parameters to be identified are c_i , k_i , β and γ ($i = 1, 2, 3, 4, 5$). The exact values of system parameters and estimated results are shown in Table 2 and convergence histories for linear and nonlinear parameters are shown in Fig. 7.

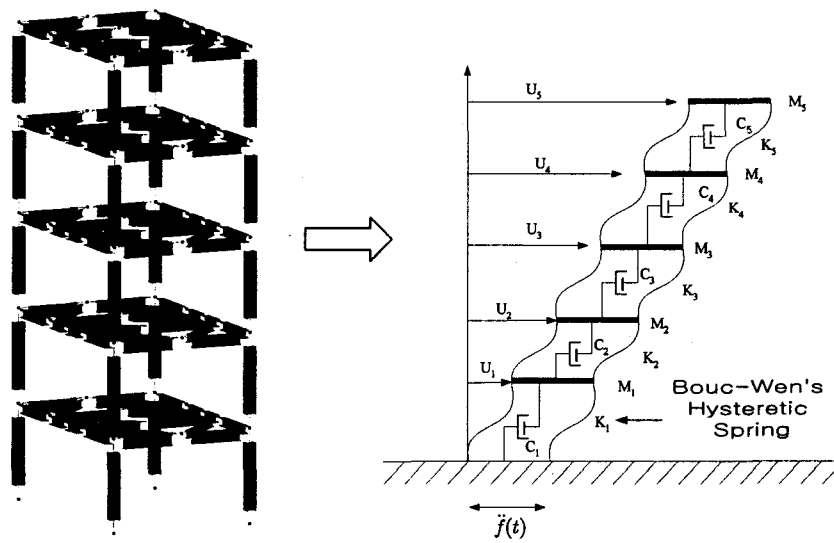


Fig.6 Five story building structure model

Table 2 The Exact Values for Simulations and Identified values

Story	C			K		
	C_{true}	C_{guess}	$C_{identified}$	K_{true}	K_{guess}	$K_{identified}$
1	0.2	0.1	0.20011	10	20	9.9949
2	0.2	0.1	0.19826	10	30	10.004
3	0.2	0.1	0.20131	10	20	9.9957
4	0.2	0.1	0.20015	10	30	10.005
5	0.2	0.1	0.19839	10	20	9.9959
Story	β			γ		
	β_{true}	β_{guess}	$\beta_{identified}$	γ_{true}	γ_{guess}	$\gamma_{identified}$
1	0.1	0.0	0.099911	0.2	0.0	0.20002

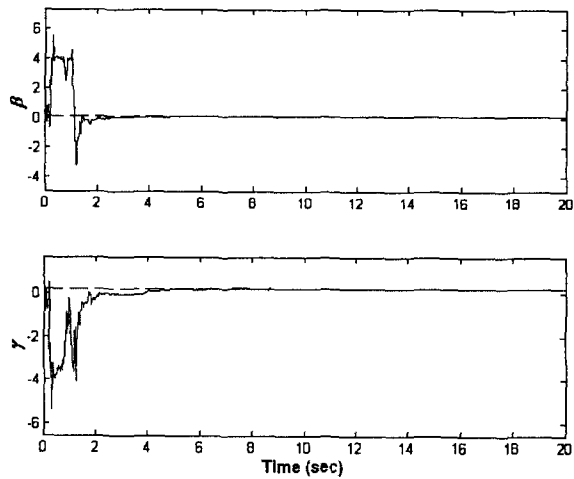
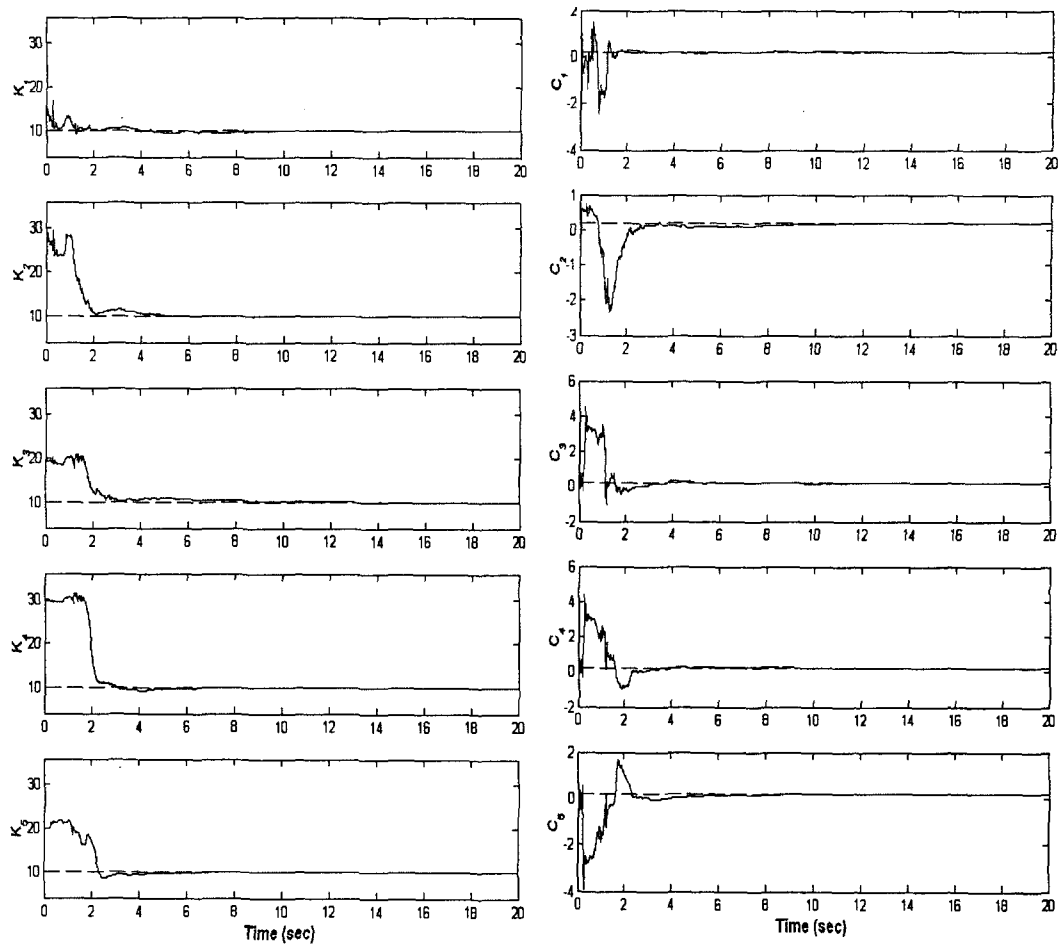


Fig.7 Convergence histories of system parameters (---: true value of system parameters)

4. CONCLUSION

In this study, the recently developed unscented Kalman Filter (UKF) as an alternative to the Extended Kalman filter (EKF) has been applied to the identification of nonlinear structural dynamic systems. The results from a series of numerical simulation studies are summarized below.

1. The UKF is superior to the EKF in terms of the expected error for non-linear systems, because it does not require crude linearization in order to predict the new state of the system.
2. The UKF is more tractable, since the UKF avoids the derivation of Jacobian matrices for linearizing nonlinear system dynamic equation and observation equation.
3. From the numerical simulation study on non-linear SDOF systems, the UKF shows robustness against initial guesses of $\hat{x}(0|0)$ and $P(0|0)$.
4. Numerical simulations for non-linear SDOF cases with Bouc-Wen's hysteretic model show that the UKF gives more accurate estimates for the parameters.
5. Numerical simulations for a five-story building structure with Bouc-Wen's hysteretic model show good estimation capability.
6. Several schemes employed to improve the performance of the EKF, such as adaptive fading algorithm and weighted global iteration, appear to be also applicable to the UKF.

REFERENCE

1. Juiler, S.J. and Uhlmann, J.K., 1994, "A new approach for the nonlinear transformation of means and covariances in linear filters", IEEE Transactions on Automatic Control
2. Koo, K.Y., 2001, Time Domain Identification of Nonlinear Structural Dynamic Systems Using Unscented Kalman Filter, M.S. Thesis, KAIST
3. Lee, K.J., 1990, Parameter Estimation and Damage Detection of Structures by System Identification Technique, M.S. Thesis, KAIST
4. Maruyama, O., Yun, C.B., Hoshiya, M. and Shinozuka, M., 1989, Program EXKAL2 for identification of Structural Dynamic Systems, Technical Report NCEER, New York
5. Shinozuka, M., Yun, C.B., and Imai, H., 1982, "Identification of linear structural dynamic systems", Journal of Engineering Mechanics Division, ASCE, 108(6), 1371-1390
6. Sorenson, H.W., 1985, Kalman filtering: Theory and Application, IEEE press, New York
7. Yun, C.B., Lee, H.J. and Lee, C.G., 1997, "Sequential Prediction-Error Method For Structural Identification", Journal of Engineering Mechanics, 115-122
8. Yun, C.B. and Shinozuka, M., 1980, "Identification of Nonlinear Structural Dynamic Systems", Journal of Structural Mechanics, 8(2), 187-203