마코프 입력 지연 시스템의 확률적 안정화

Stochastic Stabilization of TS Fuzzy System with Markovian Input Delay

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ABSTRACT

This paper discusses a stochastic stabilization of Takagi-Sugeno (TS) fuzzy system with Markovian input delay. The finite Markovian process is adopted to model the input delay of the overall control system. It is assumed that the zero and hold devices are used for control input. The continuous-time TS fuzzy system with the Markovian input delay is discretized for easy handling delay, accordingly, the discretized TS fuzzy system is represented by a discrete-time TS fuzzy system with jumping parameters. The stochastic stabilizibility of the jump TS fuzzy system is derived and formulated in terms of linear matrix inequalities (LMIs).

Keywords : TS fuzzy systems, input delay, Markovian jump systems, linear matrix inequality

Ⅰ. Introduction

Recently, as the communication systems has been more reliable, some attempts have been tried to remotely control via communication networks such as the Internet. Since the control loops of the remote-control system are closed over communication networks or field buses, time delay phenomena inevitably occur. The stability and performance of the controlled system are definitely dependent on the transmission performance of the communication networks. It is well known that the existence of time delay makes the closed-loop stabilization more difficult[3]. Some control methodologies that deal with the input-delay have been developed, using a rigorous mathematical tools such as Lyapunov-Krasovskii stability theorem and Lyapunov-Razumikhin stability theorem. Very recently, the stochastic approach to handling time-varying delay in random manner has gathered attentions of research. This is very desirable for designing a stabilizing controller for remote-control system via the Internet, since the delay in the Internet randomly varies.

Motivated by the above observations, this paper aims at studying the control problem for a class of Takagi-Sugeno (TS) fuzzy systems in the presence of randomly time-varying input delay. Although recent researches have been devoted to TS fuzzy-model-based control[4-11], this issue has not been directly tackled, thus must also be carefully handled in TS fuzzy systems for safety and improved operational performance of the nonlinear remote-control systems such as virtual laboratory (VL).

The stochastic property of the input delays are modelled using the Markov chain with the finite states, which is quite reasonable. The continuous-time TS fuzzy system is discretized for designing digital control law. The discretized TS fuzzy system is represented as a discrete-time TS fuzzy

system with jumping parameters. The sufficient condition for the stochastic stability of the jump TS fuzzy system is derived and formulated in terms of coupled linear matrix inequalities (LMIs).
The organization of this paper is as follows: Section 2 reviews the TS fuzzy system and its basic properties. The main results of this paper are discussed and explained in Section 3. To the end, Section 5 concludes this paper with some remarks.

II. Input-Delayed TS Fuzzy Systems

Consider the sampled-data TS fuzzy system described by the following fuzzy rules:

Plant Rule i
IF $z_1(t)$ is $\Gamma_1^1 \cdots z_n(t)$ is Γ_1^n
THEN $\dot{x_c}(t) = A_i x_c(t) + B_i u_c(t)$ (1)

where Γ_j^i $(j=1,\cdots,n, i=1,\cdots,q)$ is the fuzzy set, $x(t) \in R^n$ is the state, $u(t)=u(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT+T)$, $T>0$ is a sampling period, and τ_k represents the time-lag, which is governed by an underlying Markov chain. The defuzzified output of this TS fuzzy system (1) is represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t)) \left(A_i x(t) + B_i u(t-\tau_k)\right) \quad (2)$$

Assumption 1:Assume the the delay time τ_k of the control input is not larger than the sampling time T for $k=1,2,\ldots,$.

Throughout this paper, we employ a sampled-data TS fuzzy-model-based controller as follows:

$$u(t) = \sum_{i=1}^{q} \theta_i(x(kT))(K_i x(kT) + L_i u(t-T)) \quad (3)$$

on any internal $\forall t \in [kT, kT+T), k=1,2,\ldots,$.

Assumption 2:Assume that the firing strength of the ith rule, $\theta_i(z(t))$ is approximated by their values at time kT, i.e., $\theta_i(x(t)) \approx \theta_i(x(kT))$, for $kT \le t < kT+T$.

Consequently, the nonlinear matrices $\sum_{i=1}^{q} \theta(x(t))A_i$ and $\sum_{i=1}^{q} \theta(x(t))B_i$ can be approximated as constant matrices $\sum_{i=1}^{q} \theta(x(kT))A_i$ and $\sum_{i=1}^{q} \theta(x(kT))B_i$, respectively, over any interval $[kT, kT+T)$.

Theorem 1:The dynamical nonlinear behavior of the digital TS fuzzy system (2) can be efficiently approximated by

$$x_{k+1} \approx \sum_{i=1}^{q} \theta(x_k)(G_i x_k + H_{i1}(\tau_k)u_k + H_{i2}(\tau_k)u_{k-1}) \quad (4)$$

where $G_i = \exp(A_i T)$ and $H_{i1}(\tau_k) = \int_0^{T-\tau_k} e^{A\lambda} d\lambda B_i$, $H_{i2}(\tau_k) = \int_{T-\tau_k}^{T} e^{A\lambda} d\lambda B_i$, x_k and u_k are the abbreviations of $x(kT)$ and $u(kT)$, respectively.

Proof: The proof is omitted due to lack of space.

Introducing the augmented state $\chi_k = [x_k^T \ u_{k-1}^T]^T$ yields the closed-loop system as

$$\chi_{k+1} = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(x_k)\theta_j(x_k) \hat{G}_{ij}(\tau_k)\chi_k \quad (5)$$

where

$$\hat{G}_{ij}(\tau_k) = \begin{bmatrix} G_i + H_{i1}(\tau_k)K_j & H_{i2}(\tau_k)L_j \\ K_j & L_j \end{bmatrix}$$

where the time delay τ_k is not specifically determined since it varies with random fashion. To model the time delay phenomena, one possible and suitable way is to let the distribution of the delays be governed by the state of an underlying Markov chain taking values in a finite set $\hat{L} = \{1,2,\ldots,s\}$ with transition probabilities

$$\Pr\{\tau_{k+1} = m | \tau_k = l\} = p_{lm} \quad (6)$$

where $p_{lm} \ge 0$. When the system operates in the lth mode $(\tau_k = l)$, the activated TS fuzzy system is $\chi_{k+1} = \sum_{i=1}^{q} \sum_{j=1}^{q} \hat{G}_{ij}^l \chi_k$, where $\hat{G}_{ij}^l = \hat{G}_{ij}(\tau_k = l)$, and (5) is called the jump TS fuzzy system.

III. Main Result

Definition 1[1,2]: The jump TS fuzzy system is said to be stochastically stable if for all initial mode $\tau_0 \in \hat{L}$, there exists a finite number $\hat{M}(\tau_0) > 0$ such that

$$\lim_{N\to\infty} E\left\{ \sum_{k=0}^{N} x_k^T(\tau_0) x_k(\tau_0) \middle| \tau_0 \right\} < \hat{M}(\tau_0)$$

Remark 1:Definition 1 implies the asymptotic convergence to the origin, in the mean-square sense. In other words, as is obvious from, stochastically stability dictates

$$\lim_{k\to\infty} E\{x^T(\tau_0) x_k(\tau_0) big|\tau_0\} = 0$$

Theorem 2:The jump TS fuzzy system is stochastically stabilizable if there exist $P_l > 0,\ l = 1, 2\ldots, s,$ satisfying the following matrix inequalities

$$\hat{Z}_l = \begin{bmatrix} -\hat{P}_l & * \\ \hat{G}_{ij}^l & -P_l \end{bmatrix} < 0\ i, j = 1, 2, \ldots, q. \qquad (8)$$

where $\hat{P}_l = \sum_{m=1}^{s} p_{lm} P_m$ and star denotes the transposed element.

Proof: Construct the stochastic Lyapunov functional as

$$V_k(\tau_k) = \chi_k^T P_l \chi_k \qquad (9)$$

where $P_l = P_{\tau_k = l}$. Then further computation yields

$$\begin{aligned} & E\{V_{k+1}(\tau_{k+1})|\tau_k = l\} - V_k(\tau_k = l) \\ &= \sum_{m=1}^{s} p(\tau_{k+1} = m|l)(\chi_{k+1}^T P_m \chi_{k+1}) - \chi_k^T P_l \chi_k \\ &= \sum_{m=1}^{s} p_{lm} \chi_{k+1}^T P_m \chi_{k+1} - \chi_k^T P_l \chi_k \\ &= \chi_k^T \left(\left(\sum_{i=1}^{q} \sum_{j=1}^{q} \hat{G}_{ij} \right)^T \hat{P}_l \left(\sum_{i=1}^{q} \sum_{j=1}^{q} \hat{G}_{ij} \right) - P_l \right) \chi_k \\ &\le \chi_k^T \hat{Z}_l \chi_k \\ &< 0 \end{aligned} \qquad (10)$$

Thus we have

$$\begin{aligned} & \frac{E V_{k+1}(\tau_{k+1})|\tau_k - V_k(\tau_k)}{V_k(\tau_k)} \\ &\le -\frac{\chi_k^T(-\hat{Z}_l)\chi_k}{\chi_k^T P_l \chi_k} \\ &\le -\min_{l\in L}\left\{ \frac{\lambda_{\min}(-\hat{Z}_l)}{\lambda_{\max}(P_l)} \right\} \\ &= \alpha - 1 \end{aligned} \qquad (11)$$

where,

$$\alpha = 1 - \min_{l\in L}\left\{ \frac{\lambda_{\min}(-\hat{Z}_l)}{\lambda_{\max}(P_l)} \right\} < 1$$

On the other hand, from (11), we obviously have

$$\alpha \ge \frac{E V_{k+1}(\tau_{k+1})|\tau_k}{V_k(\tau_k)} > 0$$

Hence, one has

$$E\{V_{k+1}(\tau_{k+1})|\tau_k\} \le \alpha V_k(\tau_k) \qquad (12)$$

Taking iterative expectation on both sides of (12), we have

$$E\{V_k(\tau_k)\} \le \alpha^k V(\tau_0) \qquad (13)$$

Further computation gives

$$E\left\{ \sum_{k=0}^{N} V(\tau_k) \middle| \tau_0 \right\} \le \frac{1-\alpha^N}{1-\alpha} V_0(\tau_0) \qquad (14)$$

Thus

$$\lim_{N\to\infty} E\left\{ \sum_{k=0}^{N} V(\tau_k) \middle| \tau_0 \right\} < \frac{V_0(\tau_0)}{1-\alpha} \qquad (15)$$

Using Rayleigh quotient, one get

$$\lim_{N\to\infty} E\left\{ \sum_{k=0}^{N} \chi_k^T \chi_k \middle| \tau_0 \right\} < \hat{M}(\tau_0) \qquad (16)$$

where

$$\hat{M}(\tau_0) = (\min_{l\in L} \lambda_{\min}(P_l))^{-1} \frac{V_0(\tau_0)}{1-\alpha}.$$

Since $\hat{M}$ is bounded and $|x_k| \le |\chi_k|$, (16) implies the stochastically stable.

Corollary 1: If there exist positive definite matrices P_{l1}, P_{l2}, and matrices M_j, L_j satisfying the following coupled LMIs

$$\begin{bmatrix} -P_{l1}^{-1} & * & * \\ 0 & -P_{l2}^{-1} & * \\ G_i P_{l1}^{-1} + H_{l1}^l M_j^l & H_{l2}^l N_j^l & -p_{l1}^{-1} P_{11}^{-1} \\ M_j^l & N_j^l & 0 \\ \vdots & \vdots & \vdots \\ G_i P_{l1}^{-1} + H_{l1}^l M_j^l & H_{l2}^l N_j^l & 0 \\ M_j^l & N_j^l & 0 \end{bmatrix}$$

$$\left.\begin{matrix} * & \cdots & * & * \\ * & \cdots & * & * \\ * & \cdots & * & * \\ -p_{l1}^{-1}P_{12}^{-1} & \cdots & * & * \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & -p_{ls}^{-1}P_{s1}^{-1} & * \\ 0 & \cdots & 0 & -p_{ls}^{-1}P_{s2}^{-1} \end{matrix}\right] < 0$$

where $i, j = 1, 2, \ldots, q$, $l = 1, 2, \ldots, s$, then the jump TS fuzzy system is asymptotically stabilizable.

Proof: Choose the positive definite matrices P_l in (8) as $diag\{P_{l1}, P_{l2}\}$, where $P_{l1} > 0$, $P_{l2} > 0$. From Theorem 2 and by basic calculation, it is easy to obtain (17) from (8).

Remark 2: In implementing the control system, every message sent out by the plant and controller is time-stamped[3]. To calculate the delay accurately, plant and controller clocks must be synchronized.

Ⅳ. Conclusion

This paper has discussed the stabilization problem of the continuous-time TS fuzzy system with randomly time-varying input delay. The input delay was suitably modelled via Markov process with finite states. The continuous-time TS fuzzy system was discretized for easy handling of the input delay, which results in the discrete-time TS fuzzy systems with Markovian jump parameters. It has been shown that the given problems can be solved if a set of coupled LMIs has a solution.

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Ⅴ. 참고문헌

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