비국소 초기 조건을 갖는 비선형 퍼지 미분방정식에 대한 해의 존재성과 유일성

The existence and uniqueness of solution for the nonlinear fuzzy differential equations with nonlocal initial condition

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ABSTRACT. In this paper, we study the existence and uniqueness of fuzzy solution for the *nonlinear fuzzy differential* equations with nonlocal initial condition in E_N^2 by using the concept of fuzzy number of dimension 2 whose values are normal convex upper semicontinuous and compactly supported surface in R^2.

Keywords and Phrases : fuzzy number, nonlinear fuzzy solution, fuzzy process

1. Introduction

The issue of solution of fuzzy differential equation has been discussed by many researchers which plays an important role in various applications. Kaleva [5] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal convex upper semicontinuous and compactly supported fuzzy sets in R^n, and Seikkala [12] studied the fuzzy initial value problem on E^1. Also, Kwun, Kang and Kim [8] proved the existence of fuzzy solution for the fuzzy differential equations in E_N.

In this paper, we consider the existence and uniqueness of fuzzy solution for the nonlinear fuzzy differential equations with nonlocal initial condition :

(F . D . E .)

$$\dot{x}(t) = a(t)x(t) + f(t, x(t))$$

$$x_0 = x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)), \cdot \in \{t_1, t_2, \cdots, t_p\}$$

here $a: [0, T] \to E_N^2$ is a fuzzy coefficient, nonlinear function $f: [0, T] \times E_N^2 \to E_N^2$ and $g: [0, T]^p \times E_N^2 \to E_N^2$ satisfies a global Lipschitz condition.

We consider a fuzzy graph $G \subset R \times R$, that is, a functional fuzzy relation in R^2 such that its membership function

$\mu_G(x, y), (x, y) \in R^2, \ \mu_G(x, y) \in [0, 1],$

has the following properties :

1. For all $x_0 \in R$;

   $\mu_G(x_0, y) \in [0, 1]$ is a convex membership function.

2. For all $y_0 \in R$;

   $\mu_G(x, y_0) \in [0, 1]$ is a convex membership function.

3. For all $\alpha \in [0, 1]$;

   $\mu_G(x, y) = \alpha$ is a convex surface.

4. There exist $(x_1, y_1) \in R^2$ such that

   $\mu_G(x_1, y_1) = 1.$

If the above conditions are satisfied, the fuzzy subset $G \subset R^2$ is called a fuzzy number of dimension 2.

A fuzzy number of dimension 2, $G \subset R^2$ such that for all $(x, y) \in R^2$.

$$\mu_G(x, y) = \mu_A(x) \wedge \mu_B(y).$$

We see that fuzzy number of dimension 2, $G \subset R^2$ is the direct product of two fuzzy numbers A and B are called noninteracive.

The first projection of G is $\vee_y \mu_G(x, y) = \mu_A(x)$ and the second projection of G is $\vee_x \mu_G(x, y) = \mu_B(y)$.

Let E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X and Y ([7]).

We denote by fuzzy number of dimension 2 in E_N^2, $A = (a_1, a_2)$, where a_1, a_2 is projection of A to axis X and Y respectively. And a_1 and a_2 are noninteractive fuzzy number in R.

The α-level set of fuzzy number of dimension 2 in E_N^2 defined by

$[A]^\alpha = \{(x_1, x_2) \in R^2 \mid (x_1, x_2) \in [a_1]^\alpha \times [a_2]^\alpha\}$

where operation X is cartesian product of the sets.

Let $A, B \in E_N^2$.

$$A = B \Leftrightarrow [A]^\alpha = [B]^\alpha \text{ for all } \alpha \in (0, 1].$$

If $A, B \in E_N^2$, then for $\alpha \in (0, 1]$,

$[A *_2 B]^\alpha = [a_1 *_1 b_1]^\alpha \times [a_2 *_1 b_2]^\alpha$, where $*_2$ is operation in E_N^2 and $*_1$ is operation in E_N^1

We use the metric d_∞ on E_N^2 defined by

$d_\infty(A, B) = \sup\{d_H([A]^\alpha, [B]^\alpha) : \alpha \in (0, 1]\}$

for all $A, B \in E_N^2$.

In section 2, we study the existence and uniqueness of the fuzzy solution for the nonlinear fuzzy differential equation (F.D.E.) and metrics related to fuzzy numbers

2. Existence and uniqueness of fuzzy solution

In this section, we consider the existence and uniqueness of fuzzy solution for the following nonlinear fuzzy differential equation with nonlocal initial condition :

(F.D.E.)

$$\dot{x}(t) = a(t)x(t) + f(t, x(t)), \ 0 \le t \le T,$$

$$x_0 = x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)), \ \cdot \in \{t_1, t_2, \cdots, t_p\}$$

where $a: [0, T] \to E_N^2$ is a fuzzy coefficient, nonlinear function $f: [0, T] \times E_N^2 \to E_N^2$ and $g: [0, T]^p \times E_N^2 \to E_N^2$ a satisfies global Lipschitz condition.

Let I be a real interval. A mapping $x : I \to E_N^2$ is called a fuzzy process. We set

$$x^{\alpha}(t) = x_1^{\alpha}(t) \times x_2^{\alpha}(t)$$
$$= [x_{1l}^{\alpha}(t), x_{1r}^{\alpha}(t)] \times [x_{2l}^{\alpha}(t), x_{2r}^{\alpha}(t)],$$
$t \in I$, $0 < \alpha \le 1$, $x_1, x_2 \in E_N$.

The derivative $x'(t)$ of a fuzzy process x is defined by

$$(x')^{\alpha}(t) = (x_1^{\alpha})'(t) \times (x_2^{\alpha})'(t)$$
$$= [(x_{1l}^{\alpha})'(t), (x_{1r}^{\alpha})'(t)] \times [(x_{2l}^{\alpha})'(t), (x_{2r}^{\alpha})'(t)].$$

The fuzzy integral $\int_a^b x(t)dt$, $a, b \in I$ is defined by

$$[\int_a^b x(t)dt]^{\alpha}$$
$$= (\int_a^b x_1^{\alpha}(t)dt) \times (\int_a^b x_2^{\alpha}(t)dt)$$
$$= [\int_a^b x_{1l}^{\alpha}(t)dt, \int_a^b x_{1r}^{\alpha}(t)dt]$$
$$\times [\int_a^b x_{2l}^{\alpha}(t)dt, \int_a^b x_{2r}^{\alpha}(t)dt]$$

provided that the Lebesgue integrals on the right exist.

Definition 2.1.

The fuzzy process $x: [0, T] \to E_N^2$ is a fuzzy solution of the (F.D.E.)without inhomogeneous term if and only if

$$(x_{ml}^{\alpha})'(t) = \min\{a_{mi}^{\alpha}(t) x_{1j}^{\alpha}(t) : m = 1, 2, i, j = l, r\}$$
$$(x_{mr}^{\alpha})'(t) = \max\{a_{mi}^{\alpha}(t) x_{mj}^{\alpha}(t) : m = 1, 2, i, j = l, r\}$$
$$x_{ml}^{\alpha}(0) = x_{0ml}^{\alpha} - g_{ml}^{\alpha}(t_1, t_2, \cdots, t_p, x(\cdot)),$$
$$x_{mr}^{\alpha}(0) = x_{0mr}^{\alpha} - g_{mr}^{\alpha}(t_1, t_2, \cdots, t_p, x(\cdot)) \quad m = 1, 2.$$

Theorem 2.1

For every $x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)) \in E_N^2$

$$\begin{cases} \dot{x}(t) = a(t)x(t), \\ x_0 = x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)), \\ \cdot \in \{t_1, t_2, \cdots, t_p\} \end{cases}$$

has a unique fuzzy solution $x \in C([0, T]: E_N^2)$.

The (F.D.E.) is related to the following fuzzy integral equations :

(F.I.E.)

$$\begin{cases} x(t) = S(t)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot))) \\ \quad + \int_0^t S(t-s) f(s, x(s)) ds \\ x_0 = x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)) \in E_N^2, \\ \cdot \in \{t_1, t_2, \cdots, t_p.\} \end{cases}$$

We assume the following hypotheses :

(H1) The nonlinear function $f: [0, T] \times E_N^2 \to E_N^2$ satisfies a global Lipschitz condition, that is, there exists a finite constant $K > 0$ such that

$$d_H([f(s, x(s))]^{\alpha}, [f(s, y(s))]^{\alpha})$$
$$\le K d_H([x(s)]^{\alpha}, [y(s)]^{\alpha}),$$

where $x, y \in E_N^2$ and f is regular function satisfying

$$f(s, x^{\alpha}) = f(s, x_1^{\alpha} \times x_2^{\alpha})$$
$$= f_1(s, x_1^{\alpha}) \times f_2(s, x_2^{\alpha})$$
$$= f_1^{\alpha}(s, x) \times f_2^{\alpha}(s, x) = f^{\alpha}(s, x).$$

(H2) The nonlinear function $g: [0, T]^p \times E_N^2 \to E_N^2$ satisfies a Lipschitz condition, that is, there exists a finite constant $L > 0$ such that

$$d_H(g^{\alpha}(t_1, t_2, \cdots, t_p, x(\cdot)), g^{\alpha}(t_1, t_2, \cdots, t_p, y(\cdot)))$$
$$\le L d_H(x^{\alpha}, y^{\alpha})$$

where $x, y \in E_N^2$ and g is regular function satisfying

$$g(t_1, t_2, \cdots, t_p, x^{\alpha}) = g(t_1, t_2, \cdots, t_p, x_1^{\alpha} \times x_2^{\alpha})$$
$$= g_1(t_1, t_2, \cdots, t_p, x^{\alpha}) \times g_2(t_1, t_2, \cdots, t_p, x^{\alpha})$$
$$= g_1^{\alpha}(t_1, t_2, \cdots, t_p, x) \times g_2^{\alpha}(t_1, t_2, \cdots, t_p, x)$$
$$= g^{\alpha}(t_1, t_2, \cdots, t_p, x)$$

Theorem 2.2

Let $T > 0$, assume that the function f and g satisfy the hypotheses (H1) and (H2) for every $x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)) \in E_N^2$, and $c(L + KT) < 1$, then (F.D.E.) has a unique fuzzy solution $x \in C([0, T]: E_N^2)$.

3. Examples

Example 3.1. Consider the following nonlinear fuzzy differential equation with nonlocal initial condition :

$$\begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)), 0 \le t \le T, \\ x_0 = x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)), \\ \cdot \in \{t_1, t_2, \cdots, t_p\} \end{cases}$$

where the fuzzy coefficient $a(t) = (\tilde{2}, \tilde{2})t$.

If the nonlinear function $f: [0, T] \times E_N^2 \to E_N^2$ is represented by $f(t, x(t)) = \tilde{2}tx(t)^2$, it is satisfies the following inequality.

When nonlinear function

$g: [0, T]^p \times E_N^2 \times E_N^2 \to E_N^2$ is represented by

$$g(t_1, t_2, \cdots, t_p, x(\cdot)) = \sum_{k=1}^{p} c_k x(t_k)$$

where c_k is real constants, it satisfies the following inequality

$$\begin{aligned} & d_H(g^a(t_1, t_2, \cdots, t_p, \varphi_1(\cdot)), g^a(t_1, t_2, \cdots, t_p, \varphi_2(\cdot))) \\ = \ & d_H([\sum_{k=1}^{p} c_k \varphi_1(t_k)]^a, [\sum_{k=1}^{p} c_k \varphi_2(t_k)]^a) \\ \le \ & \sum_{k=1}^{p} c_k d_H(\varphi_1^a(t_k), \varphi_2^a(t_k)) \\ \le \ & \sum_{k=1}^{p} c_k \max_{t_k} d_H(\varphi_1^a(t_k), \varphi_2^a(t_k)) \\ \le \ & L d_H(\varphi_1^a(\cdot), \varphi_2^a(\cdot)) \end{aligned}$$

where constant $L = \sum_{k=1}^{p} c_k > 0$.

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