퍼지회귀계수에 관한 퍼지검정

Fuzzy Test for the Fuzzy Regression Coefficient

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Abstract

We propose fuzzy least-squares regression analysis by few error term data and test the slop by fuzzy hypotheses membership function for fuzzy number data with agreement index. Finding the agreement index by area for fuzzy hypotheses membership function and membership function of confidence interval, we obtain the results to acceptance or reject for the test of fuzzy hypotheses.

Key Words : fuzzy regression, fuzzy confidence interval, fuzzy hypotheses, fuzzy test, separability , agreement index.

I. Introduction

Fuzzy regression was proposed to deal with fuzzy data. In contrast to the ordinary regression that is based on possibility theory based on[1]. Other fuzzy regression methods have be developed using different optimal criteria for fuzzy line or curve fitting by Tanaka et al. [4]. In order to integrate both randomness an fuzziness types of uncertainty into one regression model

concepts of hybride fuzzy least-squares regression analysis was proposed Chang et al. [9]. Other hand, Grzegorzewski[8] proposed fuzzy test

For testing statistical hypotheses with vague data. He suggested a measure of fuzziness of the considered fuzzy test and also discussed the robustness of that test with possibility.

We propose fuzzy least-squares regression analysis by few error term data and slop coefficient and test the model by fuzzy hypotheses membership function with agreement index. Finding the agreement index by area for fuzzy hypotheses membership function and membership function of confidence interval, we obtain the results to acceptance or reject for the test of fuzzy hypotheses.

II. Fuzzy regression model

We begin with an experiment to determine the relation between two fuzzy variable $\tilde{X}$ and $\tilde{Y}$, we have fuzzy linear regression model

$$\tilde{Y} = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{X} \qquad (2.1)$$

where $\tilde{\beta}_0$ is indicate the intercept and $\tilde{\beta}_1$ represent the slop of the line. To estimate the linear regression, we consider $\min \tilde{Q}$ where,

$$\tilde{Q} = \sum_{i=1}^{n} \tilde{\varepsilon}_i^2 = \sum_{i=1}^{n} (\tilde{Y}_i \ominus \tilde{\beta}_0 \ominus \tilde{\beta}_1 \otimes \tilde{X}_i)^2. \quad (2.2)$$

Put

$$[\tilde{Q}]^\alpha = [\tilde{Q}_l^\alpha, \tilde{Q}_r^\alpha], \quad [\tilde{\beta}_0]^\alpha = [\tilde{\beta}_{0l}^\alpha, \tilde{\beta}_{0r}^\alpha],$$
$$[\tilde{\beta}_1^\alpha] = [\tilde{\beta}_{1l}^\alpha, \tilde{\beta}_{1r}^\alpha], \quad [\tilde{X}_i]^\alpha = [\tilde{X}_{il}^\alpha, \tilde{X}_{ir}^\alpha]$$
$$[\tilde{Y}_i]^\alpha = [\tilde{Y}_{il}^\alpha, \tilde{Y}_{ir}^\alpha] \ (i = 1, 2, \cdots, n).$$

We take for one please point p in each intervals

$$\forall \tilde{Q}_p^\alpha \in [\tilde{Q}_l^\alpha, \tilde{Q}_r^\alpha], \ \forall \tilde{\beta}_{0p}^\alpha \in [\tilde{\beta}_{0l}^\alpha, \tilde{\beta}_{0r}^\alpha],$$
$$[\tilde{\beta}_1]^\alpha = [\tilde{\beta}_{1l}^\alpha, \tilde{\beta}_{1r}^\alpha], \forall \tilde{X}_{ip}^\alpha \in [\tilde{X}_{il}^\alpha, \tilde{X}_{ir}^\alpha],$$
$$\forall \tilde{Y}_{ip}^\alpha \in [\tilde{Y}_{il}^\alpha, \tilde{Y}_{ir}^\alpha], \quad i = 1, 2, \cdots, n,$$

then we have

$$\tilde{Q}_p^\alpha = \sum_{i=1}^{n} (\tilde{Y}_{ip}^\alpha \ominus \tilde{\beta}_{0p}^\alpha \ominus \tilde{\beta}_{1p}^\alpha \otimes \tilde{X}_{ip}^\alpha)^2. \quad (2.3)$$

Therefore for $\alpha \in (0, 1]$, we can fine out the partial differentiation for $\tilde{\beta}_{0p}^\alpha$ and $\tilde{\beta}_{1p}^\alpha$ respectively, as follows

$$\frac{\delta \tilde{Q}_p^\alpha}{\delta \tilde{\beta}_{0p}^\alpha} = -2 \sum_{i=1}^{n} (\tilde{Y}_{ip}^\alpha \ominus \tilde{\beta}_{0p}^\alpha \ominus \tilde{\beta}_{1p}^\alpha \otimes \tilde{X}_{ip}^\alpha),$$
$$\frac{\delta \tilde{Q}_p^\alpha}{\delta \tilde{\beta}_{1p}^\alpha} = -2 \sum_{i=1}^{n} (\tilde{Y}_{ip}^\alpha \ominus \tilde{\beta}_{0p}^\alpha \ominus \tilde{\beta}_{1p}^\alpha \otimes \tilde{X}_{ip}^\alpha) \otimes \tilde{X}_{ip}^\alpha.$$

Let the right side of above equation equal to 0, then we have normal equation

$$n \tilde{\beta}_{0p}^\alpha \oplus \tilde{\beta}_{1p}^\alpha \otimes \sum_{i=1}^{n} \tilde{X}_{ip}^\alpha = \sum_{i=1}^{n} \tilde{Y}_{ip}^\alpha \quad (2.4)$$
$$\tilde{\beta}_{0p}^\alpha \otimes \sum_{i=1}^{n} \tilde{X}_{ip}^\alpha \oplus \tilde{\beta}_{1p}^\alpha \otimes \sum_{i=1}^{n} (\tilde{X}_{ip}^\alpha)^2 = \sum_{i=1}^{n} \tilde{X}_{ip}^\alpha \otimes \tilde{Y}_{ip}^\alpha.$$

Theorem 2.1. We can find the estimator as

$$\hat{\tilde{\beta}}_1 = \frac{\tilde{S}_{XY}}{\tilde{S}_{XX}}, \ \hat{\tilde{\beta}}_0 = \overline{\tilde{Y}} \ominus (\hat{\tilde{\beta}}_1 \otimes \overline{\tilde{X}}), \quad (2.5)$$
$$\hat{\tilde{Y}}_i = \hat{\tilde{\beta}}_0 \oplus (\hat{\tilde{\beta}}_1 \otimes \tilde{X}_i), \quad (2.6)$$

where, $\tilde{S}_{XY} = \sum_{i=1}^{n} (\tilde{X}_i \ominus \overline{\tilde{X}}) \otimes (\tilde{Y}_i \ominus \overline{\tilde{Y}}),$

$$\tilde{S}_{XX} = \sum_{i=1}^{n} \tilde{X}_i^2 \ominus n \overline{\tilde{X}}^2.$$

[proof] By normal equation (2.4)

$$\sum_{i=1}^{n} \tilde{X}_{ip}^\alpha \tilde{Y}_{ip}^\alpha = \tilde{\beta}_{0p}^\alpha \sum_{i=1}^{n} \tilde{X}_{ip}^\alpha \oplus \tilde{\beta}_{1p}^\alpha \sum_{i=1}^{n} (\tilde{X}_{ip}^2)^\alpha$$
$$= n \overline{\tilde{X}_p^\alpha} \ \overline{\tilde{Y}_p^\alpha} \oplus \tilde{\beta}_{1p}^\alpha \left\{ \sum_{i=1}^{n} (\tilde{X}_{ip}^2)^\alpha \ominus n (\overline{\tilde{X}_p^2})^\alpha \right\}$$

Hence

$$\hat{\tilde{\beta}}_{1p}^\alpha = \frac{\Sigma_{i=1}^n \tilde{X}_{ip}^\alpha \tilde{Y}_{ip}^\alpha \ominus n \overline{\tilde{X}_p^\alpha} \ \overline{\tilde{Y}_p^\alpha}}{\Sigma_{i=1}^n (\tilde{X}_{ip}^2)^\alpha \ominus n (\overline{\tilde{X}_p^2})^\alpha}$$

$$\hat{\tilde{\beta}}_{0p}^\alpha = \frac{1}{n} \sum_{i=1}^{n} \tilde{Y}_{ip}^\alpha \ominus \hat{\tilde{\beta}}_{1p}^\alpha \frac{1}{n} \sum_{i=1}^{n} \tilde{X}_{ip}^\alpha = \overline{\tilde{Y}_p^\alpha} \ominus \hat{\tilde{\beta}}_{1p}^\alpha \overline{\tilde{X}_p^\alpha}$$

Therefore for $\alpha \in (0, 1]$, we obtain the line of regression $\hat{\tilde{Y}}_{ip}^\alpha = \hat{\tilde{\beta}}_{0p}^\alpha \oplus \hat{\tilde{\beta}}_{1p}^\alpha \tilde{X}_{ip}^\alpha$.

Thus $[\hat{\tilde{Y}}]^\alpha = [\hat{\tilde{\beta}}_0]^\alpha \oplus [\hat{\tilde{\beta}}_1]^\alpha [\tilde{X}_i]^\alpha$, where

$$[\hat{\tilde{\beta}}_1]^\alpha = \frac{\Sigma_{i=1}^n \tilde{X}_i^\alpha \tilde{Y}_i^\alpha \ominus \overline{\tilde{X}^\alpha} \ \overline{\tilde{Y}^\alpha}}{\Sigma_{i=1}^n (\tilde{X}_i^2)^\alpha \ominus n (\overline{\tilde{X}^2})^\alpha}$$
$$[\hat{\tilde{\beta}}_0]^\alpha = \overline{\tilde{Y}^\alpha} \ominus \hat{\tilde{\beta}}_1^\alpha \overline{\tilde{X}^\alpha}.$$

Hence by resolution identity, we have $\hat{\tilde{Y}} = \hat{\tilde{\beta}}_0 \oplus (\hat{\tilde{\beta}}_1 \otimes \tilde{X})$, where

$$\hat{\tilde{\beta}}_1 = _\alpha \frac{\Sigma_{i=1}^n (\tilde{X}_i \otimes \tilde{Y}_i) \ominus n (\overline{\tilde{X}} \otimes \overline{\tilde{Y}})}{\Sigma_{i=1}^n \tilde{X}_i^2 \ominus n \overline{\tilde{X}}^2} \quad (2.7)$$
$$\hat{\tilde{\beta}}_0 = _\alpha \overline{\tilde{Y}} \ominus (\hat{\tilde{\beta}}_1 \otimes \overline{\tilde{X}}). \quad (2.8)$$

III. Fuzzy confidence interval

To testing the true slop $\tilde{\beta}_1$, we have the following theorem.

Theorem 3.1
The least squares estimators are unbiased and have variance :

(1) $E(\hat{\tilde{\beta}}_1) = \tilde{\beta}_1 \quad (3.1)$

(2) $Var(\hat{\tilde{\beta}}_1) = \dfrac{\tilde{\sigma}^2}{\tilde{S}_{XX}}. \quad (3.2)$

[proof] (1) From **Theorem 2.1**

$$\hat{\tilde{\beta}}_1 = \frac{\tilde{S}_{XY}}{\tilde{S}_{XX}} = [\sum_{i=1}^{n} (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes (\tilde{Y}_i \ominus \overline{\tilde{Y}})]$$
$$\oplus [\sum_{i=1}^{n} (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes (\tilde{x}_i \ominus \overline{\tilde{x}})]$$

since $\Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}) = \tilde{0}$, we have

$$\Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes (\tilde{Y}_i \ominus \overline{\tilde{Y}})$$
$$= \Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes \tilde{Y}_i \ominus \overline{\tilde{Y}} \otimes \Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}),$$
$$= \Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes \tilde{Y}_i \ominus \tilde{0}.$$

Put $\tilde{k}_i = (\tilde{x}_i \ominus \overline{\tilde{x}}) \oplus \tilde{S}_{XX}$ then $\Sigma \tilde{k}_i = \tilde{0}$

$$\Sigma \tilde{k}_i \otimes \tilde{x}_i = \Sigma \tilde{k}_i \otimes (\tilde{x}_i \ominus \overline{\tilde{x}})$$
$$= [\Sigma (\tilde{x}_i \ominus \overline{\tilde{x}}) \otimes (\tilde{x}_i \ominus \overline{\tilde{x}})] \oplus \tilde{S}_{XX} = \tilde{1}.$$

Thus we have

$$E(\hat{\tilde{\beta}}_1) = \Sigma \tilde{k}_i \otimes E(\tilde{Y}_i) = \Sigma \tilde{k}_i \otimes (\tilde{\beta}_0 \oplus \tilde{\beta}_1 \otimes \tilde{x}_i)$$
$$= \tilde{\beta}_0 \Sigma \tilde{k}_i \oplus \tilde{\beta}_1 \otimes \Sigma \tilde{k}_i \otimes \tilde{x}_i = \tilde{\beta}_0 \otimes \tilde{0} \oplus \tilde{\beta}_1 \otimes \tilde{1}$$
$$= \tilde{0} \oplus \tilde{\beta}_1 = \tilde{\beta}_1$$

(2) We assume the $Var(\tilde{\varepsilon}_i) = \tilde{\sigma}^2$, and $\tilde{Y}_i$ are independently distributed and has variance $\tilde{\sigma}^2$, so we have

$$Var(\widetilde{\beta_1}) = Var(\sum \widetilde{k_i} \otimes \widetilde{Y_i}) = \sum \widetilde{k_i^2} \otimes Var(\widetilde{Y_i})$$
$$= \sum \widetilde{k_i} \otimes \widetilde{\sigma^2} = [\sum (\widetilde{x_i} \ominus \widetilde{\bar{x}}) \oplus \widetilde{S_{XX}}]^2 \otimes \sigma^2$$
$$= \widetilde{\sigma^2}[\widetilde{S_{XX}} \otimes \widetilde{S_{XX}} \oplus \widetilde{S_{XX}}] \doteq \widetilde{\sigma^2} \oplus \widetilde{S_{XX}}.$$

Since $\widetilde{Y_i} \sim N(\widetilde{\beta_0} \oplus \widetilde{\beta_1} \otimes \widetilde{x_i}, \widetilde{\sigma^2})$, $\widetilde{\beta_1}$ is linear combination of $\widetilde{Y_i}$ to $\sum \widetilde{k_i} \otimes \widetilde{Y_i}$, $\widetilde{\beta_1}$ has also normal distribution.

Theorem 3.2

$\widetilde{\beta_1}$ is fuzzy normal distribution such as

$$\widetilde{\beta_1} \sim_a N(\widetilde{\beta_1}, \frac{\widetilde{\sigma^2}}{\widetilde{S_{XX}}}) \qquad (3.3)$$

[proof] Since $\widetilde{\beta_1}$ is a bundle of β_1, if we have a α-level and one please p in the interval then $[\widetilde{\beta_1}]_p^\alpha$ is normal distribution with $N([\widetilde{\beta_1}]_p^\alpha, [\frac{\widetilde{\sigma^2}}{\widetilde{S_{XX}}}]_p^\alpha)$, by resolution identity, we have $\widetilde{\beta_1} \sim_a N(\widetilde{\beta_1}, \frac{\widetilde{\sigma^2}}{\widetilde{S_{XX}}})$.

Replacing $\widetilde{\sigma^2}$ with its sample estimated standard errors $\widetilde{MSE}$, we obtain the unbiased estimate $\frac{\widetilde{MSE}}{\widetilde{S_{XX}}}$ for $Var(\widetilde{\beta_1})$.

We can test the null hypotheses such as

$$H_0: \widetilde{\beta_1} \simeq \widetilde{0} \qquad (3.4)$$

against a one or a two-side alternative.

We may test wether or not $\widetilde{\beta_1}$ is similarity equal some postulated value $\widetilde{\beta_0}$ with

$$\frac{[\widetilde{\beta_1}]_p^\alpha - [\widetilde{\beta_{10}}]_p^\alpha}{\sqrt{\frac{[\widetilde{MSE}]_p^\alpha}{[\widetilde{S_{XX}}]_p^\alpha}}} \sim t(n-2). \qquad (3.5)$$

In addition to testing hypotheses, we can provide a confidence interval for the parameter β_1 using the t-distribution. For instance $(1-\alpha') \times 100$ % confidence interval of $\widetilde{\beta_1}$:

$$\widetilde{\beta_1} \ominus t_{\frac{\alpha}{2}}(n-2)\sqrt{\frac{\widetilde{MSE}}{\widetilde{S_{XX}}}} < \beta_1 < \widetilde{\beta_1} \oplus t_{\frac{\alpha}{2}}(n-2)\sqrt{\frac{\widetilde{MSE}}{\widetilde{S_{XX}}}} \qquad (3.6)$$

with $d.f. = n-2$.

Ⅳ. Fuzzy statistical test of fuzzy hypotheses

Let $\widetilde{X}$ be a random sample from sample space Ω and $\{P_\theta, \theta \in \Theta\}$ be a family of fuzzy probability, where θ is a parameter and Θ is a parameter space. For each $\psi \in \Theta$, we can consider a family of hypothesis $\{(H_0(\psi), H_1(\psi)) | \psi \in \Theta\}$. We introduce the fuzzy hypothesis as a fuzzy subset.

Definition 4.1 The fuzzy hypothesis H_f is a fuzzy subset of $\{(H_0(\psi), H_1(\psi)) | \psi \in \Theta\}$ with fuzzy hypothesis membership function $\chi_{H_f}((H_0(\psi), H_1(\psi))$.

We set with simplicity

$$\chi_{H_f}(\psi) \equiv \chi_{H_f}((H_0(\psi), H_1(\psi)) \qquad (4.1)$$

also assume normality and convexity.

The fuzzy null hypothesis and the fuzzy alternative hypothesis can be defined as follows.

Definition 4.2 The fuzzy null hypothesis $H_{f,0}$ is a fuzzy subset of Θ with a membership function $\chi_H(\psi)$. The fuzzy alternative hypothesis $H_{f,1}$ is a fuzzy subset of Θ and defined by the equation

$$H_{f,1} = \overline{H}_{f,0} \cap [\{ \bigcup_{\delta \in (0,1]} \delta(\bigcup_{\{\psi | \chi_H(\psi) \geq 0\}} \Theta_{K,\psi})\}] \qquad (4.2)$$

where $\delta(A)$ stands for the fuzzy set whose membership function is product of a scalar δ and the characteristic function of a set A.

The first term of the right hand side corresponds to the negation of the null hypothesis. An example, the fuzzy hypothesis H_f can be interpreted as a hypothesis " $\theta \simeq \psi$ ", which is obtained by adding fuzziness to an ordinary null hypothesis $H_0: \theta = \psi_0$, we note that $H_f: \theta \simeq \psi_0$ for such a fuzzy hypothesis. Now we shall develop a product for testing the fuzzy hypothesis H_f. We assume the existence of a fuzzy test statistic $\widetilde{T}(\psi)$ and critical region $K(\alpha, \psi)$ for a level of significance α.

Definition 4.3 We present he function by a fuzzy subset $H \subset R$. Let us consider fuzzy number $A \subset R$, with we call the agreement index of A with regard to H, the ratio being define the following way :

$$l(A,H) = \frac{(area\ A \cap H)}{(area\ A)} \in [0,1]. \qquad (4.3)$$

Using membership function $\chi_{\tilde{T}}(\psi)$ of $\tilde{T}(\psi)$, we also define the fuzzy hypothesis membership function $\chi_{R\alpha}$ on $\{0,1\}$ as follows.

Definition 4.4 We define the real-valued function r_α on Θ as Definition 1, the maximum grade membership function of acceptance or reject is

$$\chi_{R\alpha}(0) = \sup_\psi \left\{ \frac{area\ (\chi_H(\psi) \cap \chi_{\tilde{T}}(\psi))}{area\ \chi_H(\psi)} \right\} \qquad (4.4)$$

$$\chi_{R\alpha}(1) = 1 - \chi_{R\alpha}(0). \qquad (4.5)$$

Let R_α denotes the fuzzy subset of an entire set $\{0,1\}$ defined by $\chi_{R\alpha}$, since $\{0,1\}$ corresponds {"accept","reject"}, the value $\chi_{R\alpha}(1)$ and $\chi_{R\alpha}(0)$ are equal to the grades of the judgements that the hypothesis is reject and is not rejected respectively. We show the statistical properties of our testing method.

Theorem 4.1. If θ is a element of Θ then the grade of judgements of fuzzy hypothesis

$$\sup(\{\psi \mid \tilde{T}(\psi) \dot{\subset} K(\alpha,\psi)\}: \theta) \leq \alpha \qquad (4.6)$$

is $r_\alpha(\psi)$, where $P(\cdot\,;\theta)$ denotes the probability under the distribution P_Θ.

[proof] We can easily proved from (4.3)

V. Example

We obtained five numbers artificial observation data like two variables $\tilde{X}$ and $\tilde{Y}$. We can find out the fuzzy regression line by **Theorem 2.1** .

For $\alpha \in (0,1]$, the $\alpha-level$ sets of observation data are respectively,

$[\tilde{X}_1]^\alpha = [0.01\alpha+1.99,\ 2.01-0.01\alpha]$
$[\tilde{X}_2]^\alpha = [0.01\alpha+2.99,\ 3.01-0.01\alpha]$
$[\tilde{X}_3]^\alpha = [0.01\alpha+3.99,\ 4.01-0.01\alpha]$
$[\tilde{X}_4]^\alpha = [0.01\alpha+4.99,\ 5.01-0.01\alpha]$
$[\tilde{X}_5]^\alpha = [0.01\alpha+5.99,\ 6.01-0.01\alpha]$

$[\tilde{Y}_1]^\alpha = [0.01\alpha+3.99,\ 4.01-0.01\alpha]$
$[\tilde{Y}_2]^\alpha = [0.01\alpha+6.99,\ 7.01-0.01\alpha]$
$[\tilde{Y}_3]^\alpha = [0.01\alpha+5.99,\ 6.01-0.01\alpha]$
$[\tilde{Y}_4]^\alpha = [0.01\alpha+7.99,\ 8.01-0.01\alpha]$
$[\tilde{Y}_5]^\alpha = [0.01\alpha+9.99,\ 10.01-0.01\alpha]$

For the $\alpha \in (0,1]$, the $\alpha-level$ sets, we obtain the fuzzy sample mean of $\tilde{X}$ an $\tilde{Y}$ as follows:

$[\bar{\tilde{X}}]^\alpha = [0.01\alpha+3.99,\ 4.01-0.01\alpha]$,
$[\bar{\tilde{Y}}]^\alpha = [0.01\alpha+6.99,\ 7.01-0.01\alpha]$.

Therefore for $\alpha \in (0,1]$, we have

$$[\tilde{\beta}_1]^\alpha = [1.1\alpha+11.9,\ -1.1\alpha+14.1] \oplus [0.8\alpha+9.2,\ 10.8-0.8\alpha]$$

$$[\tilde{\beta}_0]^\alpha = [0.01\alpha+6.99,\ 7.01-0.01\alpha] \ominus [1.1\alpha+11.9, -1.1\alpha+14.1] \oplus [0.8\alpha+9.2,\ 10.8-0.8\alpha] \otimes [0.01\alpha+3.99,\ 4.01-0.01\alpha].$$

For $\alpha=0$ and $\alpha=1$, we have estimaters

$$[\tilde{\beta}_1]^0 = [11.9,\ 14.1] \oplus [9.2,\ 10.8] = [1.1007,\ 1.5312],$$
$$[\tilde{\beta}_0]^0 = [6.99,\ 7.01] \ominus [1.007,\ 1.5312] \otimes [3.99,\ 4.01] = [0.8499,\ 2.6183],$$
$$\therefore [\tilde{Y}_i]^0 = [0.8499,\ 2.6183] \oplus [1.1007,\ 1.5312] \otimes [\tilde{X}_i]^0.$$

$$[\tilde{\beta}_1]^1 = [13,\ 13] \oplus [10,\ 10] = [1.3,\ 1.3],$$
$$[\tilde{\beta}_0]^1 = [7,\ 7] \ominus [1.3,\ 1.3] \otimes [4,\ 4] = [1.8,\ 1.8].$$
$$\therefore [\tilde{Y}_i]^1 = [1.8,\ 1.8] \oplus [1.3,\ 1.3] \otimes [\tilde{X}_i]^1.$$

To seek the confidence intervals, we obtain

$$[\tilde{S}^2_{XY}]^\alpha = (0.0881\alpha^2+7.1031\alpha+161.8086,\ 0.1338\alpha^2-7.5442\alpha+176.4105).$$

$$\alpha=0,\quad \widetilde{SSE} = \tilde{S}_{YY} - \frac{\tilde{S}^2_{XY}}{S_{XX}} = (19.6812,\ 20.3220) \ominus (161.8086,\ 176.4105) \oplus (9.7612,\ 10.242) = (19.6812,\ 20.3220) \ominus (15.7985,\ 18.0726) = (1.6086,\ 4.5235)$$

$$\widetilde{MSE} = \frac{\widetilde{SSE}}{n-2} = (0.5362,\ 1.5078)$$

$$\alpha=1,\quad \widetilde{SSE} = (20,\ 20) \ominus (169,\ 169) \oplus (10,\ 10) = 3.1,$$
$$MSE = \frac{SSE}{n-2} = 1.0333$$

Thus, we have a $100(1-\alpha')\%$ the confidence interval membership function by significance level 0.05 for $\tilde{\beta}_1$ with (3.6),

$$\chi_{cI}(x) = \begin{cases} -1.7381+9.8814x & x < 0.2771 \\ 1 & 0.2771 \leq x \leq 2.3229 \\ 5.7905-2.0623x & 2.3229 \leq x \end{cases}$$

If we have two fuzzy hypotheses membership functions :

(1) $\chi_H(x) = \begin{cases} 0.5+5x & x \le 1 \\ 1.5-5x & 1 < x \end{cases}$

(2) $\chi_H(x) = \begin{cases} -4+2x & x \le 2.5 \\ 6-2x & 2.5 < x \end{cases}$

then we obtain the test results by agreement index

$$R(1) = 0.340/0 + 0.660/1$$

$$R(2) = 0.6578/0 + 0.3422/1$$

respectively.

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