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On type-2 fuzzy set-valued mappings

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Abstract

In this paper, we define type-2 fuzzy mappings on $L-L$ fuzzy numbers and discuss some properties of these mappings.

1. Preliminaries and definitions

Let X be a finite set. A fuzzy set A in X is defined by $A=\{(x,\mu_A(x))|x\in X\}$, where $\mu_A:X\rightarrow[0,1]$ is the membership function of A. When $\mu_A(x)$ becomes a fuzzy set, A becomes a type-2 fuzzy set.

Now, since a type-2 fuzzy set is obtained by assigning fuzzy membership values to elements of X, we can extend the set-theoretic operations of ordinary fuzzy set theory to allow them to deal with fuzzy grades of membership.

Definition 1.1 [2] A fuzzy set M of $[0,1]$ is called a fuzzy number if
(1) M is normal;
(2) M is convex;
(3) μ_M is piecewise continuous.

Definition 1.2 [2] A fuzzy number M of $[0,1]$ is said to be $L-R$ fuzzy number, $M=(m,\alpha,\beta)_{LR}$ if its membership function is defined by

(i) when $\alpha>0$ and $\beta>0$,

$$\mu_{M(x)}=\begin{cases} L(\frac{m-x}{\alpha}) & \text{for } m-\alpha\le x\le m\le 1, x\ge 0, \\ R(\frac{x-m}{\beta}) & \text{for } m+\beta\ge x\ge m\ge 0, x\le 1, \\ 0 & \text{else,}\end{cases}$$

(ii) when $\alpha=0$ and $\beta>0$,

$$\mu_{M(x)}=\begin{cases} R(\frac{x-m}{\beta}) & \text{for } m+\beta\ge x\ge m\ge 0, x\le 1, \\ 0 & \text{else,}\end{cases}$$

(iii) when $\alpha>0$ and $\beta=0$,

$$\mu_{M(x)}=\begin{cases} L(\frac{m-x}{\alpha}) & \text{for } m-\alpha\le x\le m\le 1, x\ge 0, \\ 0 & \text{else,}\end{cases}$$

(iv) when $\alpha=0$ and $\beta=0$,

$$\mu_{M(x)}=\begin{cases} 1 & \text{for } x=m \\ 0 & \text{else.}\end{cases}$$

where, α and β are called the left and right spreads of an $L-R$ fuzzy number M, respectively, and L and R are strictly decreasing continuous functions from $[0,1]$ to $[0,1]$ such that $L(0)=R(0)=1$ and $L(1)=R(1)=0$. In this case, L and R is called the left and the right shape function, respectively.

A_{LR} will stand for the class of all $L-R$ fuzzy numbers of $[0,1]$.

Definition 1.3 [2] Let $M_x=(m_x,\alpha_x,\beta_x)_{LR}$ and $M_y=(m_y,\gamma_y,\delta_y)_{LR}$ be elements to A_{LR}. Then $\max^{\sim}(M_x,M_y)$, $\min^{\sim}(M_x,M_y)$ are defined by

$$\max{}^{\sim}(M_x,M_y)=(m_x\vee m_y,\alpha_x\wedge\gamma_y,\beta_x\vee\delta_y)_{LR},$$

$$\min{}^{\sim}(M_x,M_y)=(m_x\wedge m_y,\alpha_x\vee\gamma_y,\beta_x\wedge\delta_y)_{LR},$$

where $\vee$ and $\wedge$ are max and min, respectively.

Definition 1.4 [2] Let $M=(m,\alpha,\beta)_{LR}$ and

$M^1=(1,0,0)_{RL}$ be elements to A_{LR} and A_{LR}, respectively. The complemented M^* of M is defined by $M^* \equiv M^1 \ominus M=(1-m, \beta, \alpha)_{RL}$.

2. Main results

Let $X=\{x_1,\cdots,x_n\}$ and $Y=\{y_1,\cdots,y_n\}$ be finite sets. We consider that A_{LL} is the class of all $L-L$ fuzzy numbers of $[0,1]$.

The collection of type-2 fuzzy set-valued mappings of a set X is denoted by $F_2(X)$, i.e., $M \in F_2(X) \Leftrightarrow M: X \to A_{LL}$ by $M(x)=M_x$.

Definition 2.1 We say that ψ is type-2 fuzzy set-valued mappings on $X \times Y$ if (1) $\psi: X\times Y \to A_{LL}$ by $\psi(x,y)=M_{xy} \in A_{LL}$, $\forall (x,y) \in X \times Y$ (2) $\forall x \in X$, there exists $y \in Y$ such that $\max^{\sim}(\psi(x,y))=(1,0,0)_{LL}=M^1$.

Definition 2.2 Let ψ be type-2 fuzzy set-valued mappings on $X \times Y$. T_ψ is called the inverse image operator associated with ψ iff $\forall M \in F_2(Y)$, $\forall x \in X$,

$(T_\psi M)(x)=\max^{\sim}(\min^{\sim}(M_{xy}, M_y))$.

From the definition 2.2, it is ease to show that $T_\psi: F_2(Y) \to F_2(X)$ is a mapping from type-2 fuzzy sets of Y to type-2 fuzzy sets of X.

Definition 2.3 [2] Let $M_x=(m_x, \alpha_x, \beta_x)_{LL}$ and $N_x=(n_x, \gamma_x, \delta_x)_{LL}$ be elements of A_{LL}. Then, we define the order $\leq$ of M_x and N_x; $M_x \leq N_x$ if and only if $m_x \leq n_x$, $\alpha_x \geq \gamma_x$, and $\beta_x \leq \delta_x$.

Definition 2.4 [2] Let $M, N: X \to A_{LL}$ be type-2 fuzzy set-valued mappings of a set X. Then we define the order $\leq$ of M and N; $M \leq N$ iff $M_x \leq N_x$, $\forall x \in X$.

Proposition 2.5 Let T_ψ is the inverse image operator associated with ψ and $M^0=(0,0,0)_{LL}$. Then we have

(1) $(T_\psi M^0)(x)=(0, \min \alpha_{xy}, 0)_{LL}$,

(2) $(T_\psi M^1)(x) \leq M^1$,

(3) T_ψ is order preserving.

Proof.

(1) $(T_\psi M^0)(x)$

$$
\begin{aligned}
&= \max^{\sim}(\min^{\sim}(M_{xy}, M^0{}_y)) \\
&= \max^{\sim}(\min^{\sim}((m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}, (0,0,0)_{LL})) \\
&= \max^{\sim}((m_{xy}\wedge 0, \alpha_{xy} \vee 0, \beta_{xy} \wedge 0)_{LL}) \\
&= \max^{\sim}((0, \alpha_{xy}, 0)_{LL}) \\
&= (0, \min \alpha_{xy}, 0)_{LL}
\end{aligned}
$$

(2) By the definitions 2.1 and 2.4, we have

$(T_\psi M^1)(x)$

$$
\begin{aligned}
&= \max^{\sim}(\min^{\sim}(M_{xy}, M^1{}_y)) \\
&= \max^{\sim}(\min^{\sim}((m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}, (1,0,0)_{LL})) \\
&= \max^{\sim}((m_{xy}\wedge 1, \alpha_{xy} \vee 0, \beta_{xy} \wedge 0)_{LL}) \\
&= \max^{\sim}((m_{xy}, \alpha_{xy}, 0)_{LL}) \\
&\leq \max^{\sim}((m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}) \\
&= \max^{\sim}(\psi(x,y)) \\
&= (1,0,0)_{LL} = M^1
\end{aligned}
$$

(3) Let $M \leq N$ in $F_2(Y)$. Since $\max^{\sim}$ is nondecreasing, we have

$(T_\phi M)(x) = \max^\sim(\min^\sim(M_{xy}, M))$

$\le \max^\sim(\min^\sim(M_{xy}, N)) = (T_\phi N)(x), \ \forall x \in X.$ □

We note that if $M, N \in F_2(Y)$, then $\max^\sim(M, N)$ means $\max^\sim(M,N)(y) = \max^\sim(M_y, N_y)$ for all $y \in Y$ and $\max^\sim(T_\phi M, T_\phi N)$ means $\max^\sim(T_\phi M, T_\phi N)(x) = \max^\sim(T_\phi M(x), T_\phi N(x))$ for all $x \in X$.

Proposition 2.6

$T_\phi(\max^\sim(M, N)) = \max^\sim(T_\phi M, T_\phi N)$

Proof. Let $M_{xy} = (m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}$, $M_y = (m_y, \alpha_y, \beta_y)_{LL}$, and $N_y = (n_y, \gamma_y, \delta_y)_{LL}$.

Since $\max^\sim$, $\min^\sim$ are distributive law on A_{LL}, we have

$T_\phi(\max^\sim(M, N))(x)$

$= \max^\sim(\min^\sim(M_{xy}, \max^\sim(M, N)(y)))$

$= \max^\sim(\min^\sim(M_{xy}, \max^\sim(M_y, N_y)))$

$= \max^\sim(\min^\sim((m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}, \max^\sim((m_y, \alpha_y, \beta_y)_{LL}, (n_y, \gamma_y, \delta_y)_{LL})))$

$= \max^\sim(\min^\sim((m_{xy}, \alpha_{xy}, \beta_{xy})_{LL}, (m_y \vee n_y, \alpha_y \wedge \gamma_y, \beta_y \vee \delta_y)_{LL}))$

$= \max^\sim((m_{xy} \wedge (m_y \vee n_y), \alpha_{xy} \vee (\alpha_y \wedge \gamma_y), \beta_{xy} \wedge (\beta_y \vee \delta_y)_{LL}))$

$= \max^\sim(((m_{xy} \wedge m_y) \vee (m_{xy} \wedge n_y), (\alpha_{xy} \vee \alpha_y) \wedge (\alpha_{xy} \vee \gamma_y), (\beta_{xy} \wedge \beta_y) \vee (\beta_{xy} \wedge \delta_y))_{LL})$

$= \max^\sim(\max^\sim((m_{xy} \wedge m_y, \alpha_{xy} \vee \alpha_y, \beta_{xy} \wedge \beta_y)_{LL}, (m_{xy} \wedge m_y, \alpha_{xy} \vee \gamma_y, \beta_{xy} \wedge \delta_y)_{LL}))$

$= \max^\sim((T_\phi M)(x), (T_\phi N)(x))$

$= \max^\sim(T_\phi M, T_\phi N)(x) \quad \forall x \in X$

Therefore, we have

$T_\phi(\max^\sim(M, N)) = \max^\sim(T_\phi M, T_\phi N)$ □

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