그룹의사결정 지원을 위한 계층적 분석과정: 시뮬레이션 접근방법

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Abstract

The Analytic Hierarchy Process (AHP) is well suited to group decision making and offers numerous benefits as a synthesizing mechanism in group decisions. To date, the majority of AHP applications have been in group settings. In general, aggregation methods employed in AHP can be largely classified into two methods: geometric mean method and (weighted) arithmetic mean method. In a situation where there do not exist clear guidelines for selection between them, two methods do not always guarantee the same group decision result. Thus we suggest a simulation approach for building group consensus as a complementary tool, even when just group judgments are required. Without any efforts to make point estimates from individual diverse preference judgments, a simulation approach suggests the process how the individual preference judgments are aggregated into consensus, displaying possible disagreements as is natural in group members' different viewpoints.

1. Introduction

The Analytic Hierarchy Process (AHP), introduced by Saaty (1980), has also been applied to group decision problems. Saaty (1989) has discussed several practical and theoretical aspects of group decision making using AHP. There are at least two methods employed in AHP for aggregating group opinions. In the first, geometric mean method, as a most common group preference aggregation method in the AHP literature (Aczel and Saaty, 1983; Saaty and Kearns, 1985; Benjamin et al., 1992; Bard and Souk, 1990) utilizes geometric mean of individual evaluations as elements in pairwise comparison matrices and then priorities are then computed. In the weighted arithmetic mean method, a simple arithmetic mean of individual priorities is used to arrive at the group consensus. In viewpoint of social choice axioms, the geometric mean method of combining individual opinions has been shown to violate at least one of the axioms of group preference aggregation, namely the Pareto optimality axiom. The other method, the weighted arithmetic mean method has been found to satisfy all the axioms except the independence of irrelevant alternatives and it has been shown that this does not limit the applicability of this method (Ramanathan and Ganesh, 1994).

In this paper, we consider simulation approach as a group preference aggregation method rather than deriving group point estimates from individual pairwise judgments between criteria or between alternative on each criterion, which was adopted in many of group AHP applications. In applying a simulation approach, it is a prerequisite to have multitude of decision makers (at least the number of scales used in AHP) involved as is often case in public policy making for generating random observations from empirically observed frequency distribution which is determined from the frequency of responses. Using the simulation approach, which reflects diversification of group members' preferences as it is, analysis such as expected weights and expected ranks displays insights into group decision making context (Ahn, 2000).

2. A simulation approach when multiple decision makers are involved

Let $A_1, A_2, \ldots, A_n$ be a set of n alternatives compared in pairs according to a given criterion. We define a square matrix $A^k=(a^k_{i,j}), \forall i,j \in [1, n], k \in K$ to be a reciprocal matrix with n alternatives where $a^k_{i,j}=1/a^k_{j,i}$ and $a^k_{i,j}$ indicates that the ith alternative is $a^k_{i,j}$ times more dominant than the jth alternative on the criterion considered in kth group member's viewpoint. Similarly, let $C_1, C_2, \ldots, C_m$ be a set of m common criteria which is shared among group decision makers. We define a square matrix $C^k=(c^k_{p,q}), \forall p,q \in [1, m], k \in K$ to be a reciprocal matrix with m criteria where $c^k_{p,q}=1/c^k_{q,p}$ and $c^k_{p,q}$ indicates that the ith criterion is $c^k_{p,q}$ times more important than the jth criterion considered in kth group member's viewpoint.

Gathering K decision makers' pairwise judgments on criteria and between alternatives on criteria considered, it can be thought that variable $a_{i,j}$ ranged from $a^L_{i,j}=\min[a^1_{i,j}, a^2_{i,j}, \ldots, a^K_{i,j}]$ to $a^U_{i,j}=\max[a^1_{i,j}, a^2_{i,j}, \ldots, a^K_{i,j}]$ and $c_{p,q}$ ranged from $c^L_{p,q}=\min[c^1_{p,q}, c^2_{p,q}, \ldots, c^K_{p,q}]$ to $c^U_{p,q}=\max[c^1_{p,q}, c^2_{p,q}, \ldots, c^K_{p,q}]$ can be regarded as variables bounded between 1/9 and 9 respectively. Let $f(a_{i,j})$, $f(c_{p,q})$ be the empirically observed relative frequency distribution and $F(a_{i,j})$, $F(c_{p,q})$ the cumulative frequency distribution on $a_{i,j}$ and $c_{p,q}$ respectively.

Let $a^{(r)}_{i,j}$ and $c^{(r)}_{p,q}, r=1,2,\ldots,R,$ $i,j=1,2\ldots,n$, $p,q=1,2,\ldots,m$ be pairwise comparisons of size r generated from the cumulative frequency distribution $F(a_{i,j})$ and $F(c_{p,q})$ respectively. Let $\Psi^{(r)}$ be matrix of which elements are eigenvectors calculated from generated pairwise comparison, $a^{(r)}_{i,j}$ and $C^{(r)}$ be the eigenvectors associated with generated pairwise comparison, $c^{(r)}_{p,q}$ in rth simulation run. Then final priorities of alternatives considered can be determined as displaying descending order of magnitude of $C^{(r)} * \Psi^{(r)}$ The simulated final priorities are sometimes obtained in case simulated pairwise judgment matrices have high inconsistency ratio. To avoid this case, we consider generated pairwise comparison matrices with inconsistency ratio less than or equal to 0.1.

To illustrate the aforementioned simulation process, we consider an artificial example with three alternatives evaluated on three criteria and then the example is extended to illustrate more general case of four alternatives with five criteria. At first, let the hierarchy to be used in the example be as shown in Figure 1. It has three alternatives (A_1, A_2, A_3) to be compared using three criteria, (C_1, C_2, C_3). And the group members' pairwise judgment for this hierarchy is shown in Figure 1.

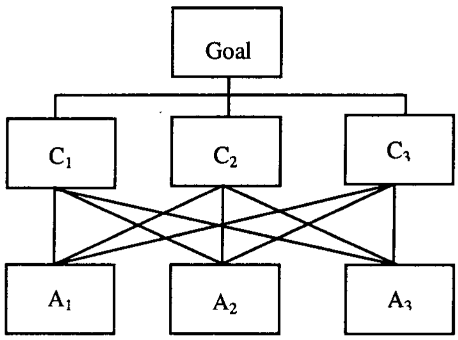

Figure 1. A typical AHP model

There is a continuum of decision making contexts ranging from (1) common objectives – contexts where all parties have (basically) the same objectives, to (2) non-common objectives – contexts in which parties (or groups of parties) have non-shared (and sometimes hidden) objectives, to (3) conflict – contexts in which parties seek concessions from opposing parties. Further the common objectives context can be decomposed into following three situations – consensus building, vote or compromise and separate models or players (Dyer and Forman, 1992). In our example, we assume that the participants involved in decision making process share common objectives for group consensus building. This approach is especially useful when judgments are elicited using (Web based) questionnaires as the group members will not have the chances to interact with each other so that the judgments are not influenced.

The preference judgments from group members $(K=25)$ are shown in Table 1, where frequencies of preference judgments about pairs of criteria and alternatives on each criterion are denoted.

Before analyzing simulation results, let us scrutinize the preference frequency in Table 1. At first, we can find the group's strong tendency which says criterion C_1 is most preferred, C_3 is next, and finally C_2, that is $C_1 \phi C_3 \phi C_2$. And we can infer $A_1 \phi A_3 \phi A_2$ on criterion C_1, $A_3 \phi A_2 \phi A_1$ on criterion C_2, and $A_1 \phi A_3 \phi A_2$ on criterion C_3 from the frequency between alternatives on criterion although there exist some disagreements. However roughly aggregated group preference, $A_3 \phi A_2 \phi A_1$ on criterion C_2 does not have much power on deciding the final priority because the weight of criterion C_2 is evaluated less important than the other two criteria. Consequently, we are strongly confident as a group opinion that A_1 is most preferred, A_2 is secondly, and A_3 is the least preferred, which is the result we want to show in a simulation run.

For each of the matrices of the example, discrete values for the judgments were generated from the empirically observed distribution. For example, let us consider the C_{12} column in Table 1. According to the relative frequency distribution, we can construct the cumulative distribution as shown in Figure 2.

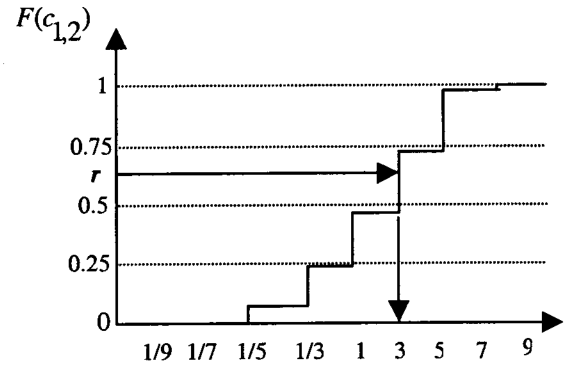

Figure 2. A cumulative distribution on $C_{1,2}$

Thus the random observation (i.e., pairwise ratio comparison) is generated from the equation, $r = F(c_{1,2})$, where $r \in [0,1]$ is a random number. In this manner, we can generate random observations C_{13} and C_{23}, which are used for components of matrix for calculating eigenvectors. After all matrices were determined, the overall synthesized priorities were calculated and thus the rank was recorded. This process was repeated 500 times. Of the 500 runs, 64% resulted in the first alternative with the first rank, 53% in the third alternative with the second rank, and 77% in the second alternative with the third rank (See Table 2 for the details).

Table 1. Preference frequency from group decision makers

Scale	Between criteria			Between alternatives on criterion								
				C_1			C_2			C_3		
	C_{12}	C_{13}	C_{23}	A_{12}	A_{13}	A_{23}	A_{12}	A_{13}	A_{23}	A_{12}	A_{13}	A_{23}
9				3						1		
7	1	4		5	3			1	2	5	6	
5	6*	5		7	4	1	1	1	3	3	7	1
3	7	7	1	3	5	3	5	4	3	6	3	3
1	5	3	4	2	8	5	7	6	2	4	4	2
1/3	4	2	3	4	3	7	7	3	6	3	4	5
1/5	2	3	10	1	2	7	5	5	4	2	1	7
1/7		1	7			2		4	5	1		5
1/9								1				2

*: six of 25 persons response that criterion 1 is strongly more important than criterion 2.

Table 2. Composite of 500 runs based on empirically observed distribution function (frequency of each rank)

	Composite results					
Rank	One	Two	Three	One	Two	Three
1	320	25	155	64%	5%	31%
2	142	91	267	28%	18%	53%
3	38	384	78	8%	77%	16%

In the AHP with single decision maker, priorities with an IR greater than 0.10 are considered to have judgments which are too random-like (Vargas, 1982). In the group decision context, each preference judgment with small inconsistency is combined to build group consensus which can be ended with large inconsistency. Hence, pairwise comparison judgments with an IR less than with 0.10 for simulated criteria matrices are considered in Table 3. Although each alternative has seen each possible rank, it is clear that alternative one is inclined to be positioned in the first rank, 81% of the time. Likewise, alternative two is inclined to be positioned in the third rank, 89% of the time, and alternative three, 73% of the time. However, how much confidence can we have in first two rankings and others? In order to address this question, we consider the notion of *expected rank* and *expected weight* which were suggested by Hauser and Tadikamalla (1996).

Table 3. Composite of 500 runs with IR$\leq$0.1 based on empirically observed distribution function (frequency of each rank)

	Composite results					
	One	Two	Three	One	Two	Three
1	405	15	90	81%	3%	18%
2	85	40	365	17%	8%	73%
3	10	445	45	2%	89%	9%

Expected score which can be defined by (1) implies that we will sum together the product of the fraction of time each rank occurred and n+1 minus the rank itself instead of the rank itself because the rank and the fraction of the time each rank occurred is inversely correlated.

$$ES_i = \sum_{k=1}^{n}(p_{i,k})(n+1-k), \quad \forall i \in [1, n], \quad (1)$$

where ES_i is the expected score of the ith alternative and $p_{i,k}$ is the proportion of the trials that the ith alternative had rank k.

Next, let the expected weight to be the normalized expected scores. When alternatives are placed in descending order of the expected weights, the results reveal the expected rank of alternatives. Hence, we define

$$EW_i = \frac{ES_i}{\sum_{k=1}^{n} ES_k}, \quad \forall i \in [1, n]. \quad (2)$$

The expected weights of (2) are determined from the frequency that each rank occurred for each alternative. Hence these weights are statistical weights indicating a composite frequency or a mean of feasible weights around which we expect the actual weight to be scattered.

Table 4. Expected weight and rank by formula (1) and (2)

Alternative	Expected rank	Expected weight
One	1	0.4637
Two	3	0.1883
Three	2	0.3479

3. Conclusions

To date, the majority of AHP applications have been in group settings. One reason for this may be that groups often have an advantage over individuals when there exists a significant difference between the importance of quality in the decision and the importance of time in which to obtain the decision. Another reason may be the best alternative is selected by comparing alternative solutions, testing against selected criteria, a task ideally suited for AHP.

In general, group decision making methods employed in the AHP can be largely classified into two ways: geometric mean method and arithmetic mean method. In the geometric mean method, as a most common group preference aggregation method in AHP literature, geometric mean of individual evaluations is used as elements in pairwise comparison matrices and then priority are computed. In the arithmetic mean method, a simple arithmetic mean of the individual priorities is used to arrive at the group consensus. Making group point estimates from individual judgments on each attribute is a solution alternative reflecting group members' diverse preferences. However, widely adopted aggregation methods adopted in AHP literatures do not guarantee the same group decision result and there do not exist clear guidelines for selection between two alternatives. In a situation where exact solutions are sometimes more important than probabilistic ones and thus combining judgments for a group working together is so important and can not be replaced by a statistical approach, aforementioned aggregation method is recommended to implement for deriving group judgments. Even in that case, a simulation approach which reflects diversification of group members' preference as it is, is useful as a complementary tool to get some detailed analysis.

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