

# System and Disturbance Identification for Model-Based Learning and Repetitive Control\*

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**Abstract** An extension of interaction matrix formulation to the problem of system and disturbance identification for a plant that is corrupted by both process and output disturbances is presented. With only an assumed upper bound on the order of the system and an assumed upper bound on the number of disturbance frequencies, it is shown that both the disturbance-free model and disturbance effect can be recovered exactly from disturbance-corrupted input-output data without direct measurement of the periodic disturbances. The rich information returned by the identification can be used by a performance-oriented model-based learning or repetitive control system to eliminate unwanted periodic disturbances.

## 1. Introduction

Iterative learning control and repetitive control improve the tracking error of a repetitive process by compensating for unwanted periodic disturbances that are present in a repetitive process [1]-[3]. Since there are two independent variables (repetition and time), learning and repetitive control can be viewed from the perspective of 2-D system theory [4]. At one end of the spectrum are learning and repetitive controllers that can guarantee convergence to zero tracking error without requiring little knowledge of the plant and the disturbances. Such a general approach, although attractive in theory, may have limited applications in practice because these model-independent controllers may exhibit unacceptable learning behavior while converging to zero tracking

error. At the other end of the spectrum are performance-oriented model-based controllers. These controllers require knowledge of the plant and possibly of the disturbance environment in their design. One does not expect that such information can be derived accurately from analytical modeling alone, especially when the disturbances may not be predicted accurately before hand. Consequently, system identification under one form or another is used to provide the needed information. System identification has a unique advantage that the identified model reflects the true dynamics of the system under consideration. For example, an experimentally identified model naturally incorporates actuator or sensor dynamics that may be left out in an analytical model. Thus in the context of designing model-based learning or repetitive

controllers, one asks the following question: To what extent the system disturbance-free dynamics and the disturbance environment can be identified from input-output data that are corrupted by unknown periodic disturbances? It is clear that a comprehensive answer to this question will significantly contribute towards making learning and repetitive control attractive in the real world.

Such system and disturbance identification has been addressed in [5]–[7] in the context of "clear-box" adaptive control. In the vibration control of flexible spacecraft, it can easily be too demanding to ask for zero tracking error. When the actuator is near a node of a specific vibration mode, the amount of control energy needed to eliminate disturbance effects that excite the mode can be so large that it saturates the actuator and cripples the performance of the controller. The term "clear-box" distinguishes the approach from a black-box approach in that "clear-box" brings out useful information for the control problem that is normally hidden or left unused by a typical black-box approach. Using only disturbance-corrupted data (without direct measurement of the disturbances themselves) the method extracts the system disturbance-free dynamics and the disturbance effect. It develops information that describes how serious the disturbances are for each frequency, and simultaneously determines what part of the control budget would be required to cancel each frequency. This strategy not only avoids preventable failures in difficult problems but also allows efficient use of limited control resources. The Clear-Box

algorithm has been successfully demonstrated on a flexible structure at Princeton [8] and on an Ultra Quiet Platform at the Naval Postgraduate School [9].

Central to the clear-box system and disturbance identification method is the derivation of an equation that relates the excitation or control input to the disturbance-corrupted output. Through a so-called "interaction matrix", the unknown disturbance inputs are eliminated from this input-output model. This matrix describes how the coefficients of the identified model become corrupted by the unknown disturbances. Through this interaction matrix both system disturbance effect can be correctly recovered.

The derivation in [5]–[7] is for a linear system with state disturbances and without a direct transmission term in the output equation. In practice, a direct transmission term may be present. For example in the case of an active engine mount where the objective is to minimize the force transmission from an unbalanced engine to the vehicle body, the presence of a direct transmission term in the output equation is automatic. Road disturbances also appear directly at the output equation. In these cases, it is not immediately clear how the interaction matrix, which operates on the state equation as derived in [5]–[7], can be used to eliminate the output disturbances as well. It is the objective of this paper to provide the necessary extension of the interaction matrix method to allow for these possibilities. The identification results

presented here can then be used in the design of smart repetitive and learning controllers to eliminate unwanted periodic disturbances.

## 2. Problem Statement

Consider an  $n$ -th order,  $r$ -input,  $q$ -output discrete-time system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d d(k) \\ y(k) &= Cx(k) + Du(k) + Hv(k) \end{aligned} \quad (1)$$

where  $d(k)$  and  $v(k)$  are unknown periodic plant and output disturbances, respectively. One set of sufficiently rich data consisting of excitation input  $u(k)$  and disturbance-corrupted output  $y(k)$  is given. We wish to recover the following information:

- the disturbance-free state-space model  $A, B, C, D$ ,
- the disturbance frequencies,
- the disturbance contribution on the output data (total and from each frequency),
- the disturbance-rejection control signal (total and from each frequency).

Other than the given set of disturbance-corrupted data, nothing else is known about the system, except an upper bound on the order of the system and an upper bound on the number of disturbance frequencies can be assumed. Furthermore, the disturbance frequencies may coincide with any number of the system flexible modes.

## 3. Disturbance-Corrupted Input-Output Model

In the following we derive an input-output equation that relates the excitation input data to disturbance-corrupted output data with all disturbance input terms absent. By repeated substitution, we have

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{I}_p u_p(k) + \mathcal{I}_d d_p(k) \\ y_p(k) &= \mathcal{O} x(k) + \tau u_p(k) + \tau_d d_p(k) + \tau_v v_p(k) \end{aligned} \quad (2)$$

where

$$u_p(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+p-1) \end{bmatrix}, \quad y_p(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+p-1) \end{bmatrix}$$

$$d_p(k) = \begin{bmatrix} d(k) \\ d(k+1) \\ \vdots \\ d(k+p-1) \end{bmatrix}, \quad v_p(k) = \begin{bmatrix} v(k) \\ v(k+1) \\ \vdots \\ v(k+p-1) \end{bmatrix}$$

$$\mathcal{I} = [A^{p-1}B, \dots, AB, B],$$

$$\mathcal{I}_d = [A^{p-1}B_d, \dots, AB_d, B_d]$$

$$\tau = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{p-2}B & \dots & CB & D \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-2} \\ CA^{p-1} \end{bmatrix}$$

$$\tau_d = \begin{bmatrix} 0 & 0 & \dots & 0 \\ CB_d & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{p-2}B_d & \dots & CB_d & 0 \end{bmatrix}$$

$$\tau_v = \begin{bmatrix} H & 0 & \dots & 0 \\ 0 & H & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & H \end{bmatrix}$$

The interaction matrix  $M$  is introduced by adding and subtracting the product  $M y_p(k)$  to the right hand side of (2) to produce

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{I}_p u_p(k) + \mathcal{I}_d d_p(k) \\ &\quad + M[\mathcal{O} x(k) + \tau u_p(k) + \tau_d d_p(k) + \tau_v v_p(k)] \\ &\quad - M y_p(k) \\ &= [A^p + M\mathcal{O}]x(k) + [\mathcal{I} + M\tau]u_p(k) \\ &\quad + [\mathcal{I}_d + M\tau_d]d_p(k) + MT_v v_p(k) - M y_p(k) \end{aligned}$$

We do not need to determine the interaction matrix  $M$  but only concern with  $M$  its existence at the moment. With  $m$  introduced, the output equation becomes

$$\begin{aligned} y(k+p) = & [CA^p + CM\theta]x(k) + [C_d + CM\tau]v_p(k) \\ & + [C_d + CM\tau_d]d_p(k) + CM\tau_v v_p(k) \\ & - CM y_p(k) + Du(k) + Hv(k+p) \end{aligned} \quad (3)$$

We now impose the conditions for  $M$ , or more precisely, for the product  $CM$  such that explicit dependence of the disturbance-corrupted output  $y(k)$  on the state  $x(k)$  and unknown plant and output disturbance inputs  $d(k)$  and  $v(k)$  is eliminated for all  $k$ , i.e.,

$$\begin{aligned} CA^p + CM\theta &= 0 \\ [C_d + CM\tau_d]d_p(k) + CM\tau_v v_p(k) + Hv(k+p) &= 0 \end{aligned} \quad (4)$$

Let us examine the second equation in (4) in more details. It can be re-written as

$$[(C_d + CM\tau_d), CM\tau_v, H]D_f = 0$$

where  $D_f$  is a matrix of time-shifted state and output disturbance time histories,

$$D_f = \begin{bmatrix} d_p(0) & d_p(1) & \dots \\ v_p(0) & v_p(1) & \dots \\ v(p) & v(p+1) & \dots \end{bmatrix}$$

Since we are dealing with periodic disturbances, the rank of  $D_f$  is limited by the number disturbance frequencies present in the data. Specifically, if the number of distinct disturbance frequencies is  $f$  then the rank of  $D_f$  is  $2f+1$  where the 1 accounts for any possible constant bias in the disturbances. Let  $D$  denote a matrix formed by  $2f$  or  $2f+1$  linearly independent columns of  $D_f$ . The equation that  $CM$  must satisfy is

$$CM[\theta, \tau_d D_1 + \tau_v D_2] = -[CA^p, CC_d D_1 + HD_3] \quad (5)$$

where  $D_1, D_2, D_3$  are the three row partitions of  $D$  corresponding to the combinations  $C_d + CM\tau_d$ ,  $CM\tau_v$  and  $H$  respectively. Since (5) is a set of linear equations, the existence of  $CM$  is guaranteed as long as the matrix  $[\theta, \tau_d D_1 + \tau_v D_2]$  is full rank (which is generally the case), and the number of unknowns in the elements of the product  $CM$  is at least equal to the number of equations. Counting the number of equations and unknowns, the condition on  $p$  can be easily shown to be

$$pq \geq n + 2f + 1 \quad (6)$$

Thus from an assumed upper bound on the order of the system and an assumed upper bound on the number of distinct disturbance frequencies,  $p$  can be chosen such that the above condition holds. As long as  $CM$  exists that satisfies (5), Eq. (3) becomes

$$y(k+p) = [(C_d + CM\tau), D] \begin{bmatrix} u_p(k) \\ u(k+p) \end{bmatrix} - CM y_p(k) \quad (7)$$

Equation (7) is an input-output equation that relates excitation input data to disturbance-corrupted data. Information about the disturbance is embedded partly in the product  $CM$ . Given a set of sufficiently rich input excitation and disturbance-corrupted output, the parameter combinations  $D$ ,  $-CM$ , and  $C_d + CM\tau$  can be easily identified because (7) is simply a set of linear equations with these parameter combinations as coefficients.

#### 4. Recovery of Disturbance-Free State Model

We are interested in recovering the disturbance-free model but the coefficients identified from (7) can be thought of as disturbance-corrupted. In the following we show that it is possible to recover the disturbance-free state-space model from the disturbance-corrupted coefficients without actually knowing the disturbances themselves. This is a two-step process. We first recover the system Markov parameters from the identified coefficients, then the state-space model is factorized from the recovered Markov parameters. The second step is straightforward and can be easily handled by any realization procedure such as the one described in [7] or [10]. In this paper only the first step is shown. Let the identified parameter combinations  $-CM$ ,  $C_k^+ + CM\tau$  be partitioned as follows,

$$\begin{aligned} [\alpha_p, \alpha_{p-1}, \dots, \alpha_1] &= -CM \\ [\beta_p, \beta_{p-1}, \dots, \beta_1] &= C_k^+ + CM\tau, \quad \beta_0 = D \end{aligned} \quad (8)$$

The first  $p$  Markov parameters can be recovered as follows,

$$\begin{aligned} D &= \beta_0 \\ CB &= \beta_1 + \alpha_1 D \\ CAB &= \beta_2 + \alpha_2 D + \alpha_1 CB \\ &\vdots \\ CA^{p-1}B &= \beta_p + \alpha_p D + \alpha_{p-1}CB + \dots + \alpha_1 CA^{p-2}B \end{aligned} \quad (9)$$

To recover the additional Markov parameters, we make use of the condition  $CA^p + CM\theta = 0$  by postmultiplying it by  $B$ , then  $AB$ , etc. so that

$$CA^p B = \alpha_1 CA^{p-1}B + \dots + \alpha_p CB$$

$$CA^{p+1}B = \alpha_1 CA^p B + \dots + \alpha_p CAB \quad (10)$$

Any additional Markov parameters can be recovered in the same manner. Once a sufficient number of Markov parameters is obtained, a realization of system state-space model  $A, B, C$  can be found by any realization procedure [7], [10]

#### 5. Recovery of Disturbance Contribution

Once the system disturbance-free model is recovered, it is relatively straightforward to recover the disturbance contribution on the output data : One can create a model of the form, [5], [7]

$$\begin{aligned} y(k) &= \bar{a}_1 y(k-1) + \dots + \bar{a}_p y(k-p) + \bar{\beta}_0 u(k) \\ &\quad + \bar{\beta}_1 u(k-1) + \dots + \bar{\beta}_p u(k-p) + \eta(k) \end{aligned} \quad (11)$$

where the coefficients  $\bar{a}_i, \bar{\beta}_i$  can be found from the now known disturbance-free state-space model as follows. Let  $\bar{A}, \bar{B}, \bar{C}$  denote a realization of  $A, B, C$  from the recovered Markov parameters. The coefficients  $\bar{a}_i, \bar{\beta}_i$  are then given as

$$[\bar{a}_p, \bar{a}_{p-1}, \dots, \bar{a}_1] = -\bar{C}\bar{M} \quad (12)$$

$$[\bar{\beta}_p, \bar{\beta}_{p-1}, \dots, \bar{\beta}_1] = \bar{C}(\bar{C} + \bar{M}\tau), \quad \bar{\beta}_0 = \bar{D}$$

where the product  $\bar{C}\bar{M}$  satisfies

$$\bar{C}\bar{A}^p + \bar{C}\bar{M}\theta = 0$$

so that only explicit dependence on the system state is eliminated. For example,  $\bar{C}\bar{M}$  can be found from

$$\bar{C}\bar{M} = -\bar{C}\bar{A}^p \theta^+ \quad (13)$$

where  $+$  denotes the pseudo-inverse operation through the singular value decomposition. Such an  $\bar{C}\bar{M}$  does not eliminate the dependence on the plant and output disturbances, which is the intention here. All the disturbance contribution is now grouped in a term called disturbance effect, denoted by  $\eta(k)$ ,

$$\eta(k) = [C\bar{\tau}_d + CM\tau_d]d_p(k-p) + CM\tau_v v_p(k-p) + Hv(k) \quad (14)$$

Although the right hand side of (14) is not known, the disturbance effect  $\eta(k)$  can still be computed from

$$\eta(k) = y(k) - \bar{\alpha}_1 y(k-1) - \dots - \bar{\alpha}_p y(k-p) - \bar{\beta}_0 u(k) - \bar{\beta}_1 u(k-1) - \dots - \bar{\beta}_p u(k-p) \quad (15)$$

Once  $\eta(k)$  is known, the disturbance contribution on the output, denoted by  $y_d(k)$ , can be solved from

$$y_d(k) = \bar{\alpha}_1 y_d(k-1) + \dots + \bar{\alpha}_p y_d(k-p) + \eta(k) \quad (16)$$

## 6. Computation of Disturbance-Rejection Control

The needed feedforward control signal to cancel the disturbance present in the data, denoted by  $u_f(k)$  can be found from

$$\bar{\beta}_0 u_f(k) + \bar{\beta}_1 u_f(k-1) + \dots + \bar{\beta}_p u_f(k-p) = -\eta(k) \quad (17)$$

Up to this point we have dealt with only one set of disturbance-corrupted input-output data. Hence  $y_d(k)$  is the portion of the given output data corrupted by the disturbances and

$u_f(k)$  is the control signal that one could use (but did not use) to cancel the effect of  $y_d(k)$ .

Thus the emphasis so far has been on the identification and after-the-fact analysis problem. It should be noted here, however, that the same equations can be used for on-line disturbance identification and rejection as well.

The readers are referred to [5]-[7] for more details.

## 7. Partial Disturbance Models

As derived here, the disturbance effect  $\eta(k)$  includes all disturbance frequencies present in the data.

It is also possible to use the interaction matrix formulation to create partial disturbance models where  $\eta(k)$  contains information of one or more interested disturbance frequencies, and information about the rest of the disturbance frequencies remains embedded in the coefficients  $\alpha_i$ ,  $\beta_i$ .

Such partial disturbance models are particularly useful in the computation of the contribution of each disturbance frequency on the system output and the corresponding disturbance-rejection control signal needed. This information is then used in a smart disturbance rejection system where only disturbance frequencies that are deemed worthwhile to cancel are cancelled in view of limited control resources. In such cases, partial disturbance models can be used for partial disturbance cancellation. The readers are referred to [5] for more details as to how this

can be accomplished.

## 8. Conclusions

This paper presents a formal extension of the interaction matrix formulation to the problem of system and disturbance identification for a plant that has a direct transmission term and is corrupted by both process and output disturbances. We have shown that under rather mild assumptions, both the disturbance-free model and disturbance effect can be recovered exactly from disturbance-corrupted data. The identification results can be used to design a smart model-based learning or repetitive control system to eliminate unwanted periodic disturbances.

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