

# Adaptive Algorithm with Time-Varying Step-Size Using Orthogonality Principles

Jung-Hoon Park, Jang Sik Park\*, Kyung Sik Son

Dept. Electronics Eng, Pusan National University

\* Dept. Visual Tech., Dongeui Institute of Technology

email: jsipark@dit.ac.kr

## Abstract

Adaptive signal processing is used to acoustic echo canceller, adaptive noise canceller and adaptive algorithm among adaptive algorithms is mainly used because the structure is simple and computation. LMS algorithm has trade-off between the converge speed and the steady state error. In this paper, step-size of adaptive algorithm is varied with orthogonality principles of optimal filter to get fast though small steady state error. Time varying step-size is determined proportional to the maximum vector of LMS algorithm. As results of simulations, the adaptive algorithm with proposed time-v compared with conventional ones.

## I. Introduction

Time varying adaptive signal processing followed by signals path or change of input signal is used acoustic echo canceller, adaptive noise canceller, adaptive equalizer, and etc. Especially, adaptive signal processing can be effectively adapted acoustic echo canceller used tele-conference system because primary input signal and reference input signal can be separated clearly. Not only tele-conference system and hands-free system but also preprocessing of speech recognition require acoustic echo canceller.

Practically, NLMS(normalized least mean square) algorithm transposed from LMS algorithm is used. Though convergence speed is slow to colored signal, NLMS algorithm has small computation because of the simple structure. Steady state error and convergence speed is determined by adaptive constant. However, there is a trade-off between the steady state error and the convergence speed of NLMS algorithm. To solve demerit of the one, FRLS(fast recursive least square) or AP(affine

projection) algorithm was proposed. And to solve demerit of the other, adaptive algorithms with variable step-size was proposed.

In this brief, a new time varying step-size is proposed to improve the trade-off between steady state error and convergence speed. Determination of the step-size is basically followed to orthogonality principles. This principles says when adaptive filter converge to optimal filter, then the error signal is uncorrelated with the input signals to the weights. At the beginning of adaptation, step-size maintains large value and converges fastly because reference input signal is included in estimated error signal. When adaptive filter is completely converged, the cross-correlation between input signals and estimated error signal crosses to zero. Therefore excess mean square error is smaller because adaptive constant is small, and misadjustment of the coefficients of adaptive filter caused by perimeter errors. In this brief, cross-correlation vector is estimated by the tangent vector of the LMS algorithm and time-varying adaptive constants is determined on the maximum value

among the cross-correlation vectors which is estimated by considering of computation. Adaptive algorithm with proposed time-varying step-size is compared with conventional ones.

## II. Acoustic Echo Canceller

In speech communication using hands-free set, the speech signal of far-end speaker,  $x(k)$ , which is outputted from speaker, is inputted to mike by the particular sound route and generate echo signal,  $y(k)$ . Echo signal is retransmitted with near-end speaker,  $n(k)$ . This makes unnatural speech communication through the echo route and the delay of the communication system, so speaker will listen the oneself's speech. Therefore acoustic echo should be removed for natural communication. Acoustic echo canceller using adaptive filter method, remove the echo by  $d(k)$ , mike input signal, minus  $\hat{y}(k)$ , echo signal which is estimated from adaptive filter.

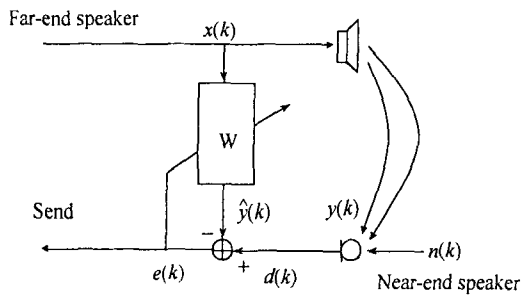


Fig. 1. Acoustic echo canceller using adaptive filtering

Generally NLMS algorithm is used for adaptive algorithm of the acoustic echo canceller because it has stable converge and simple structure. NLMS algorithm is normalization of adaptive constant of the LMS algorithm as the power of input signal. LMS algorithm is used in many field because it is derived from stationary circumstance, though, it adapts well in nonstationary circumstance, too. Like fig. 1, NLMS algorithm, which adapts the

adaptive filter, adapts the coefficients of the adaptive filter using estimate and input signal like equation(1)~(4).

$$d(k) = y(k) + n(k) = \mathbf{W}_o^T(k) \mathbf{X}(k) + n(k) \quad (1)$$

$$e(k) = d(k) - \hat{y}(k) = d(k) - \mathbf{W}^T(k) \mathbf{X}(k) \quad (2)$$

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \mu(k)e(k)\mathbf{X}(k) \quad (3)$$

$$\mu(k) = \frac{\alpha}{\mathbf{X}^T(k)\mathbf{X}(k)} \approx \frac{\alpha}{L\sigma_x^2} \quad (4)$$

$d(k)$  is sampled signal of the summation of acoustic echo and surrounding noises, and also, is main input signal of the adaptive filter.  $n(k)$  is surrounding noise signal including near-end speech signal and noise.  $\mathbf{W}_o(k)$  is optimal weight vector, no other than the coefficients vector of the route of acoustic echo.  $\mathbf{X}(k)$  is input vector and  $\mathbf{W}(k)$  is coefficients vector, respectively.  $[\cdot]^T$  is the transpose of the vector. In equation(4)  $\mu(k)$  is adaptive constant,  $\alpha$  is normalization adaptive constant of the adaptive filter.  $L$  represent the order of the adaptive filter,  $\sigma_x^2$  is the variance of the input signal.  $e(k)$  is the estimated error signal, besides the signal will be transmitted to the other. And, estimated error signal is the residual acoustic echo signal in acoustic echo canceller.

NLMS algorithm converges fast than LMS algorithm because NLMS algorithm normalizes the adaptive constant of the LMS algorithm as the power of the input signal. Coefficient vector of the adaptive filter  $\mathbf{W}(k)$  is applied to minimize  $E[e^2(k)]$  using  $e(k)$  and  $\mathbf{X}(k)$ . As the coefficient vector  $\mathbf{W}(k)$  estimates the optimal coefficient vector  $\mathbf{W}_o(k)$ , the output of the adaptive filter  $\hat{y}(k)$  estimate the echo signal  $y(k)$ . Like equation (2), mike input signal  $d(k)$  minus  $\hat{y}(k)$ , we can separate and transmit the near-end speaker's speech signal  $n(k)$ .

NLMS algorithm has demerit, coefficients of the adaptive filter is misadjusted by near-end speaker in same time communication. In same time communication, these signals is included in estimated error signal, equation (1) substitute to (2)

is

$$\begin{aligned} e(k) &= y(k) + n(k) - \mathbf{W}^T(k) \mathbf{X}(k) \\ &= \mathbf{W}_0^T(k) \mathbf{X}(k) - \mathbf{W}^T(k) \mathbf{X}(k) + n(k) \end{aligned} \quad (5)$$

Equation (5) substitute to (3) is

$$\begin{aligned} \mathbf{W}(k+1) &= \mathbf{W}(k) + \mu(k) (y(k) - \mathbf{W}^T(k) \mathbf{X}(k)) \mathbf{X}(k) \\ &\quad + \mu(k) n(k) \mathbf{X}(k) \end{aligned} \quad (6)$$

The coefficients vector of the adaptive filter  $\mathbf{W}(k)$  is misadjusted by  $n(k)$ , has no correlation. And, near -end speaker's speech signal, which has large energy, mis adjustment is larger. And a good many residual echo exist because it is difficult the adaptive filter estimate echo path by surrounding noises and estimated error signal.

### III. Proposed Time-Varying Step-Size

Adaptive filter converge to the optimal filter, the cross-correlation vector between input signal vector and estimated error signal become  $E[e(n) \mathbf{X}(n)] = 0$ , and this is orthogonality principles. Using orthogonality principles, the cross-correlation between reference input signal vector and estimated error signal is used as the filter's adaptive constant, adaptive constant keep large value and converge fast because there are many elements of the reference input signal in estimated error signal. When adaptive filter converge, the cross-correlation between reference input signal vector and estimated error signal is nearly 0. Therefore adaptive constant, and the coefficient misadjustment is smaller, and excess mean square error is smaller, too.

The cross-correlation between reference input signal vector and estimated error signal  $E[e(k) \mathbf{X}(k)]$  is inferred as instantaneous gradient vector like equation(7).

$$\delta_n(k) = \beta \delta_n(k-1) + (1-\beta) x_n(k) e(k) \quad (7)$$

$$n = 0, 1, \dots, L-1$$

$\delta_n(k)$  is running power estimate which infer the cross-correlation between reference input signal vector and estimated error signal from instantaneous gradient vector.  $\beta$  is forgetting factor,  $x_n(k)$  is input signal at  $n$  tap, none other than  $x(k-n)$ .  $L$  is order of the adaptive filter. Equation is (8)

$$\begin{aligned} \delta_n(k) &= \beta \delta_n(k-1) + (1-\beta) x_n(k) e(k) \\ &= \beta^k \delta_n(0) + (1-\beta) \sum_{i=0}^k \beta^{k-i} x_n(i) e(i) \end{aligned} \quad (8)$$

Assume  $\delta_n(0) = 0$  and adjust the past estimated cross-correlation from 0 to  $k_0$  sample, the latest estimated cross-correlation from  $k_0+1$  to  $k$ , then

$$\begin{aligned} \delta_n(k) &= \beta^{k-k_0} (1-\beta) \sum_{i=0}^{k_0} \beta^{k_0-i} x_n(i) e(i) \\ &\quad + (1-\beta) \sum_{i=k_0+1}^k \beta^{k-i} x_n(i) e(i) \end{aligned} \quad (9)$$

$k-k_0$  is large enough, the first part of equation (9) is almost decreased, the second part remain mostly. Therefore, estimate as instantaneous gradient of LMS algorithm, the latest cross-correlation can be reflected. The more  $\beta$  is approach to 1,  $k-k_0$  should have the larger value.

Equation (7) is normalized as equation (10) to cross-correlation is smaller than 1 by summation of the reference input signal and estimated error signal.

$$\delta_n(k) = \beta \delta_n(k-1) + (1-\beta) \frac{x_n(k) e(k)}{\sigma_x^2(k) + \sigma_e^2(k)} \quad (10)$$

Fig 2 represents the value in first tap among the instantaneous vector which estimated the cross-correlation between reference input signal vector and estimated error signal. When the cross-correlation vector is decreased by orthogonality principles, and became steady-state, we can see the cross-correlation vector is small

considerably than the beginning of the converge. Like fig 2, the cross-correlation or adaptive constant converge so fast, the converge speed in the beginning adaptation is slow, and adaptive filter cannot infer the echo path, exactly. And each adaptive constant, like equation(7), is used adapt at all coefficients of the adaptive filter, the computation is proportion to the filter's order, the multi- plication increase 3L.

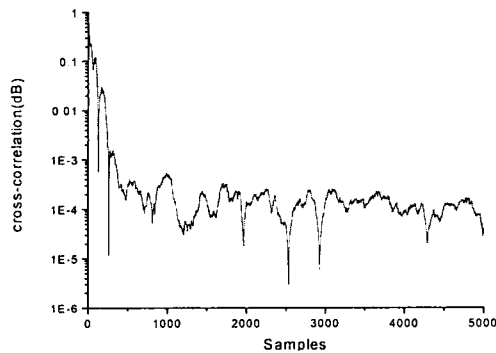


Fig. 2. The cross-correlation of input and estimated error signals.

For computations saving, the instantaneous gradient of the maximum value of the adaptive filter's coefficient is defined adaptive constant.

$$N = \max_k W(k)$$

$$\delta(k) = \beta\delta(k-1) + (1-\beta) x_M(k) e(k) \quad (11)$$

Therefore adaptive constants should be kept large many and few. The cross-correlation,  $\delta(k)$  is low pass filtered like equation(12), the converge speed of the adaptive filter should be kept fast.

$$c(k) = \beta c(k) + (1-\beta) \delta(k) \quad (12)$$

$c(k)$  is low pass filtered cross-correlation.  $c(k)$  is normalized by the summation of the reference input signal and estimated error signal like equation(13) for the coefficient of the adaptive filter is not misadjusted by the surrounding errors.

$$\mu(k) = \frac{d|c(k)|}{L(\sigma_x^2(k) + \sigma_e^2(k))} \quad (13)$$

$\mu(k)$  is adaptive constant which adapts the coefficient of the adaptive filter,  $\alpha$  is normalization adaptive constant. The reason absoluteness of the cross-correlation is if the adaptive constant is smaller than 0, the system can be unstable. The reson normalization using the summation of the input signal and estimated error signal is for reduce oscillation of the adaptive constant by the input signal and estimated error signal. Fig 3 represent the instantaneous gradient of front and rear of low pass filtered.

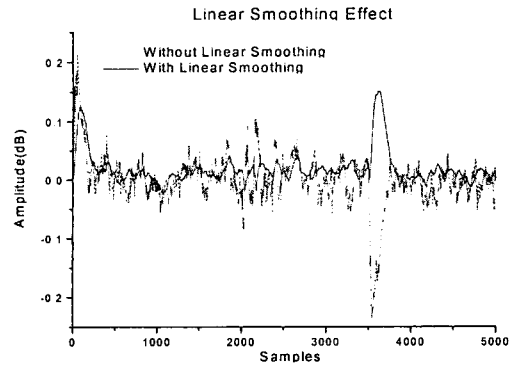


Fig. 3. low pass filtered gradient.

#### IV. Simulation Results and Considera

Through simple simulations, the performance of the adaptive algorithm which has proposed time-varying adaptive constant is appraised. For surrounding noisesignal correspondent with the input signal of the adaptive filter, use the independent, mean is 0, variance is 1, and white gaussian noise is used. And acoustic echo path is modeled by 32-order FIR filter, change the echo path at 5000 sample for compare the performance when the acoustic echo path is changed.

Fig 4 represents the coefficients misadjustment of

the adaptive algorithms. Short dashed-line represents of the results of the NLMS algorithm, long dashed-line represents of the results of the coefficients misadjustment of the robust to noise which is proposed by Greenberg, etc. Line represents the coefficients misadjustment of the adaptive algorithm which has proposed time-varying adaptive constant.

NLMS algorithm has more misadjustment to the near -end speaker's speech and surrounding errors and the coefficients misadjustment is small compare to the adaptive algorithm which is proposed by Greenberg, etc.

And, after 5000 sample, proposed adaptive algorithm is converge fast compared with the algorithm which is proposed by Greenberg, etc, though slow compared with NLMS algorithm. We can see the performance of the proposed algorithm is better than the adaptive algorithm which is used until now.

## V. Conclusion

In this paper, we proposed the adaptive algorithm, which has the time-varying adaptive constant using orthogonality principles of the optimal filter for the improvement of the trade-off between the convergence speed of algorithm and the performance in steady state.

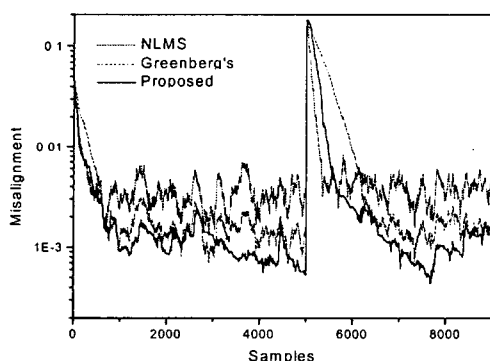


Fig. 4. Comparion misalignments of adaptive alg

The maximum instantaneous gradient is found among the coefficients of the adaptive filter, is low pass filtered and normalized by the summation of the input signal and the estimated error signals. It is shown that the proposed algorithm have good performance compare to the conventioanl algorithms through simulations. Quantitative analysis and experiment using DSP is needed.

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