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Observability for the nonlinear fuzzy neutral functional differential equations

*이창근, 권영철, 최재룡,
*부산시 사하구 하단동 동아대학교 수학과,
* C. K. Lee, Y. C. Kwun and J. R. Choi,
Dept. of Mathematics, Dong-a University, Pusan 604-714, Korea,
Dept. of Mathematics, Dong-a University, Pusan 604-714, Korea,
yckwun@daunet.donga.ac.kr,
Dept. of Mathematics, Dong-a University, Pusan 604-714, Korea,
jrchoi@daunet.donga.ac.kr.

ABSTRACT

In this paper, we consider the observability conditions for the following nonlinear fuzzy neutral functional differential equations :

$$\begin{cases} \frac{d}{dt}[x(t) - f(t, x_t)] = Ax(t) + k(t, x_t) + u(t), \ t \in [0, T], \\ x(t) = \phi(t), \ t \in [-\infty, 0], \\ y = \Lambda x(t), \end{cases}$$

where $x(t)$ is state function on E_N^2, A is fuzzy number on E_N^2, $u(t)$ is control function on E_N^2 and nonlinear continuous functions $f: J \times C_0 \rightarrow E_N^2$, $k: J \times C_0 \rightarrow E_N^2$ are satisfies global Lipschitz conditions.

Keywords and Phrases : observability, fuzzy neutral functional differential equations, fuzzy number, fuzzy process

1. Introduction

In general, several systems are primarily related to uncertainty and unexactness.
The problem of unexactness is generally considered to be an exact science whereas, that of uncertainty is considered as being vague, fuzzy and accidental. The problems of accident has been studied in probability theories and have made much progress, but that of the fuzzy is relatively new theory and has many possibilities on development.

In particular, Kloeden([5]) studied the fuzzy dynamic system on the research of the fuzzy system, Kaleva([3]) researched the fuzzy differential equations and Cauchy problem, and Seikkala([9]) studied the initial value problems, Kwun and Park([7]) studied optimal control problem for fuzzy differential equa tions, Subrahmanyam and Sudarsanam([10]) researched the fuzzy volterral integral equa tions. Balasubramaniam and Muralisankar([1]) researched the existence and uniqueness of fuzzy solution for the nonlinear fuzzy neutral functional differential equation and Kwun and Kang([6]) researched the exact controllability of the nonlinear fuzzy differential system.

The purpose of this paper is to investigate the continuously initial observability for the nonlinear fuzzy neutral differential equation.

Let E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X and Y([4]).

2. Preliminary

We consider a fuzzy graph $G\subset R\times R$, that is, a functional fuzzy relation in R^2 such that its membership function

$\mu_G(x,y),\ (x,y)\in R^2,\ \mu_G(x,y)\in[0,1]$,

has the following properties :

1. For all $x_0\in R$; $\mu_G(x_0,y)\in[0,1]$ is a convex membership function.
2. For all $y_0\in R$; $\mu_G(x,y_0)\in[0,1]$ is a convex membership function.
3. For all $\alpha\in[0,1]$; $\mu_G(x,y)=\alpha$ is a convex surface.
4. There exist $(x_1,y_1)\in R^2$ such that $\mu_G(x_1,y_1)=1$.

If the above conditions are satisfied, the fuzzy subset $G\subset R^2$ is called a fuzzy number of dimension 2.

A fuzzy number of dimension 2, $G\subset R^2$ such that for all $(x,y)\in R^2$.

$$\mu_G(x,y)=\mu_A(x)\wedge\mu_B(y).$$

We see that fuzzy number of dimension 2, $G\subset R^2$ is the direct product of two fuzzy numbers A and B are called noninteracive. The first projection of G is $\vee_y\mu_G(x,y)=\mu_A(x)$ and the second projection of G is $\vee_x\mu_G(x,y)=\mu_B(y)$.

We denote by fuzzy number of dimension 2 in E_N^2, $A=(a_1,a_2)$, where a_1,a_2 is projection of A to axis X and Y respectively. And a_1 and a_2 are noninteractive fuzzy number in R.

3. Continuously initial observability

In this chapter we consider the continuously initial observability conditions for the following nonlinear fuzzy neutral functional differential equations:

$$(3.1)\quad \begin{cases} \frac{d}{dt}[x(t)-f(t,x_t)]=Ax(t)+k(t,x_t),\ t\in[0,T],\\ x(t)=\phi(t),\ t\in(-\infty,0],\\ y(t)=\tilde{\Pi}(x(t)), \end{cases}$$

where $x(t)$ is state function on E_N^2, A is fuzzy number on E_N^2 and nonlinear continuous functions $f:J\times C_0\to E_N^2$, $k:J\times C_0\to E_N^2$ satisfies the global Lipschitz conditions $C_0=C((-\infty,0]:E_N^2)$. Let $\tilde{\Pi}:C((-\infty,T]:E_N^2)\to Y$ is a given fuzzy mapping where Y is in E_N^2.

Instead of (3.1), we consider the following fuzzy integral system:

$$(3.2)\quad \begin{cases} x(t)=S(t)[\phi(0)-f(0,\phi)]+f(t,x_t)\\ \qquad +\int_0^t AS(t-s)f(s,x_s)ds\\ \qquad +\int_0^t S(t-s)k(s,x_s)ds,\ t\in[0,T],\\ x(t)=\phi(t),\ t\in(-\infty,0],\\ y(t)=\tilde{\Pi}(x(t)), \end{cases}$$

where $S(t)$ is the fuzzy number of dimension 2 and

$[S(t)]^\alpha=[S_1(t)]^\alpha\times[S_2(t)]^\alpha$

$=[S^{\alpha}_{1l}(t)\,,\,S^{\alpha}_{1r}(t)]\times[S^{\alpha}_{2l}(t)\,,\,S^{\alpha}_{2r}(t)]$

where $S^{\alpha}_{il}(t)=\exp\{\int_0^t A^{\alpha}_{il}ds\}$ and

$S^{\alpha}_{ir}(t)=\exp\{\int_0^t A^{\alpha}_{ir}ds\},\ (i=1,2).$

And $S^{\alpha}_{ij}(t)\,(i=1,2,j=l,r)$ is continuous. That is, there exists a constant $c>0$ such that

$$|\,S^{\alpha}_{ij}(t)\,|\le c \text{ for all } t\in[0,T].$$

Definition 3.1. The (3.2) is continuously initial observable if, for any initial state $\phi(0)$ there exists a fuzzy mapping $\widehat{H}$ such that the fuzzy out put $y(t)$ satisfies $\widehat{H}(\phi(0))=y(t)$.

Define the fuzzy mapping $\widehat{H}\colon \widehat{P}(R^2)\to E^2_N$ by

$$[\,\widehat{H}(v(t))]^{\alpha}=\begin{cases}[\,\widetilde{\Pi}(S(t)v(t))]^{\alpha}, & v(t)\subset\overline{\Gamma_{\phi(0)}},\\ 0, & \text{otherwise.}\end{cases}$$

where $\widehat{P}(R^2)$ is a set of subsets in R^2 and $\Gamma_{\phi(0)}$ is in support of the fuzzy initial value $\phi(0)$.

Then there exists $\widehat{H}_i\colon \widehat{P}(R)\to E_N\,(i=1,2)$ such that

$$[\,\widehat{H}_i(v_i(t))]^{\alpha}=\begin{cases}[\,\widetilde{\Pi}_i(S_i(t)v_i(t))]^{\alpha}, & v_i(t)\subset\overline{\Gamma_{\phi_i(0)}},\\ 0, & \text{otherwise}\end{cases}$$

where v_i is projection of v to axis x and y respectively and there exists $\widehat{H}_{ij}^{\alpha}$ $(i=1,2,\ j=l,r)$ such that

$$\widehat{H}_{il}^{\alpha}(v_{il}(t))=\widetilde{\Pi}^{\alpha}_{il}(S^{\alpha}_{il}(t)v_{il}(t)),$$
$$v_{il}(t)\in[\phi^{\alpha}_{il}(0),\phi^{1}_{i}(0)],$$
$$\widehat{H}_{ir}^{\alpha}(v_{ir}(t))=\widetilde{\Pi}_{ir}^{\alpha}(S^{\alpha}_{ir}(t)v_{ir}(t)),$$
$$v_{ir}(t)\in[\phi^{1}_{i}(0),\phi^{\alpha}_{ir}(0)].$$

We assume that $\widehat{H}_{il}^{\alpha}$, $\widehat{H}_{ir}^{\alpha}$ are bijective mappings.

Thus we can be introduced to $\phi(0)$ of (3.2)

$$\phi(0)=\ \widehat{H}^{-1}[y(t)+\widetilde{\Pi}S(t)f(0,\phi)$$
$$-\widetilde{\Pi}\{f(t,x_t)+\int_0^t AS(t-s)f(s,x_s)ds$$
$$+\int_0^t S(t-s)k(s,x_s)ds\}].$$

Then substituting this expression into the (3.2) yields α- level of $x(t)$ as

$$[x(t)]^{\alpha}=\ [S(t)\{\ \widehat{H}^{-1}\{y(t)+\widetilde{\Pi}S(t)f(0,\phi)$$
$$-\widetilde{\Pi}(f(t,x_t)+\int_0^t AS(t-s)f(s,x_s)ds$$
$$+\int_0^t S(t-s)k(s,x_s)ds)\}-f(0,\phi)\}\ .$$
$$+f(t,x_t)+\int_0^t AS(t-s)f(s,x_s)$$
$$+\int_0^t S(t-s)k(s,x_s))ds\ ]^{\alpha}$$

Thus

$$[\,\widetilde{\Pi}(x(t))]^{\alpha}=\prod_{i=1}^{2}[\,\widetilde{\Pi}_i(x_i(t))]^{\alpha}$$
$$=\ \prod_{i=1}^{2}[\ \widehat{H}^{\alpha}_{il}\{(\ \widehat{H}^{\alpha}_{il})^{-1}\{y^{\alpha}_{il}(t)+\widetilde{\Pi}^{\alpha}_{il}S^{\alpha}_{il}(t)f^{\alpha}_{il}(0,\phi)$$
$$-\ \widetilde{\Pi}^{\alpha}_{il}(f^{\alpha}_{il}(t,x_t)+\int_0^t A^{\alpha}_{il}S^{\alpha}_{il}(t-s)f^{\alpha}_{il}(s,x_s)ds$$
$$+\int_0^t S^{\alpha}_{il}(t-s)k^{\alpha}_{il}(s,x_s)ds)\}\}$$
$$-\ \widetilde{\Pi}^{\alpha}_{il}S^{\alpha}_{il}(t)f^{\alpha}_{il}(0,\phi)+\widetilde{\Pi}^{\alpha}_{il}f^{\alpha}_{il}(t,x_t)$$
$$+\ \widetilde{\Pi}^{\alpha}_{il}\int_0^t A^{\alpha}_{il}S^{\alpha}_{il}(t-s)f^{\alpha}_{il}(s,x_s)$$
$$+\ \widetilde{\Pi}^{\alpha}_{il}\int_0^t S^{\alpha}_{il}(t-s)k^{\alpha}_{il}(s,x_s))ds\ ,$$
$$\widehat{H}^{\alpha}_{ir}\{(\ \widehat{H}^{\alpha}_{ir})^{-1}\{y^{\alpha}_{ir}(t)\ +\widetilde{\Pi}^{\alpha}_{ir}S^{\alpha}_{ir}(t)f^{\alpha}_{ir}(0,\phi)$$
$$-\ \widetilde{\Pi}^{\alpha}_{ir}(f^{\alpha}_{ir}(t,x_t)+\int_0^t A^{\alpha}_{ir}S^{\alpha}_{ir}(t-s)f^{\alpha}_{ir}(s,x_s)ds$$
$$+\int_0^t S^{\alpha}_{ir}(t-s)k^{\alpha}_{ir}(s,x_s)ds)\}\}$$
$$-\ \widetilde{\Pi}^{\alpha}_{ir}S^{\alpha}_{ir}(t)f^{\alpha}_{ir}(0,\phi)$$
$$+\ \widetilde{\Pi}^{\alpha}_{ir}f^{\alpha}_{ir}(t,x_t)+\ \widetilde{\Pi}^{\alpha}_{ir}\int_0^t A^{\alpha}_{ir}S^{\alpha}_{ir}(t-s)f^{\alpha}_{ir}(s,x_s)$$
$$+\ \widetilde{\Pi}^{\alpha}_{ir}\int_0^t S^{\alpha}_{ir}(t-s)k^{\alpha}_{ir}(s,x_s))ds]$$
$$=\ \prod_{i=1}^{2}[y^{\alpha}_{il}(t)\ ,\ y^{\alpha}_{il}(t)]=[y_1(t)]^{\alpha}\times[y_2(t)]^{\alpha}=[y(t)]^{\alpha}.$$

Define

$$\Phi x(t)=\ S(t)\{\ \widehat{H}^{-1}\{y(t)+\widetilde{\Pi}S(t)f(0,\phi)$$
$$-\widetilde{\Pi}(f(t,x_t)+\int_0^t AS(t-s)f(s,x_s)ds$$
$$+\int_0^t S(t-s)k(s,x_s)ds)\}-f(0,\phi)\}+f(t,x_t)$$
$$+\int_0^t AS(t-s)f(s,x_s)+\int_0^t S(t-s)k(s,x_s))ds,$$

where the fuzzy mapping $\widehat{H}^{-1}$ satisfies the above conditions.

We assume the following hypotheses:

(H1) Nonlinear continuous fuzzy mappings $f\colon[0,T]\times C_0\to E^2_N,\ \ k\colon[0,T]\times C_0\to E^2_N$ satisfy the global Lipschitz condition, there exist a finite constants $p>0$, $q>0$ such that

$$d_H([f_i(s,(x_s(\theta))_i)]^\alpha,[f_i(s,(y_s(\theta))_i)]^\alpha)$$
$$\leq p d_H([(x_s(\theta))_i]^\alpha,[(y_s(\theta))_i]^\alpha),$$
$$d_H([k_i(s,(x_s(\theta))_i)]^\alpha,[k_i(s,(y_s(\theta))_i)]^\alpha)$$
$$\leq q d_H([(x_s(\theta))_i]^\alpha,[(y_s(\theta))_i]^\alpha)$$

for all $(x_s)_i, (y_s)_i \in C((-\infty,T]:E_N)$, $f_i:[0,T]\times C((-\infty,T]:E_N)\to E_N$ $(i=1,2)$ is a projection of f and $k_i:[0,T]\times C((-\infty,T]:E_N)\to E_N(i=1,2)$$ is a projection of k.

(H2) There exists a positive constant $l_1>0$ such that

$$d_H([\tilde{H}_i^{-1}(\tilde{H}_i(f_i(t,\xi_1)))]^\alpha,[\tilde{H}_i^{-1}(\tilde{H}_i(f_i(t,\xi_2)))]^\alpha)$$
$$\leq l_1 d_H([(\xi_1)_i]^\alpha,[(\xi_2)_i]^\alpha)$$

where $\xi_1,\xi_2\in C_0$.

(H3) There exists a positive constant $l_2>0$ such that

$$d_H([\tilde{H}_i^{-1}(\tilde{H}_i(\int_0^t A_i S_i(t-s)f_i(t,\xi_1)ds))]^\alpha,$$
$$[\tilde{H}_i^{-1}(\tilde{H}_i(\int_0^t A_i S_i(t-s)f_i(t,\xi_2)))]^\alpha)$$
$$\leq l_2\int_0^t d_H([f_i(s,\xi_1)]^\alpha,[f_i(s,\xi_2)]^\alpha)ds,$$

where $\xi_1,\xi_2\in C_0$.

(H4) There exists a positive constant $l_3>0$ such that

$$d_H([\tilde{H}_i^{-1}(\tilde{H}_i(\int_0^t S_i(t-s)k_i(t,\xi_1)ds))]^\alpha,$$
$$[\tilde{H}_i^{-1}(\tilde{H}_i(\int_0^t S_i(t-s)k_i(t,\xi_2)))]^\alpha)$$
$$\leq l_3\int_0^t d_H([k_i(s,\xi_1)]^\alpha,[k_i(s,\xi_2)]^\alpha)ds,$$

where $\xi_1,\xi_2\in C_0$.

<u>Theorem 3.1.</u>

Suppose that hypotheses (H1)-(H4) are held. Then the state of the (3.2) is continuously initial observable.

Proof. Omitted.

<u>Example 3.1.</u>

Consider the following fuzzy neutral functional differential equation:

$$\begin{cases} \frac{d}{dt}[x(t)-f(t,x_t)] = Ax(t)+k(t,x_t), & t\in(0,T], \\ x(t)=\phi(t), & t\in(-\infty,0], \\ y(t)=\tilde{H}(x(t)). \end{cases}$$

Let $f(t,x_t)=(\tilde{2}tx(t+\theta),\tilde{2}tx(t+\theta))$, $k(t,x_t)=(\tilde{2}t^2x(t+\theta)^2,\tilde{2}t^2x(t+\theta)^2)$, $\theta\in(-\infty,0]$, fuzzy number $A=(\tilde{2},\tilde{2})$ and fuzzy output $y(t)=(\tilde{3},\tilde{3})$.

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