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헬리콥터 시스템의 퍼지 분산 제어기 설계

A Decentralized Fuzzy Controller for Experimental Nonlinear Helicopter Systems

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ABSTRACT

This paper proposes a decentralized control technique for 2-dimensional experimental helicopter systems. The decentralized control technique is especially suitable in large-scale control systems. We derive the stabilization condition for the interconennected Takagi-Sugeno (TS) fuzzy system using the rigorous tool - Lyapunov stability criterion and formulate the controller design condition in terms of linear matrix inequality (LMI). To demonstrate the feasibility of the proposed method, we include the experiment result as well as a computer simulation one, which strongly convinces us the applicability to the industry.

Keywords : fuzzy control, decentralized control, linear matrix inequality, experimental helicopter system

1. Introduction

Most plants in the industry have severe nonlinearity and uncertainties. They thus post additional difficulties to the control theory of general nonlinear systems and the design of their controllers. In order to overcome this kind of difficulties in the design of a controller for an uncertain nonlinear system, various schemes have been developed in the last two decades, among which a successful approach is fuzzy control. Recently, fuzzy control has attracted increasing attention, essentially because it can provide an effective solution to the control of plants that are complex, uncertain, ill-defined, and have available qualitative knowledge from domain experts for their controllers design. There have been many successful applications in the industry to date. In spite of the usefulness of fuzzy control, its main drawback comes from the lack of a systematic control design methodology. To resolve these problems, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the innovative Takagi-Sugeno (TS) fuzzy systems, which becomes quite popular today[2-8]

On the other hand, although there have been successful applications to real industry, we have wittiness that the conventional control technique is hard to apply to large-scale nonlinear systems, such as power systems spread over distant geographic areas [1]. When a conventional centralized technique is applied for control, the complexity of the analysis grows rapidly as the order of the system increases. This situation motivates us to look for ways to simplify the analysis. If the large-scale system can be viewed as an interconnection of subsystems some of which are strongly connected, while others being weakly connected, we can deals with the simple subsystems with perturbations only.

Motivated by the above observations, this paper discusses a decentralized control technique for a large-scale systems representable by several sub-TS fuzzy systems. The stabilizing controller design condition is derived based on the Lyapunov stability criterion and formulated in terms of linear matrix inequalities (LMIs). An application to the experimental helicopter system is also included to show the effectiveness of the proposed method.

2. Interconnected TS Fuzzy Systems

Consider a class of interconnected nonlinear dynamical system of the following form:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{r} h_i(u_j(t)) \quad (1)$$

where $i=1,2,\cdots,r$, $x_i(t) \in R^n$ is the state vector, $u_i(t) \in R^m$ is the control input vector in the ith subsystem. The term, $h_i(u_j(t))$ represents interactions among the nonlinear subsystems. The central spirit of the TS fuzzy inference system is quantified by 'IF-THEN' rule base by virtue of the available qualitative knowledge from domain experts. More precisely, the k th rule of the ith sub-TS fuzzy system is formulated in the following form:

IF $z_{i1}(t)$ *is about* Γ_1^{ik} and ⋯ and $z_{in}(t)$ *is about* Γ_n^{ik} (2)
THEN $\dot{x}_i = A_{ik}x_i(t)B_{ik}u_i(t)$

where R_i^k denotes the kth fuzzy inference rule, $z_k(t)$ is the premise variable, Γ_l^{ik}, $k=1,\cdots,q$, $l=1,\cdots,n$, is the fuzzy set of the lth premise variable in the kth fuzzy inference rule.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this TS fuzzy system(2) is described by

$$\dot{x}_i(t) = \sum_{\substack{j=1 \\ j\neq i}}^{r} \sum_{k=1}^{q} \alpha_k(z_i(t))(A_{ik}x_i(t) + B_{ik}u_k(t)) + h_{ij}(u_j(t)) \quad (3)$$

Assumption 1 The term $h_{ij}(u_j(t))$, which represents the interconnections from ith TS fuzzy system to j th one, is assumed to satisfy the inequality

$$|h_{ij}(u_j(t))| \le \gamma_{ij}|u_j(t)| \quad (4)$$

3. Decentralized Controller Design

A decentralized fuzzy-model-based control law is adopted for the stabilization of the interconnected TS fuzzy system of the form:

$$u_i(t) = \sum_{k=1}^{q} \alpha(z_i(t))K_k x_i(t) \quad (4)$$

The closed-loop system of (3) with (4) is describe by

$$\begin{aligned}\dot{x}_i(t) &= \sum_{\substack{j=1 \\ j\neq i}}^{r} \sum_{k=1}^{q} \sum_{l=1}^{q} \alpha_k(z_i(t))\alpha_l(z_i(t))(A_{ik}+B_{ik}K_l)x_i(t) + h_{ij}(u_j(t)) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{r} \sum_{k=1}^{q} \alpha_k(z_i(t))(A_{ik}+B_{ik}K_k)x_i(t) \\ &\quad + 2\sum_{\substack{j=1 \\ j\neq i}}^{r} \sum_{k<l}^{q} \alpha_k(z_i(t))\alpha_l(z_i(t)) \times \left(\frac{A_{ik}+B_{ik}K_l+A_{il}+B_{il}K_k}{2}\right)x_i(t) + h_{ij}(u_j(t))\end{aligned}$$

Theorem 1 *If there exist symmetric and positive definite matrices* P_i *, some matrices* K_{ik} *, and some positive scalars* ε_{ikl}, $i,j=1,\cdots,r.$, k, $l=1,\cdots,q.$, *such that the following LMIs are satisfied, then the interconnected TS fuzzy system (3) is globally asymptotically stabilizable in the sense of Lyapunov, by employing the decentralized TS fuzzy-model-based state-feedback controller(4)*

a) $$\begin{bmatrix} \left(\begin{array}{c} W_iA_{ik}^T + A_{ik}W_i \\ +M^{T_{ik}}B^{T_{ik}} + B_{ij}M_{ik} \\ +\varepsilon_{ikk}\sum_{\substack{j=1 \\ j\neq i}}^{r}\gamma_{ij}^2 I \end{array}\right) & * \\ W_i & -\varepsilon_{ikl}I \end{bmatrix} < 0$$

$k=1,\cdots,q-1.$,

b) $$\begin{bmatrix} \left(\begin{array}{c} W_iA_{ik}^T + A_{ik}W_i \\ +M^{T_{il}}B^{T_{ik}} + B_{ik}M_{il} \\ +W_iA_{il}^T + A_{il}W_i \\ +M^{T_{il}}B^{T_{ik}} + B_{il}M_{ik} \\ +\varepsilon_{ikl}\sum_{\substack{j=1 \\ j\neq i}}^{r}\gamma_{ij}^2 I \end{array}\right) & * \\ W_i & -\varepsilon_{ikl}I \end{bmatrix} < 0$$

$k=1,\cdots,q-1.$, $l=k+1,\cdots,q.$

where $M_{ij} = K_{ij}P_i^{-1}$, *and* $*$ *denotes the transposed elements in the symmetric positions.*

Proof: Proof is omitted due to lack of space.

4. Experimental Helicopter System and Its Fuzzy Modelling

Consider the following nonlinear dynamic equations

$$\begin{aligned} J_p\ddot{p}(t) + B_p\dot{p}(t) &= R_pF_p(V_{p(t)}) - M_eg(h\sin(p(t)) + R_c\cos(p(t))) + G_p(\tau_y(V_{y(t)}), p(t)) \\ J_y\ddot{y}(t) + B_y\dot{y}(t) &= R_yF_y(V_{y(t)}) + G_y(\tau_p(V_{p(t)})) \end{aligned} \quad (6)$$

which describe the dynamic behavior for a

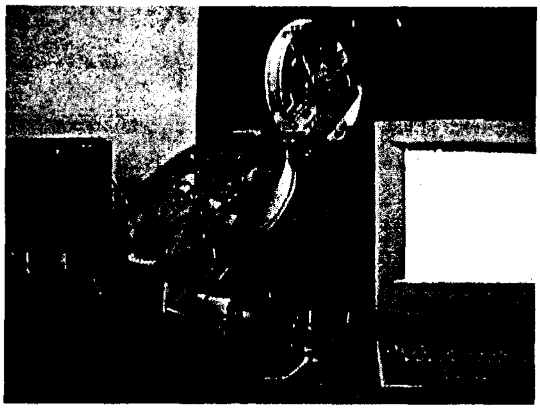

Figure 1 Experimental 2 dimensional helicopter system.

2-dimensional helicopter system. The experimental system is shown in Fig. 1. This nonlinear system can be viewed as smaller isolated subsystems and their interconnections, thus the decentralized control technique can successfully applied. Let the state vectors and input vectors for subsystems be $x_1(t)=\left[p(t)\ \ \dot{p}(t)\ \ 0.2\int_0^t p(\tau)d\tau\right]$ $x_2(t)=\left[y(t)\ \ \dot{y}(t)\ \ 0.2\int_0^t y(\tau)d\tau\right]^T$ and $u_1(t)=[V_{p(t)}]$, $u_2(t)=[V_y(t)]$, then (6) can be represented the interconnected systems

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(t) + h_i(u_j(t)) \tag{7}$$

where $i, j = 1, 2., i \neq j$.

$$f_1(x_1(t)) = \begin{bmatrix} x_{12}(t) \\ -\frac{B_p}{J_p}x_{12}(t) - \frac{M_e g(h\sin(x_{11}(t)) + R_c\cos(x_{11}(t)))}{J_p} \\ 0.2x_{11}(t) \end{bmatrix}$$

$$g_1(x_1(t)) = \begin{bmatrix} 0 \\ \frac{R_p K_{pp} g}{J_p} \\ 0 \end{bmatrix} \quad h_1(u_2(t)) = \begin{bmatrix} 0 \\ -\frac{K_{py}}{J_p}u_2(t) \\ 0 \end{bmatrix}$$

$$f_2(x_2) = \begin{bmatrix} x_{22}(t) \\ -\frac{B_y}{J_y}x_{22}(t) \\ 0.2x_{21}(t) \end{bmatrix} \quad g_2(x_2(t)) = \begin{bmatrix} 0 \\ \frac{R_y K_{yy} g}{J_y} \\ 0 \end{bmatrix}$$

$$h_2(u_1(t)) = \begin{bmatrix} 0 \\ -\frac{K_{yp}}{J_y}u_1(t) \\ 0 \end{bmatrix}$$

The composite system has one nonlinear term in subsystem $1 - \frac{M_e g(h\sin(x_{11}(t)) - R_c\cos(x_{11}(t)))}{J_p}$. Thus Subsystem 1 can be represented as a TS fuzzy system and Subsystem 2 as a linear system.

Subsystem 1

Plant Rules:

R1 : IF $x_{11}(t)$ is about Γ_1,

THEN $\dot{x}_1(t) = A_{11}x_1(t) + B_{11}u_1(t) + d_{11} + h_1(u_2(t))$

R2 : IF $x_{11}(t)$ is about Γ_2,

THEN $\dot{x}_1(t) = A_{12}x_1(t) + B_{12}u_1(t) + d_{12} + h_1(u_2(t))$

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mga}{J_p} & -\frac{B_p}{J_p} & 0 \\ 0.2 & 0 & 0 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mgb}{J_p} & -\frac{B_p}{J_p} & 0 \\ 0.2 & 0 & 0 \end{bmatrix}$$

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ \frac{R_p K_{pp} g}{J_p} \\ 0 \end{bmatrix} \quad d_{11} = \begin{bmatrix} 0 \\ -\frac{Mga\beta}{J_p} \\ 0 \end{bmatrix}$$

$$d_{12} = \begin{bmatrix} 0 \\ -\frac{Mgb\beta}{J_p} \\ 0 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B_y}{J_y} & 0 \\ 0.2 & 0 & 0 \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 0 \\ \frac{R_y K_{yy} g}{J_y} \\ 0 \end{bmatrix}$$

and the membership functions for Subsystem 1 are

$$\Gamma_1(x_1(t)) = \frac{a\sin(x_{11}(t)+\beta) - b(x_{11}(t)+\beta)}{(a-b)(x_{11}(t)+\beta)}$$

$$\Gamma_2(x_1(t)) = \frac{a(x_{11}(t)+\beta) - a\sin(x_{11}(t)+\beta)}{(a-b)(x_{11}(t)+\beta)}$$

5. Simulation and Experiment

This section shows a design procedure of the proposed decentralized controller and its experimental validation. In order to avoid the failure of the DC motors, the input voltage limitations is applied, i.e.,

$|V_p(t)| \le 2.6, |V_y(t)| \le 1.$

Based on Theorem 1, we obtain the following control gain matrices.

$K_{11} = [-1.8748\ \ -2.3129\ \ -11.0607]$

$K_{12} = [-6.2136\ \ -3.5934\ \ -16.4346]$

$K_{21} = [-5.8513\ \ -3.3977\ \ -6.1159]$

The initial value of the system is set to $x_1(0) = [0\ 0\ 0]^T, x_2(0) = [0\ 0]^T$, and the simulation time is 20 sec.. Figure 2 shows the computer simulation of the 2 dimensional helicopter system. The pitch and yaw angles are well guided to zero immediately. The simulation result shows the effectiveness of

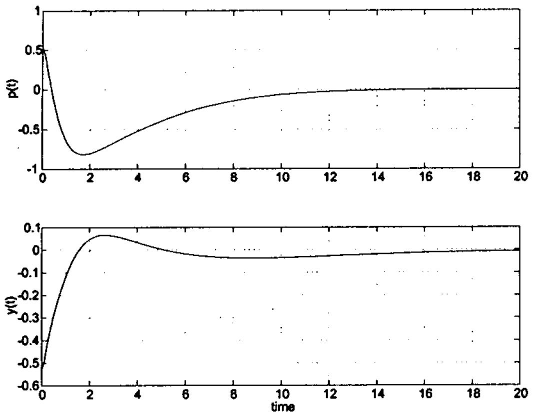

Figure 2 Simulation result of the control of the 2 dimensional helicopter system

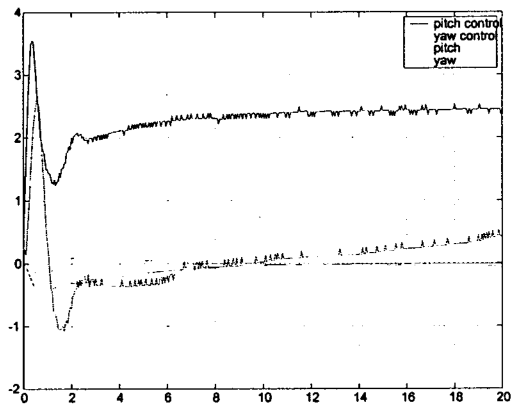

Figure 3 Experiment result of the control of the 2 dimensional helicopter system.

the proposed control technique. The experiment of the real apparatus is also carried out. The control program is made by using Microsoft Visual Studio. The experiment time is also 20 sec., and the result is shown in Fig. 3. As is expected from the simulation result, the pitch and yaw angles are directly go to origin.

5. Conclusion

This paper has discussed a decentralized control method for an interconnected TS fuzzy systems. The proposed control strategy is suitable for interconnected large-scale nonlinear systems. The stabilizing controller design condition is derived based on the Lyapunov stability criterion and formulated in terms of linear matrix inequalities (LMIs). The experimental results have successfully validated the theoretical discussion.

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6. 참고문헌

[1] M. Akar and Uozguner, "Decentralized techniques for the analysis and control of Takagi-Sugeno fuzzy systems," IEEE Trans. on Fuzzy Systems, Vol. 8, No. 6, pp. 691-704, Dec., 2000.

[2] H. J. Lee, J. B. Park and G. Chen, "Robust fuzzy control of nonlinear systems with parametric uncertainties," IEEE Trans. on Fuzzy Systems, Vol. 9, No. 2, pp. 369-379, April, 2001.

[3] W. Chang, J. B. Park, Y. H. Joo, and G. Chen, "Design of robust fuzzy-model-based controller with sliding mode control for SISO nonlinear systems," Fuzzy Sets and Systems, Vol. 125, No. 1, pp. 1-22, 2001.

[4] Y. H. Joo, G. Chen, and L. S. Shieh, "Hybrid state-space fuzzy model-based controller with dual-rate sampling for digital control of chaotic systems,'' IEEE Trans. on Fuzzy Systems, vol. 7, pp. 394-408, Aug, 1999.

[5] W. Chang, J. B. Park, Y. H. Joo, and G. Chen, "Design of sampled-data fuzzy-model-based control systems by using intelligent digital redesign," IEEE Trans. on Circ. and Syst.-1, to appear, 2001.

[6] Z. Li, J. B. Park, and Y. H. Joo "Chaotifying continuous-time TS fuzzy systems via discretization," IEEE Trans. on Circ. and Syst.-1, Vol. 48, No. 10, pp. 1237-1243, 2001.

[7] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," IEEE Trans. on Fuzzy Systems, Vol. 6, No. 2, pp. 250-265, 1998.

[8] G. Feng, "Approaches to quadratic stabilization of uncertain fuzzy dynamic systems," IEEE Trans. on Circ. and Syst.-1, Vol. 48, No. 6, pp. 760-779, 2001.