# Tablerock 사암에서 수압속도와 공극압이 파쇄압에 미치는 영향연구

# Effect of Pressurization Rate and Initial Pore Pressure on Hydraulic Fracturing Breakdown Pressure in Tablerock Sandstone

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## 1. INTRODUCTION

Hydraulic fracturing (HF) tests for the determination of *in situ* stress consist of injecting fluid into an isolated segment of a wellbore until a tensile fracture develops. Breakdown pressure $P_c$ is defined as the wellbore pressure necessary to induce a hydraulic fracture. If the wellbore and one of the principal stresses are both vertical, the fracture is typically vertical and extends along the direction of the maximum horizontal principal stress. There are two classical HF criteria relating $P_c$ to the *in situ* horizontal stresses; one based on elastic theory for impermeable rocks (Hubbert and Willis, 1957), and the other based on the poroelastic theory for permeable rocks (Haimson and Fairhurst, 1967). The Hubbert and Willis (1957) criterion (H-W) is given by:

$$P_c - P_o = T_{hf} - 3\sigma_h - \sigma_H - 2P_o \tag{1}$$

The Haimson and Fairhurst HF criterion (H-F) is represented by:

$$P_c - P_o = \frac{T_{hf} + 3\sigma_h - \sigma_H - 2P_o}{2 - 2\eta} \tag{2}$$

where $\sigma_h$ and $\sigma_H$ are the least and largest horizontal principal stresses, respectively; $P_o$ is initial pore pressure; $T_{hf}$ is the tensile strength of rock subjected to hydraulic pressure; and $\eta$ is a poroelastic parameter given by:

$$\eta = \frac{\alpha(1-2\nu)}{2(1-\nu)} \qquad 0 \le \eta \le 0.5 \tag{3}$$

where $\alpha$ is the Biot parameter (Biot and Willis, 1957) and $\nu$ is the Poisson's ratio. Two assumptions are made in both H-W and H-F criteria. One is that a tensile fracture initiates when the effective tangential stress at the wellbore wall becomes tensile and reaches the HF tensile strength ($T_{hf}$) of the rock. The other assumption is that Terzaghi's (1943) effective stress law governs the effect of pore fluid on rock stress, *i.e.*, $\sigma_{ij}^{eff} = \sigma_{ij} - \delta_{ij} P_o$ ($i, j$ = 1, 2, 3), where $\sigma_{ij}^{eff}$ is the effective stress; $\delta_{ij}$ is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$). In addition, these two HF criteria do not incorporate the wellbore pressurization rate effect on breakdown pressure, which is substantial in some experimental results (*e.g.* Haimson and Zhao, 1991).

We carried out laboratory hydrofracturing tests in hollow cylinders of high porosity sandstones. The ultimate goal of our project is to establish whether the HF technique is appropriate for estimating *in situ* stress magnitudes in highly permeable sandstones. This paper describes some results on Tablerock sandstone (porosity: 26%) with particular reference to (1) pressurization rate effect and (2) pore pressure effect on $P_c$. We determined several important mechanical properties of the rock, as listed in Table 1.

Table 1. Physical properties of the Tablerock sandstone tested.

| Porosity (%) | Permeability (Darcy) | Uniaxial Compressive strength (MPa) | Brazilian Tensile Strength (MPa) | Tangential Young Modulus (GPa) | Poisson's ratio $\nu$ | Biot parameter $\alpha$ |
|---|---|---|---|---|---|---|
| 26.0±0.9 | 0.12 | 42.0±1.0 | 4.4±0.22 | 15.3 | 0.2 | 0.71 |

## 2. EXPERIMENTAL SETUP AND PROCEDURE

We carried out laboratory hydrofracturing tests in thick walled, hollow cylinders of Tablerock sandstone having a hole diameter of 12 mm, an external diameter of 100 mm, and a length of 130 mm. Specimens were placed in a pressure vessel and subjected to predetermined confining pressure $P_{conf}$ (representing the far-field horizontal stress $\sigma_h$), vertical pressure (representing vertical far-field stress $\sigma_v$), and initial pore fluid pressure ($P_o$). Borehole fluid (viscosity: 2.5 Pa s at 20° C), which was the same as the pore fluid, was injected at a constant flow rate until a peak pressure was reached. Fluid injection was stopped as soon as borehole pressure ceased to increase. Pressure cycles were typically repeated several times. A four-channel servo-control system enabled the continuous control of the pressures applied ($\sigma_h$, $\sigma_v$, and $P_o$), and the borehole injection-fluid flow rate. A commercial software and a personal computer equipped with digital-to-analog and analog-to-digital converters were employed for test control and data acquisition (Fig. 1). The entire loading process, and borehole pressurization were automated, including injection halting upon the onset of pressure decline.

## 3. PRESSURIZATION RATE EFFECT ON BREAKDOWN PRESSURE

We conducted a series of HF tests on unjacketed specimens, in which the pore pressure $P_o$ was equal to the confining pressure $P_{conf}$ at the outer boundary. In our tests $P_{conf}$ simulates the far-field uniform horizontal stress $\sigma_h$. Unjacketed tests provided simple boundary conditions for identifying the hydrofracturing parameters that affect breakdown pressure. In these tests, borehole fluid was injected at flow rates that varied from test to test, while the confining/pore pressure and the vertical stress $\sigma_v$ were maintained constant at 20 MPa and 30 MPa, respectively. This series of tests was aimed at determining the pressurization rate effect on breakdown pressure. The result revealed that $P_c - P_o$ increased significantly with the wellbore pressurization rate $dP/dt$ (Fig. 2). This behavior suggested that the classical HF criteria (H-W and H-F) are disabled if surrounding rock is highly permeable.

Unlike Hubbert and Willis (1957) and Haimson and Fairhurst (1967), Detournay and Cheng (1992) proposed a HF model (D-C) that considers fluid diffusion through pores and microcrack lengthscale around the wellbore boundary. This model incorporates wellbore pressurization rate, explicitly, as follows:

$$P_c - P_o = \frac{T_{hf} + 3\sigma_h - \sigma_H - 2P_o}{1 + (1 - 2\eta)h(\gamma)} \tag{4}$$

where $\gamma$ is a dimensionless pressurization rate given by:

$$\gamma = \frac{A\lambda^2}{4cS} \qquad 0 \le \gamma \le \infty \tag{5}$$

where $S = T_{hf} + 3\sigma_h - \sigma_H - 2P_o$; and $A$, $\lambda$, and $c$ are borehole pressurization rate, microcrack lengthscale, and diffusivity coefficient, respectively. The function $h(\gamma)$ varies between 0 for $\gamma = \infty$ (the fast limit of wellbore pressurization rate), and 1 for $\gamma = 0$ (the slow limit).

As shown in Figure 2, the D-C criterion, which incorporates the pressurization rate effect, is probably a promising model for Tablerock sandstone. We compared our test results with the theoretical curve by Detournay and Cheng (1992). For axisymmetric stress condition ($\sigma_H = \sigma_h$), the D-C criterion can be expressed as (modified from equation 4):

$$P_c - P_o = \frac{T_{hf} + 2(\sigma_h - P_o)}{1 + (1 - 2\eta)h(\gamma)} \tag{6}$$

In Tablerock sandstone $\eta = 0.27$ (equation 3), and for our test configuration $\sigma_h - P_o = 0$. Hence, $P_c - P_o$ can be expressed as a function of $\gamma$. We graphically superimposed the D-C criterion (solid line) as a function of $\gamma$ on our experimental results (open circles) in as a function of *dP/dt* (Fig. 2). The good coincidence suggests that $P_c - P_o$ asymptotically tends to an upper and lower bound corresponding to fast and slow rates, respectively.

## 4. PORE PRESSURE EFFCT ON BREAKDOWN PRESSURE AND THE MODIFIED D-C CRITERION

In a second series of tests conducted in unjacketed specimens, the injection flow rate was kept constant in all tests at 5 $\mathrm{cm}^3$/sec yielding the same borehole pressurization rate *dP/dt* (= 15 ~ 20 MPa/sec at breakdown). The confining/pore pressure, however, was varied from test to test between 0 and 30 MPa. In this series of HF tests, the resulting $P_c - P_o$ linearly increased with the confining/pore pressure (Fig. 3), and did not remain constant as predicted by the D-C criterion (equation 6). We note, however, that equation 6 employs the Terzaghi effective stress law. This law has been found to be ineffective in another case of hydraulic fracturing (Schmitt and Zoback, 1989, 1992).

We thus considered a more general effective stress law for tensile failure as proposed by Schmitt and Zoback (1989):

$$\sigma_{ij}^{eff} = \sigma_{ij} - \delta_{ij}\beta P_o \qquad (0 \leq \beta \leq 1) \tag{7}$$

where β is an effective stress coefficient depending on rock type. We modified the D-C hydraulic fracturing criterion based on the assumption that a breakdown occurs when the effective tangential stress governed by the generalized effective stress law (equation 7) equals the HF tensile strength. The modified D-C criterion for the uniform horizontal stress $\sigma_h$ is represented as:

$$P_c - P_o = \frac{T_{hf} + 2\sigma_h - (1+\beta)P_o}{1 + (\beta - 2\eta)h(\gamma)} \tag{8}$$

For HF tests in unjacketed hollow cylinders, equation 8 becomes:

$$P_c - P_o = \frac{T_{hf} + (1-\beta)\sigma_h}{1 + (\beta - 2\eta)h(\gamma)} \tag{9}$$

This equation shows that by employing the generalized effective stress law, $P_c - P_o$ is a linear function of $\sigma_h$ if γ (corresponding to *dP/dt*) is constant, and if $\beta \neq 1$. From the linear approximation of our unjacketed experimental data (Fig. 3), we obtained $\beta = 0.72$.

## 5. VERIFICATION OF THE MODIFIED D-C CRITERION

We conducted several series of hydrofracturing tests in jacketed specimens. The uniform far-field horizontal stress applied was kept in the range of 15 - 45 MPa and the pore pressure was between 1 - 20 MPa. We plotted all experimental data in the domain of $[P_c - P_o]$ versus $[2\sigma_h - (1 + \beta)P_o]$ and found that they fit rather well a linear relationship (Fig 4):

$$P_c - P_o = 7.5 + 0.88\,[2\sigma_h - (1 + \beta)P_o] \tag{10}$$

Combining this relationship with the modified D-C criterion (equation 8) gives $h(\gamma) = 0.75$ and $T_{hf} = 8.5$ MPa for $\eta = 0.27$ and $\beta = 0.72$ (as determined from the unjacketed tests). Thus the modified D-C criterion suitable for Tablerock sandstone becomes:

$$P_c - P_o = \frac{8.5 + 2\sigma_h - 1.72P_o}{1.135} \tag{11}$$

This equation allows us to calculate the expected values of $P_c - P_o$ for given $\sigma_h$ and $P_o$. We were also able to calculate the expected $P_c - P_o$ based on the H-F, the H-W and the D-C hydraulic fracturing criteria using appropriate parameters such as $h(\gamma)$ and $T_{hf}$ for Tablerock sandstone. Comparisons between the different HF criteria for two series of tests, one in which the only variable was $\sigma_h$ and the other in which the only variable was $P_o$, were shown in Figures 5. In both series the modified D-C criterion is clearly best fitting our experimental results. Using equation 11 we are able to calculate $\sigma_h$ for the applied pore pressure and the tested breakdown pressure.

## 6. CONCLUSIONS

We carried out laboratory hydraulic fracturing tests in hollow cylinders of Tablerock sandstone to establish a correct relationship between the breakdown pressure and the far-filed stress. Based on a series of tests in which we examined wellbore pressurization rate effect on breakdown pressure, the Detournay-Cheng (1992) hydrofracturing criterion could be generally accepted as our result coincided with the theoretical curve. However, another series of tests in which the confining/pore pressure varied from test to test for given pressurization rate revealed that the D-C criterion requires a modification in order to appropriately interpret our HF test results. Incorporating a general effective stress law into the D-C criterion, we were able to correctly describe the relationship between breakdown pressure and far-field stress in high-porosity Tablerock sandstone. Our results are of significance in the petroleum industry, since many reservoirs are found in high-porosity sandstones, and knowledge of the *in situ* stress conditions gained from hydrofracturing tests is essential to borehole stability and oil field design.

## ACKNOWLEDGEMENTS

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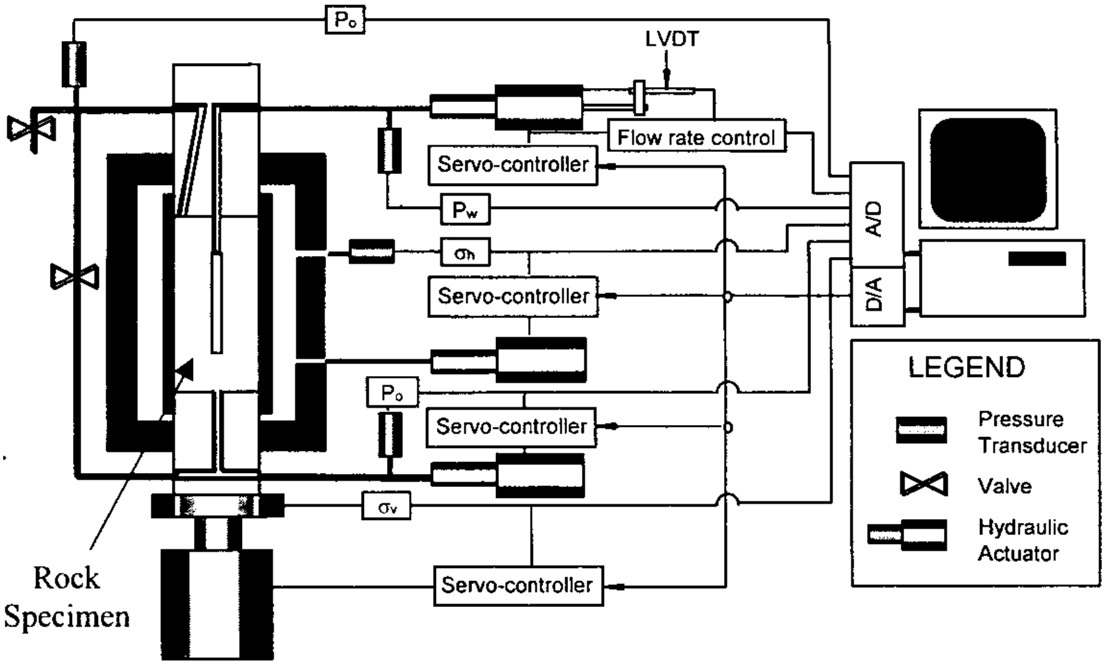


Fig. 1. Schematic of experimental setup

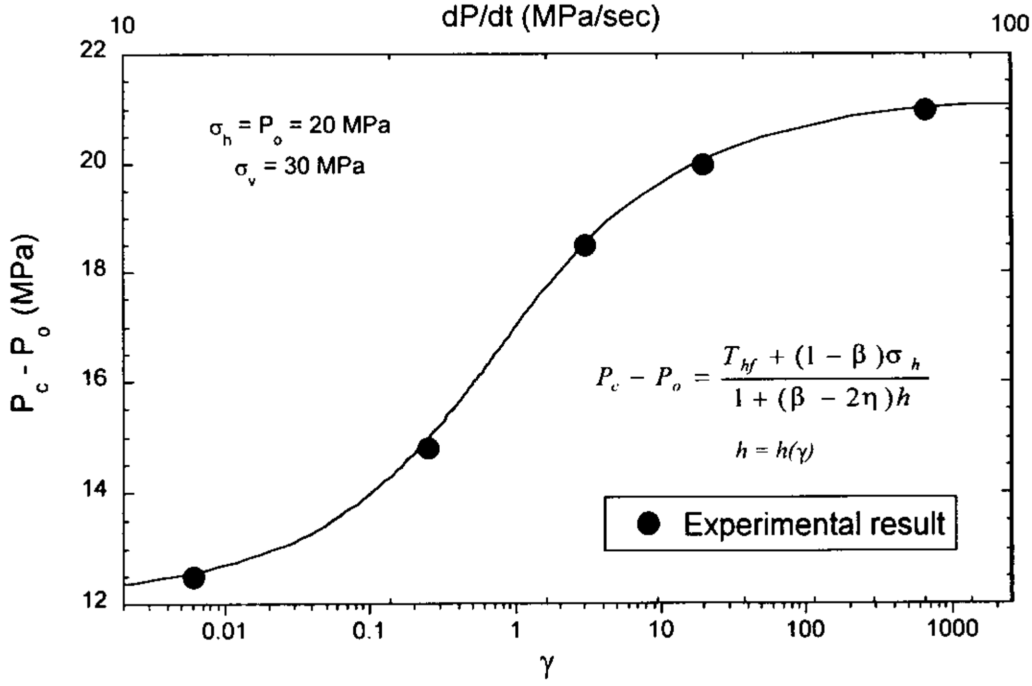


Fig. 2. Hydrofracturing test results under identical $\sigma_h$ (= $P_o$) but different flow rates in unjacketed specimens.

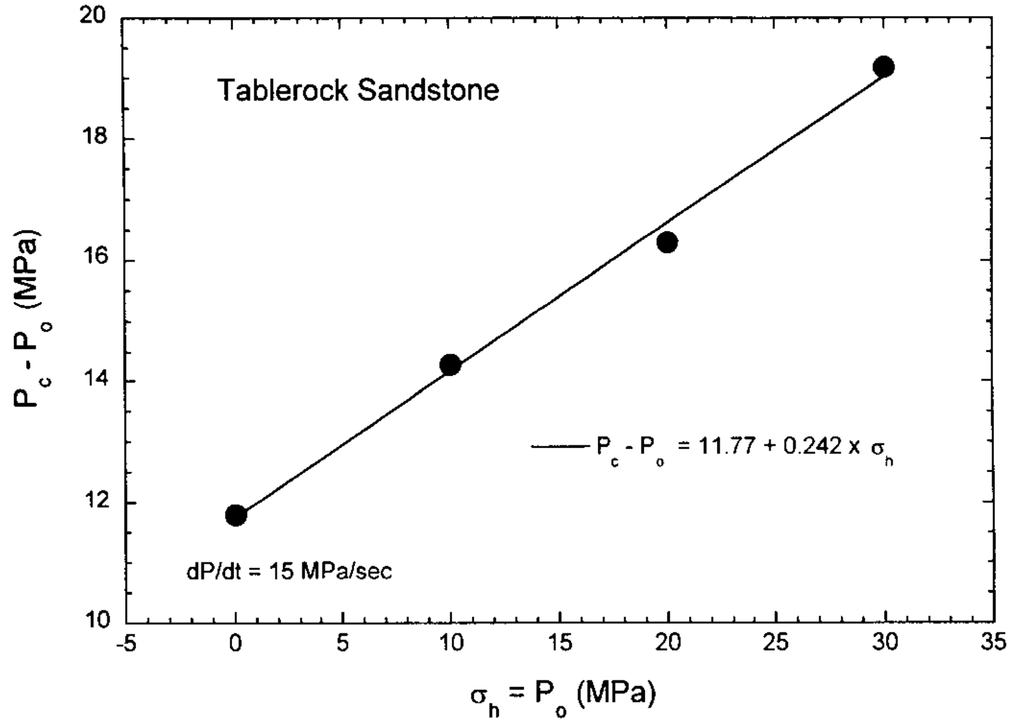


Fig. 3. Hydrofracturing test results under a constant flow rate but various $\sigma_h$ (= $P_o$) in unjacketed specimens.

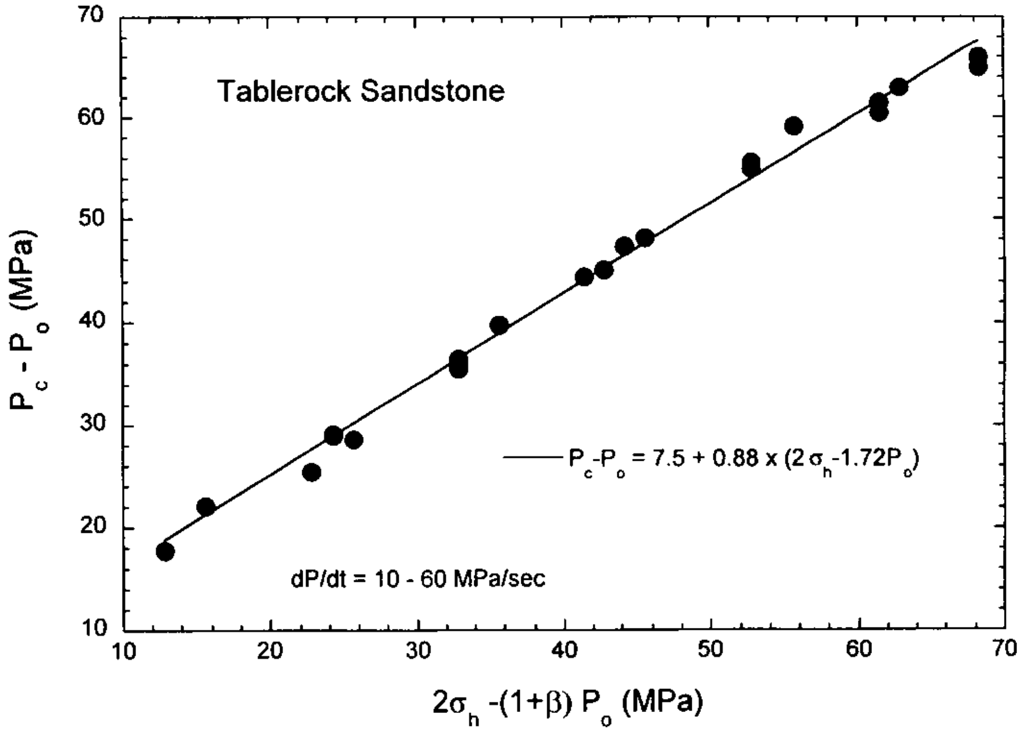
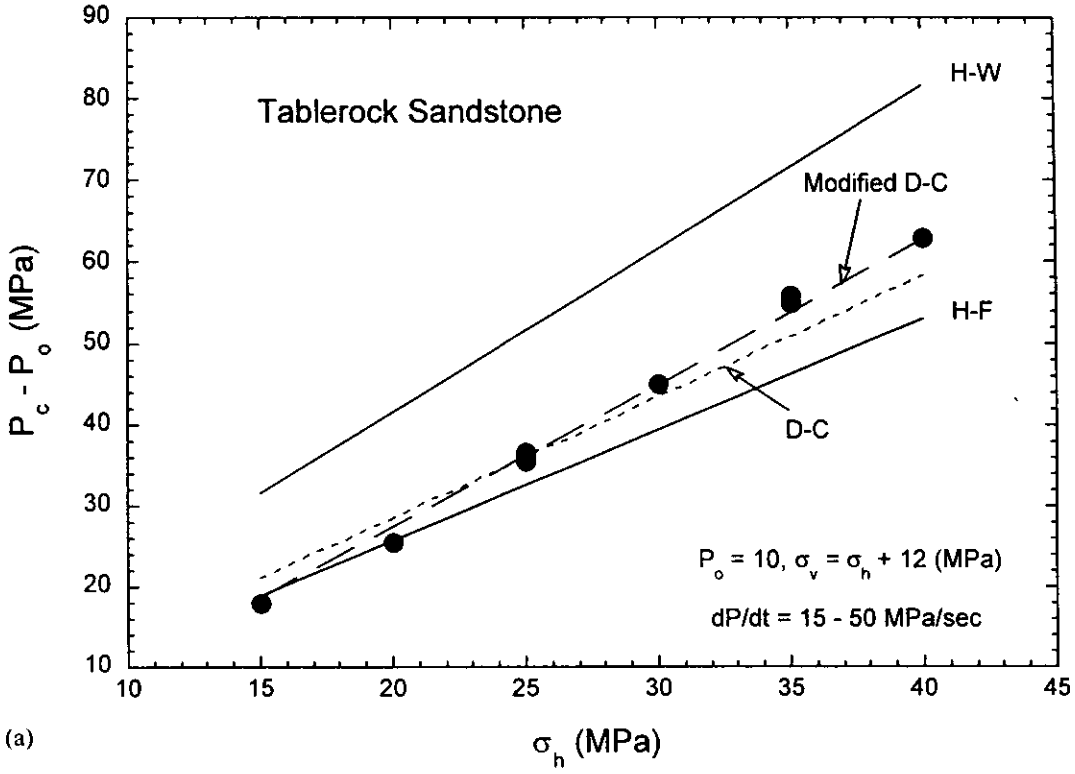
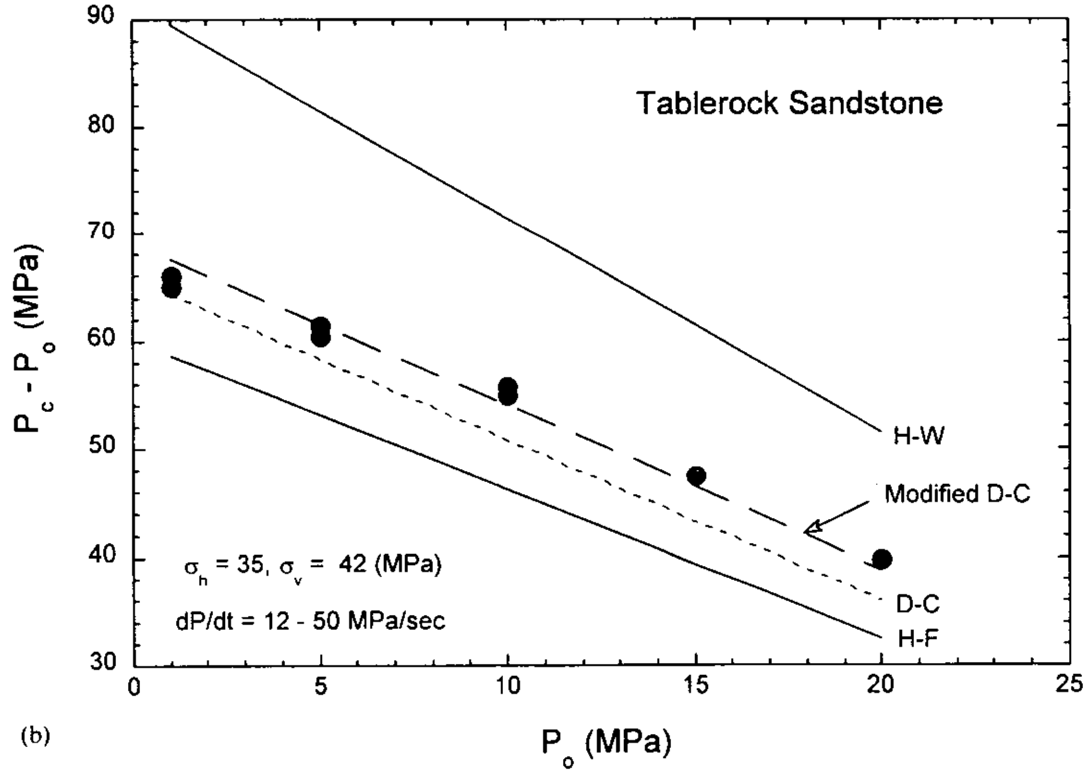


Fig. 4. Compilation of all test results in jacketed specimens plotted as $P_c - P_o$ versus $2\sigma_h - (1 + \beta)P_o$, where $\beta = 0.72$.



(a)



(b)

Fig. 5. A comparison between different hydraulic fracturing criteria in a series of jacketed tests in which (a) $P_o$ was held constant $P_o$ (10 MPa) and $\sigma_h$ was varied from test to test, and (b) $\sigma_h$ was held constant (35 MPa) and $P_o$ was varied from test to test.