3차원 다층 반무한상 구조물의 동적 간섭에 관한 해석적 연구

Three Dimensional Dynamic Interaction of Foundations on Layered Half-Space

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ABSTRACT

A dynamic interaction analysis of an adjacent surface foundation on a layered half-space is performed in the frequency domain. A semi-analytical approach is employed to reduce the integration range of the wavenumber in the surface fundamental solution for a layered half-space in boundary element (BE) formulations. The present study then adopts a combined boundary and finite element method to analyze the dynamic behavior of a system of flexible surface foundations on an elastic homogeneous and layered half-space. Numerical examples are presented to demonstrate the accuracy of the developed method. The examples show the feasibility of an extended application for the complicated dynamic interaction of foundations on layered media.

1. INTRODUCTION

Recently, the soil-structure interaction plays a fundamental role in the analysis and design of structures subjected to dynamic excitation. Dealing with adjacent structures or supports, the dynamic-interaction through the soil media might also be important for the responses of practical problems. Since Warburton et al.[1] studied the problem of cross-interaction between two circular rigid foundations problem, many researches have been investigated dynamic-interaction problems. Iguchi and Luco[2] have obtained the dynamic responses of a square flexible foundation. Qian et al.[3] proposed a boundary element method in combination with the finite element method for a flexible foundation to study the cross-interaction.

In this study, dynamic interaction between foundations on layered half-space is performed. The adoption of a coupling method that combines finite element for the foundations and boundary element for layered half-space makes it feasible. Particularly, boundary element that accounts for the layered half-space is possible by expanding the two-dimensional fundamental solutions[4] to three-dimensional fundamental solutions. The numerical integration algorithm using the semi-analytical methods over wavenumber in the frequency domain is

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employed because the solutions do not have analytic forms. In order to verify the accuracy of previously described fundamental solution response of rigid circular foundation is analyzed. Then the dynamic response of two identical flexible square foundation resting on the layered half-space is examined to demonstrate the capability of the developed method.

2. BOUNDARY ELEMENT FORMULATION

When the finite portion (Γ_f) of soil is occupied by a surface foundation, the equilibrium conditions and the compatibility conditions for the displacements are imposed over the foundation surface. Since the horizontal flat surface is stress free, the traction Green's functions automatically vanish on Γ_f and the boundary integral equation reduces as[5]

$$C(\xi)U_k(\xi,\omega)=\int_{\Gamma_f} T_i(\xi,\omega;n)U^*_{ik}(\xi,x,\omega)d\Gamma_\xi \qquad ;\ (i,k\ =\ 1,2,3) \tag{1}$$

where $\xi=(\xi_x,\xi_y,0)$, $\boldsymbol{x}=(x,y,0)$ represent the source point and the observation point located on the traction free surface and U^*_{ik} is Fourier transformed fundamental solution with transformed displacement and traction terms U_k and T_i. The coefficient $C(\xi)$ is defined with domain. Using BEM formulation for the equation (1), the surface Γ_f is discretized into NE subregions Γ_f^j ($j=1,2,...,NE$), and using quadratic shape functions at each element, Eq. (1) is transformed into a system of linear algebraic equations

$$U_{kp}=\sum_{j=1}^{NE}T_i^j\overline{U}_{ik}^j \qquad ;\ (i,k\ =\ 1,2,3) \tag{2}$$

with

$$\overline{U}_{ik}^j=\int_{\Gamma_f^j}U^*_{ik}\Phi\,|\,J\,|\,d\eta_1 d\eta_2 \tag{3}$$

here, $\Phi=[\phi_1\ \phi_2\ \ldots\ \phi_9]$ is the matrix of quadratic shape function in the local coordinates (η_1,η_2) and $|J|$ is the Jacobian of the transformation between global and local systems. Also, p is the index for the each element of surface elements. The value U_{kp} represents the k-th component of the displacement of the p-th element, and T_i^j represents the i-th component of the surface traction in the j-th surface element. equation (2) can be conveniently written in a matrix form as

$$\{U^{BE}\}=[G]\{T\} \tag{4}$$

where the matrix $[G]$ with dimension $3NE\times 3NE$ represents the force-displacement relationship between the elements of Γ_f and is called the compliance matrix for the flat surface Γ_f.

For the three dimensional fundamental solution in layered half-space, owing to the complex behavior of the integrands involve, a direct evaluation of the improper integrals is difficult both analytically and numerically. The situation is further complicated by the expected singular behavior of some of the Green's functions in the layer containing the source. To deal with such problems, it is useful to employ the method of asymptotic decomposition[4] wherein the leading asymptotic expansions of the featured integrands (responsible for singular behavior) are extracted and integrated analytically so that the remaining parts with strong decay can be evaluated numerically. Mathematically, one may write

$$\hat{u}_i^p = \left(\hat{u}_i^p\right)_A + \left(\hat{u}_i^p\right)_N \tag{5}$$

where the subscripts A and N denote the analytically and numerically evaluated parts of the fundamental solutions and the subscript i and p mean the response and source direction.

3. COMBINATION OF BEM AND FEM

On the assumption that no slippage or separation occurs at the soil-structure interface, the conditions of nodal force equilibrium and nodal displacement compatibility can be used to match the two independently modeled substructures. The total number of boundary elements used in the problem using shape function can be written in the following form[5]

$$\{u\} = \{u^{BE}\} = [N]\{u^{FE}\} \tag{6}$$

$$\{t\} = [N]\{P_r\} \tag{7}$$

where $\{u^{BE}\}$ and $\{t\}$ are the displacement vector and traction vector of the boundary element substructure. $\{u^{FE}\}$ and $\{P_r\}$ are the displacement vector and nodal reaction force vector of the finite element substructure. In modeling the foundation structure, the plate element, which has five degrees of freedom at each node, is used. Therefore, the matrix $[N]$ is introduced to match the different degrees of freedom between boundary element and finite element. Considering the applied nodal force on the structure and displacement equilibrium requirements dictate that

$$P_{\text{int}} + P_r = P_{ext} \tag{8}$$

so that

$$\left[\overline{K}\right]\{u\} + [N]^T[G]^{-1}[N]\{u\} = \{P_{ext}\} \tag{9}$$

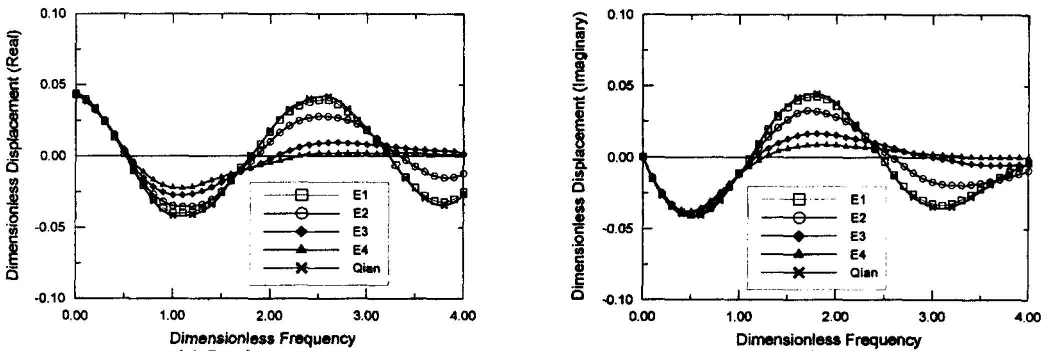
where, $\left[\overline{K}\right]$ is the modified stiffness of the finite element substructure correspond of the plate elements. If equation (9) is arranged about the displacement vector $\{u\}$ as follows, then;

$$\{u\} = \left[[\bar{K}] + [N]^T [G]^{-1} [N]\right]^{-1} \{P_{ext}\} = [S]\{P_{ext}\}. \quad (10)$$

Therefore, the relationship between external forces and nodal displacements is established.

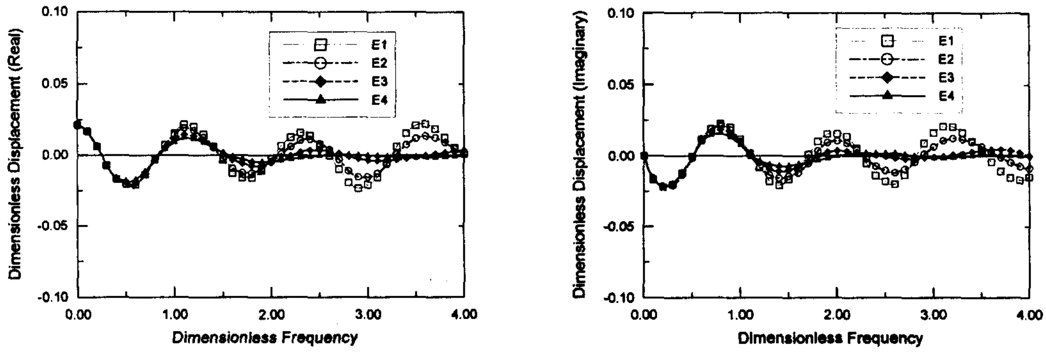
4. VERIFICATION

In order to demonstrate the validity of developed procedure following example is shown. The dynamic response of two identical square foundations of side $2a$ and thickness h resting on the half-space with a Poisson's ratio $\nu_s = 1/3$ is considered in this section. The two foundations are placed side by side a center to center distance d. The Poisson's ratio for the foundation is $\nu_f = 0.30$. A 6×6 element and 10-layer discretization is employed for both the source foundation, which is subjected to an external excitation force, and the secondary foundation which is load-free. Four values of relative rigidity are considered with $E_i = E_f h^3 / 12 E_s a^3$, where E_f is Young's modulus of foundation and E_s is that of half-space: $E_1 = 0.01, E_2 = 0.02, E_3 = 0.15, E_4 = 1.75$. The vertical compliance function C_{vv} for the active foundation under a central point load P is shown in Figure 8, where, $C_{vv} = aE_s W / 2P(1+\nu_s)$ and dimensionless frequency $a_0 = \omega a / C_s$.



(a) Real part (b) Imaginary part

Figure 1 Vertical compliance at $d/a = 2.5$



(a) Real part (b) Imaginary part

Figure 2 Vertical compliance at $d/a = 5.0$

The interaction will generate contact forces as well as displacements on the secondary foundation. The compliance functions at the center point of the secondary foundation are given in Figure 1 and 2 for the dimensionless separation distance $d/a = 2.5, 5.0$ respectively. First, the results obtained from this study closely agree with results by Qian et al.[3] in $d/a = 2.5$ and $E_1 = 0.01$ condition. In addition, it has been shown that the amplitude of the coupling compliance function decreases with increasing separation distance d. For relatively rigid foundations, it decays rapidly at higher frequency.

5. DYNAMIC INTERACTION BETWEEN TWO FLEXIBLE FOUNDATIONS

The developed method is applied to the dynamic problems of two identical square foundations, shown in Figure 3. The dimensions of the square foundation are a for sides and $h\,(0.3m)$ for thickness resting on the layered half-space. The two foundations are placed side by side with a center-to-center distance d. Details of material properties are presented in Table 1.

Thirty-six square elements are employed for both active and passive foundations, which have an external excitation force, and a receiver point without any applied load, respectively. The vertical compliance C_{vv} at the receive point under a unit harmonic point load P on the active foundation is shown in Figure 4 and 5, where $C_{vv} = aE_1W/2P(1+v_1)$ and dimensionless frequency $a_0 = \omega a / C_{s1}$.

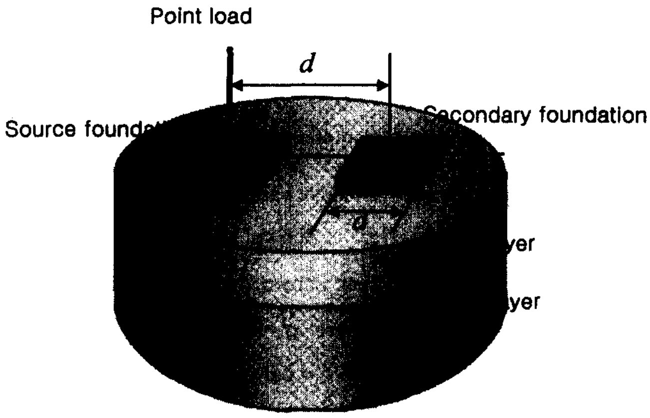
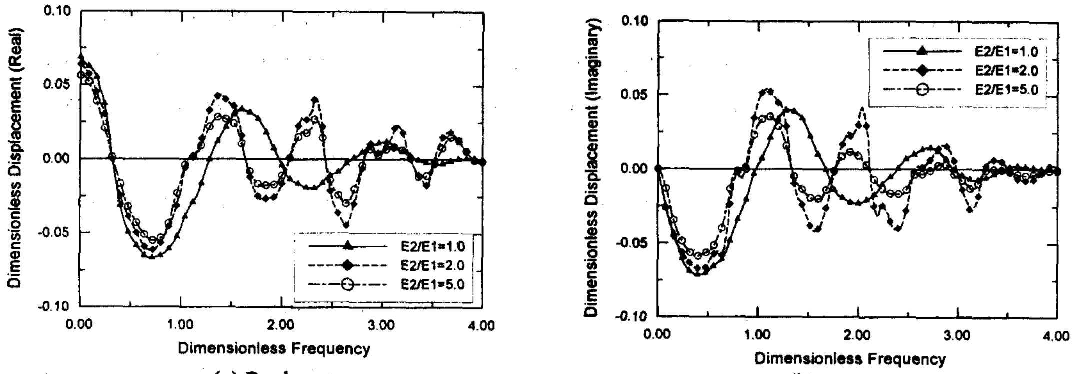


Figure 3 Two square foundations on a layered media

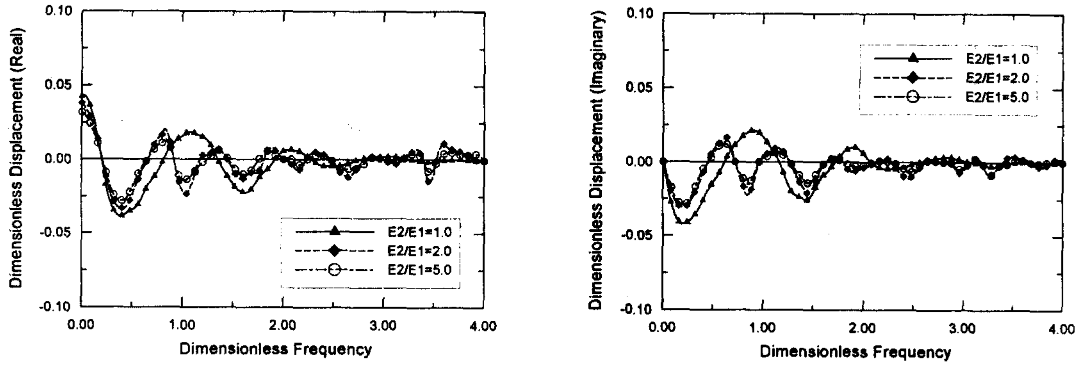
Table 1 Material properties of layered media

Layer \ Prop.		Young's Modulus (kN/m²)	Density (kg/m³)	Poisson's Ratio	Damping Ratio
1		1.0×10^6	2.0×10^3	0.333	0.05
2	E_{S1}	1.0×10^6			
	E_{S2}	2.0×10^6			
	E_{S3}	5.0×10^6			
Foundation	E_F	23.8×10^6	2.3×10^3	0.167	0.00



(a) Real part (b) Imaginary part

Figure 4 Vertical compliance at $d/H = 2.0$



(a) Real part (b) Imaginary part

Figure 5 Vertical compliance at $d/H = 4.0$

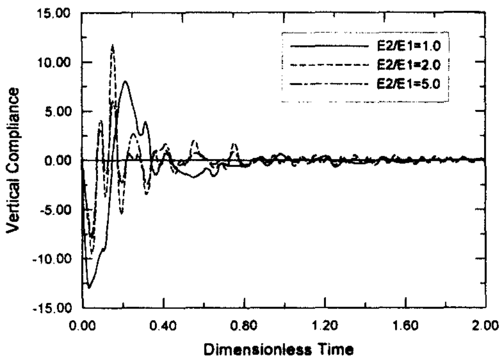
In Figure 4 and 5, the vertical compliance at two relative values of center-to-center distance versus layer depth ($d/H = 2.0,\ 4.0$) for the $H = 4m$ and $a = 4m$ is shown. As the results from the responses of two foundations on homogeneous half-space, it has been shown that the amplitude of the coupling compliance function decreases with increasing separation distance d in an oscillatory pattern. Also, more frequent oscillation is appeared in $d/H = 4.0$ than in $d/H = 2.0$. Not to same as that of homogeneous case, however, the apparent decay phenomena is not appeared at higher frequency for relatively rigid foundations, which caused by the reflection of sub layer. The sublayer's stiffness ratio variances affect more to the results of $d/H = 2.0$ than those of $d/H = 4.0$. As in Figure 4 and 5, the compliance of the center point decreases significantly with increasing stiffness ratio (E_2/E_1) in $d/H = 2.0$ but the decreasing magnitudes of compliance are relatively small in $d/H = 4.0$. In half-space, the radiation characteristic of radial direction induces a significant reduction of response and there is enough radiation in $d/H = 4.0$ reduction, so the increasing stiffness ratio (E_2/E_1) affect little for the compliance. It means that the separation distance of foundations is more important than the stiffness increase of subsoil for preventing vibration.

In order to represent the response of the structure in time variance, the inversion of the FFT is used. When the time function $h(t)$ is divided in M piecewise constant segments whose heights are h_m and base $\Delta T = T / M$, the equation (11) shows how the inversion of FFT is obtained[7].

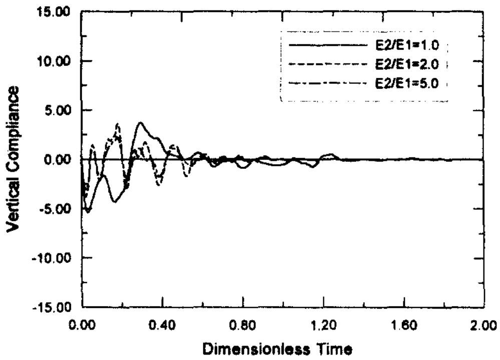
$$h_m = h(t_m) \approx \frac{1}{T}\sum_{n=0}^{M-1} H_n e^{+i\omega_n t_m} = \frac{1}{T}\sum_{n=0}^{M-1} H_n e^{+i2\pi nm/N} \quad (11a)$$

$$H_n = H(\varpi_m) \approx \Delta T\sum_{m=0}^{M-1} h_m e^{-i\omega_n t_m} = \Delta T\sum_{m=0}^{M-1} h_m e^{-i2\pi nm/N} \quad (11b)$$

Where both m and n range from 0 to $M-1$. After these inversions of the FFT, the response of the center point on the structure is represented as a time history in Figure 6. The reflection effects are also represented in a disturbance of responses in time domain which are transformed by inverse FFT. As in Figure 5, the sublayer causes disturbed responses which are induced by the high frequency component, and these are different to the results of homogeneous case ($E_2 / E_1 = 1.0$). As indicated in Figure 7, the maximum compliance reduces consistently as E_2 / E_1 become higher but the separation distance is more critical to the responses.



(a) Vertical compliance at $d / H = 2.0$



(b) Vertical compliance at $d / H = 4.0$

Figure 6 Vertical compliance in time domain

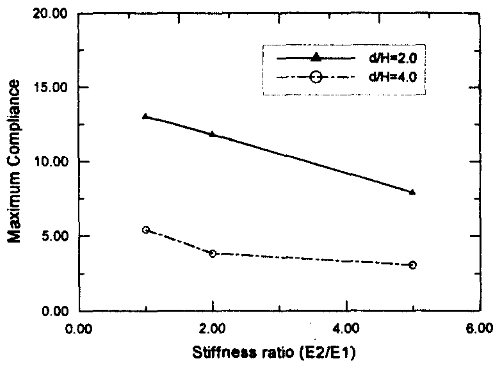


Figure 7 Comparison of maximum compliance with stiffness ratio

5. CONCLUSIONS

An dynamic interaction of a foundation on a layered half-space is performed in the frequency domain. The foundation is modeled by finite elements and the layered half-space is modeled by boundary elements based on the surface fundamental solutions of the layered half-space using a semi-analytical approach. Numerical examples are given to demonstrate the feasibility of the presented method.

As a result of application of this method to the dynamic interaction between two flexible surface foundations, it was found that the separation distance of foundations is more important than the stiffness increase of subsoil. The amplitude of the coupling compliance decreases with increasing separation distance d. In addition, sublayer's stiffness ratio variance affects more to the results of $d/H = 2.0$ than those of $d/H = 4.0$. The compliance of the center point decreases significantly with increasing stiffness ratio (E_2 / E_1) in near separation distance but the decreasing magnitudes of compliance are relatively small in far separation distance.

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