Definition of Power Quality Factors at The Point of Common Coupling in Single-Phase Systems and Three-Phase Systems

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Abstract: This paper proposes an unified definition of powers for various circuit conditions such as balanced/unbalanced, sinusoidal/non-sinusoidal, and linear/nonlinear, for single-phase systems and three-phase systems. Conventional reactive power is more classified into an interactive power and a scattering power. These powers are defined both in the time domain and the frequency domain consistently, and agree well with the conservation law. Several important power quality factors are defined to measure and evaluate the power quality for the various circuits in the single-phase and three-phase systems. Simulation results show the power quality factors can evaluate and classify the various circuit conditions clearly.

I. INTRODUCTION

Traditionally, active power has been recognized as a useful power that affects energy transfer between sub-systems, while reactive power has been regarded as a useless power that only increases the apparent power.

Although the definition of powers is explicit and meaningful in sinusoidal single-phase systems, it becomes ambiguous and ineffective, when the power system becomes multi-phased, distorted, unbalanced, and nonlinear [1]-[3]. Many new ideas have been proposed for these new circuit conditions without getting explicit nor unified definitions [4]-[9].

A three-phase circuit can be handled as three single-phase circuits by transforming a-b-c coordinates to p-q-r coordinates with the use of p-q-r theory [10]. Thus, an equivalent approach can be applied in defining powers through single-phase systems and three-phase systems. The instantaneous powers defined in p-q-r coordinates have been analyzed spectrally in the frequency domain for single-phase systems and three-phase systems [11].

This paper proposes unified definition of powers both in the time domain and in the frequency domain for single-phase systems and three-phase systems. Several power quality factors are defined to classify and evaluate the various circuit conditions such as single-phase systems, three-phase systems, balanced voltages, unbalanced voltages, zero-sequence components, harmonic components, unbalanced loads, reactive loads, and nonlinear loads. Simulations of these circuit conditions are done by use of PSIM V4.1.

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II. POWERS IN TIME DOMAIN

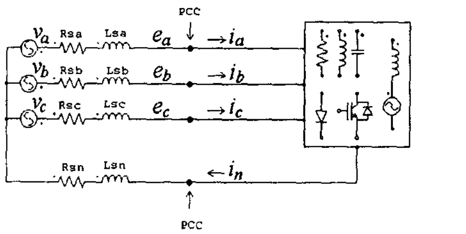

Fig. 1. Analyzed circuit diagram for three-phase four-wire systems.

Fig. 1 shows the generalized circuit that will be analyzed in this paper. The system voltages may be balanced, unbalanced or distorted by harmonics. Loads can be any types such as pure resistive loads, reactive loads, single-phase rectifiers, three-phase rectifiers, balanced or unbalanced. The instantaneous powers will be analyzed at the PCC (point of common coupling).

A. Coordinate Transformation

Voltages in Cartesian a-b-c coordinates can be transformed to Cartesian 0-α-β coordinates as (1).

$$\begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1)$$

In Cartesian p-q-r coordinates, the system currents are defined in [10]. The p-q-r coordinates are rotating along with the system voltage space vector.

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \frac{1}{e_{0\alpha\beta}}\begin{bmatrix} e_0 & e_\alpha & e_\beta \\ 0 & -\frac{e_{0\alpha\beta}e_\beta}{e_{\alpha\beta}} & \frac{e_{0\alpha\beta}e_\alpha}{e_{\alpha\beta}} \\ e_{\alpha\beta} & -\frac{e_0 e_\alpha}{e_{\alpha\beta}} & -\frac{e_\beta e_0}{e_{\alpha\beta}} \end{bmatrix}\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \quad (2)$$

, where $e_{0\alpha\beta} = \sqrt{e_0^2+e_\alpha^2+e_\beta^2}$, $e_{\alpha\beta} = \sqrt{e_\alpha^2+e_\beta^2}$.

In p-q-r coordinates, the system voltages are defined by (3). The voltage exists only in the p-axis.

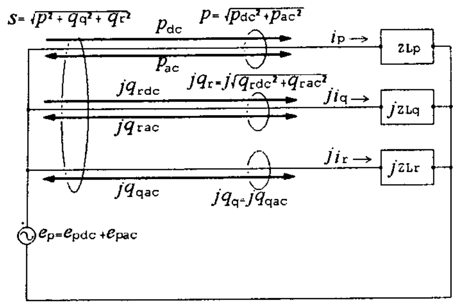

Fig. 1. Equivalent circuit diagram and power flow in p-q-r coordinates.

$$\begin{bmatrix} e_p \\ e_q \\ e_r \end{bmatrix} = \frac{1}{e_{0\alpha\beta}} \begin{bmatrix} e_0 & e_\alpha & e_\beta \\ 0 & -\frac{e_{0\alpha\beta}e_\beta}{e_{\alpha\beta}} & \frac{e_{0\alpha\beta}e_\alpha}{e_{\alpha\beta}} \\ e_{\alpha\beta} & -\frac{e_0 e_\alpha}{e_{\alpha\beta}} & -\frac{e_\beta e_0}{e_{\alpha\beta}} \end{bmatrix} \begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \begin{bmatrix} e_{0\alpha\beta} \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

In a single-phase system, the voltage and current in the p-q-r coordinates can be calculated as (4) through (6). In this case, the voltage e_p and current i_p have only odd-order frequency components without a dc component. A single-phase system has only a p-circuit.

$$e_p = e_a \quad (4)$$

$$i_p = i_a \quad (5)$$

$$e_q = e_r = i_q = i_r = 0 \quad (6)$$

B. Definition of Powers In Time Domain

Instantaneous real power p and the instantaneous imaginary power q_q, q_r are defined as (7).

$$\begin{bmatrix} p \\ q_q \\ q_r \end{bmatrix} = \begin{bmatrix} e_p i_p \\ -e_p i_r \\ e_p i_q \end{bmatrix} \quad (7)$$

The instantaneous imaginary powers q_q, q_r can be compensated without using any energy storage element such as capacitors [10], [11].

Fig. 1 shows the equivalent circuit diagram and the power flow in p-q-r coordinates. The three-phase system voltages are transformed into the single-phase source voltage e_p that comprises a dc component e_{pdc} and an ac component e_{pac}. The three circuits, completely separated from each other, are just connected in parallel to the source voltage e_p. The notation "j" means the imaginary part that is perpendicular to p-axis.

Since the three instantaneous powers are linearly independent of each other, each instantaneous power can be analyzed in the same way as for single-phase systems. Instantaneous powers can be defined in the time domain as given in Table I. Instantaneous active power $\overline{p}$ effectively transfers between two sub-systems.

Three-phase reactive currents interact with other phase voltages and are producing an instantaneous interactive power $\overline{q}$ in a balanced/sinusoidal three-phase system . The instantaneous interactive power $\overline{q}$ increases the instantaneous apparent power s. If the circuit-components are inductive dominant, the sign of $\overline{q}$ becomes positive. Conversely, if the circuit-components are capacitive dominant, the sign of $\overline{q}$ becomes negative. The instantaneous interactive power $\overline{q}$ can be exchanged among the three phases and compensated to zero without using any energy storage element.

Instantaneous scattering power st is useless power that only increases the instantaneous apparent power s. It comes from various non-ideal circuit conditions such as unbalanced or distorted source voltages and/or unbalanced or nonlinear loads. Because of p-axis instantaneous scattering power st_p, the instantaneous scattering power st can be compensated only by using energy storage element such as power capacitors.

TABLE I
DEFINITION OF INSTANTANEOUS POWERS

Definition	Description
$\overline{p} \equiv p_{dc}$	Instantaneous active power
$\overline{q} \equiv q_{rdc}$	Instantaneous interactive power
$st_p \equiv p_{ac}$	p-axis instantaneous scattering power
$st_q \equiv q_q$	q-axis instantaneous scattering power
$st_r \equiv q_{rac}$	r-axis instantaneous scattering power
$st \equiv \sqrt{p_{ac}^2+q_q^2+q_{rac}^2}$	Instantaneous scattering power
$s \equiv \lVert \vec{e} \rVert \cdot \lVert \vec{i} \rVert = \sqrt{p^2+q_q^2+q_r^2} \equiv \sqrt{\overline{p}^2+\overline{q}^2+st^2}$	Instantaneous apparent power

III. POWERS IN FREQUENCY DOMAIN

A. Fourier Analysis of Instantaneous Power

When DC and harmonic components are considered, the voltage and current in a single-phase system can be described as (8) and (9). All the variables in the frequency domain or constants are described as upper cases. Instantaneous variables in the time domain are described as lower cases.

$$e(t) = E_{DC} + \sum_{n=1}^{N_U} \sqrt{2}E_n \sin(n\omega t - \Phi_{en}) \quad (8)$$

TABLE II
SPECTRAL ANALYSIS OF THE GENERATED POWER CELLS WITH A NONLINEAR LOADS

Frequency Order of Power Components	Frequency Order of Current components																		
	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9
8																			1
7																		1	2
6																	1	2	3
5																1	2	3	4
4															1	2	3	4	5
3														1	2	V	4	5	6
2													1	2	3	4	5	6	
1												1	2	3	4	5	6		
0										0	1	2	3	I	5	6			
1								II*(q_q)=>	0		2	3	4	5	6				
2							II*(p, q_r)=>	0	1	2	3	IV	5	6					
3							0	1	2		4	5	6						
4						0	1	2	3	4	5	6							
5					II	1	2	3	4	III	6								
6				0	1	2	3	4	5	6									
7			0	1	2	VI	4	5	6										
8		0	1	2	3	4	5	6											
9	0	1	2	3	4	5	6												
10	1	2	3	4	5	6													

$$i(t) = I_{DC} + \sum_{n=1}^{N_I} \sqrt{2} I_n \sin(n\omega t - \Phi_{in}) \tag{9}$$

With the voltage and current described in (8) and (9), the instantaneous power can be calculated as (10).

$$\begin{aligned} p &= e(t) \cdot i(t) \\ &= E_{DC} I_{DC} + \sum_{n=1}^{N_L} E_n I_n \cos\{\Phi_{en} - \Phi_{in}\} \\ &+ E_{DC} \left(\sum_{n=1}^{N_I} \sqrt{2} I_n \sin\{n\omega t - \Phi_{in}\} \right) \\ &+ I_{DC} \left(\sum_{n=1}^{N_L} \sqrt{2} E_n \sin\{n\omega t - \Phi_{en}\} \right) \\ &+ \sum_{k=2}^{N_L} \sum_{n=1}^{k-1} E_k I_n \cos\{(k-n)\omega t - (\Phi_{ek} - \Phi_{in})\} \\ &+ \sum_{k=1}^{N_L} \sum_{n=k+1}^{N_I} E_k I_n \cos\{(n-k)\omega t + (\Phi_{ek} - \Phi_{in})\} \\ &- \sum_{k=1}^{N_L} \sum_{n=1}^{N_I} E_k I_n \cos\{(k+n)\omega t - (\Phi_{en} + \Phi_{in})\} \end{aligned} \tag{10}$$

When up to the 6th harmonic voltages and the 9th harmonic currents are considered, the generated power cells are distributed spectrally as in Table II [11]. Region I through Region VI correspond to the upper term through the lower term of (10) respectively. The generated power cells marked by shading are mainly related with the unbalanced voltages or the unbalanced currents.

The numbers written on the vertical axis describe the frequency orders of the generated power component. The numbers written on the horizontal axis describe the frequency orders of the current components. The number written on each power cell describes the frequency order of the voltage component.

In the case of single-phase systems, only the instantaneous real power p exists, and the power cells are generated only on even order frequencies since the current i_p and the voltage e_p have only odd order frequency components.

In the case of three-phase systems, the voltage e_p has only even order frequency components. The instantaneous real power p and the r-axis instantaneous imaginary power q_r contains only even order frequency components since i_p and i_q have also only even order frequency components. But the q-axis instantaneous imaginary power q_q contains only odd order frequency components since i_r has only odd order frequency components.

If the loads are linear elements such as resistors, inductors or capacitors, the current has limited orders of frequency. But if the loads are nonlinear such as rectifiers, the current has unlimited orders of frequencies so that the number of the power cells become infinite.

B. *Definition of Powers In Frequency Domain*

Based on the Fourier series analysis before, some important power components can be defined in the frequency domain as shown in Table III for single-phase systems and three-phase systems.

An active power P is defined as the average value of the instantaneous real power p_{dc}, that is equal to the instantaneous active power $\bar{p}$. The active power P comprises a forward-sequence (or fundamental) active power P_f and a distortion active power P_d. The forward-sequence active power is defined as P_f in three-phase systems. However, since there is no forward-sequence active power, the fundamental active power is defined as P_f in single-phase systems. The distortion active power P_d comprises a reverse-sequence active power P_r and a harmonic active power P_h in three-phase systems, but it comprises only the harmonic active power P_h in single-phase systems.

In normal three-phase systems, the forward-sequence active power P_f is the major part. The forward-sequence active power P_f produces a pure rotating-torque on three-phase rotating machines. The reverse-sequence active power P_r results in a torque fluctuation with a double rotating frequency. The harmonic active power exerts also bad effects on three-phase rotating-machines such as over-heating, vibrating, and fluctuating. Only when the loads are resistive such as heaters or light bulbs, the distortion active power is also useful.

A reactive power R is defined as a geometrical sum of an interactive power Q and a scattering power T. The interactive power Q is defined by the average value of the r-axis instantaneous imaginary power

TABLE III
DEFINITION OF SOME IMPORTANT POWER COMPONENTS

*	Definition	Description
	$P \equiv p_{dc}$	Active power.
	$P_f \equiv E_{pdc} \cdot I_{pdc}$	Forward-seq. active power in three-phase systems.
	$P_f \equiv E_{p1} \cdot I_{p1} \cdot \cos\Phi_{p1}$	Fundamental active power in single-phase systems.
	$P_d \equiv P - P_f$	Distortion active power.
	$P_r \equiv E_{p2} \cdot I_{p2} \cdot \cos\Phi_{p2}$	Reverse-seq. active power.
I	$P_h \equiv P_d - P_r$	Harmonic active power.
	$Q \equiv q_{rdc}$	Interactive power.
	$Q_f \equiv E_{pdc} \cdot I_{qdc}$	Forward-seq. interactive power.
	$Q_d \equiv Q - Q_f$	Distortion interactive power
	$Q_r \equiv E_{p2} \cdot I_{q2} \cdot \cos\Phi_{q2}$	Reverse-seq. interactive power.
	$Q_h \equiv Q_d - Q_r$	Harmonic interactive power
II III IV V VI	$T_p \equiv \sqrt{\sum_{k=2,4,6} P_k^2}$	Scattering real power.
	$T_q \equiv \sqrt{\sum_{k=1,3,5} Q_{qk}^2}$	q-axis scattering imaginary power.
	$T_r \equiv \sqrt{\sum_{k=2,4,6} Q_{rk}^2}$	r-axis scattering imaginary power.
	$T \equiv \sqrt{T_p^2 + T_q^2 + T_r^2}$	Scattering power.
All	$R \equiv \sqrt{Q^2 + T^2}$	Reactive power.
	$S \equiv \sqrt{P^2 + Q^2 + T^2}$	Apparent power.

* Related regions shown in Table II.
** Φ : Phase angle between the voltage and current components at the same frequency order.
*** Subscript number describes the frequency order.

q_{rdc}, that is equal to the instantaneous interactive power $\tilde{q}$. The scattering power T is defined by the geometric sum of the ac components of all the instantaneous powers p_{ac}, q_q $(=q_{qac})$, and q_{rac}. The reactive power in single-phase systems comprises only a scattering real power T_p.

The interactive power Q exists only in three-phase systems, and it comprises a forward-sequence interactive power Q_f and a distortion interactive power Q_d. The forward-sequence interactive power Q_f provides necessary reactive currents to the reactive loads such as inductors, capacitors, or induction motors/generators in three-phase systems. The distortion interactive power Q_d can be classified into a reverse-sequence interactive power Q_r that results from unbalance, and a harmonic interactive power Q_h that results from harmonics.

It is interesting to note that the traditional reactive power defined in sinusoidal single-phase systems is associated in the scattering real power T_p, whereas the traditional reactive power defined in sinusoidal balanced three-phase systems is associated in the forward-sequence interactive power Q_f. This is physically true since the traditional reactive power defined in sinusoidal balanced three-phase systems can be compensated without using any energy storage element, while the traditional reactive power defined in sinusoidal single-phase systems can be compensated only by using energy storage elements such as power capacitors.

In fact, single-phase systems can be regarded as unbalanced systems since all the three symmetrical components (forward-sequence, reverse-sequence, and zero-sequence) of a voltage or current are equal to 1/3 times of the rms voltage and rms current respectively in single-phase systems.

C. Decomposing Scattering Power

The scattering power T can be further decomposed into an unbalanced scattering power T_u and a harmonic scattering power T_h. In three-phase systems, the instantaneous unbalanced scattering real power $\tilde{p}_{ub}$ occurs from the interaction between the DC and 2ND order frequency components of the voltage e_p and the current i_p. So that the instantaneous unbalanced scattering real power $\tilde{p}_{ub}$ can be calculated in the time domain as (11) according to (10). Then the unbalanced scattering real power T_{pub} can be calculated in the frequency domain as (14) by the magnitude of the instantaneous unbalanced scattering real power $\tilde{p}_{ub}$. In the same way, the r-axis unbalanced scattering imaginary powers $\tilde{q}_{rub}$ and T_{rub} can be calculated as (13) and (16) respectively.

The q-axis instantaneous unbalanced scattering imaginary power $\tilde{q}_{qub}$ occurs from the interaction between the DC and 2ND order frequency component of the voltage e_p and the 1ST order frequency component of the current i_r. Thus, the q-axis unbalanced scattering imaginary powers $\tilde{q}_{qub}$ and T_{qub} can be calculated as (12) and (15) respectively.

$$\tilde{p}_{ub} = \sqrt{2}E_{pdc}I_{p2}\sin(2\omega t - \Phi_{p2}) + \sqrt{2}E_{p2}I_{pdc}\sin 2\omega t - E_{p2}I_{p2}\cos(4\omega t - \Phi_{p2}) \quad (11)$$

$$\tilde{q}_{qub} = \sqrt{2}E_{pdc}I_{r1}\sin(\omega t - \Phi_{r1}) + E_{p2}I_{r1}\cos(\omega t + \Phi_{r1}) - E_{p2}I_{r1}\cos(3\omega t - \Phi_{r1}) \quad (12)$$

$$\tilde{q}_{rub} = \sqrt{2}E_{pdc}I_{q2}\sin(2\omega t - \Phi_{q2}) + \sqrt{2}E_{p2}I_{qdc}\sin 2\omega t - E_{p2}I_{q2}\cos(4\omega t - \Phi_{q2}) \quad (13)$$

$$T_{pub} = \sqrt{2(E_{pdc}I_{p2})^2 + 2(E_{p2}I_{pdc})^2 + 4(E_{pdc}I_{p2})(E_{p2}I_{pdc})\cos\Phi_{p2} + (E_{p2}I_{p2})^2} \cong P_2 \quad (14)$$

$$T_{qub} = \sqrt{2(E_{pdc}I_{r1})^2 + 2(E_{p2}I_{r1})^2 - 2\sqrt{2}(E_{pdc}I_{r1})(E_{p2}I_{r1})\sin 2\Phi_{r1}} \cong Q_{q1} \quad (15)$$

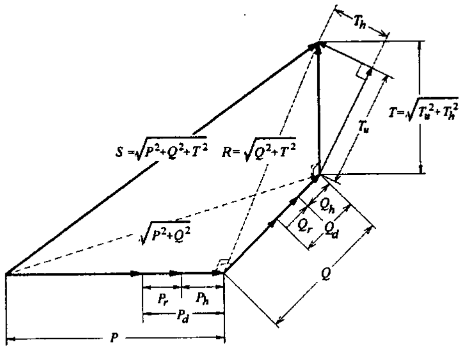

Fig. 2. Physical meaning of the power definition in the frequency domain.

$$T_{rub} = \sqrt{\begin{aligned}&2(E_{pdc}I_{q2})^2 + 2(E_{p2}I_{qdc})^2 \\ &+4\,(E_{pdc}I_{q2})(E_{p2}I_{qdc})\cos\Phi_{q2} + (E_{p2}I_{q2})^2\end{aligned}} \tag{16}$$
$$\cong Q_{r2}$$

In single-phase systems, the instantaneous unbalanced scattering real power component $\tilde{p}_{1ub}$ occurs from the interaction between the 1[ST] frequency components of the voltage e_p and the current i_p. So that the instantaneous unbalanced scattering real power $\tilde{p}_{1ub}$ can be calculated in the time domain as (17) according to (10). The unbalanced scattering real power T_{pub} that is equal to the unbalanced scattering power T_u can be calculated in the frequency domain as (18) by the magnitude of the instantaneous unbalanced scattering real power $\tilde{p}_{1ub}$.

$$\tilde{p}_{1ub} = -E_{p1}I_{p1}\cos(2\omega t - \Phi_{p1}) \tag{17}$$

$$T_u = T_{pub} = E_{p1}I_{p1} \cong P_2 \tag{18}$$

Now, the unbalanced scattering power T_u and the harmonic scattering power T_h can be defined as (19) and (20) respectively in both single-phase and three-phase systems.

$$T_u \equiv \sqrt{T_{pub}^2 + T_{qub}^2 + T_{rub}^2} \tag{19}$$

$$T_h \equiv \sqrt{T^2 - T_u^2} \tag{20}$$

Fig. 2 shows the physical meaning of the proposed definition of powers in the frequency domain. The reactive power R comprises the interactive power Q and the scattering power T in three-phase systems. There is only a scattering power T in the reactive power Q in single-phase systems.

IV. Power Quality Factors

Based on the power definition in the frequency domain so far, several important power quality factors are defined as in Table IV.

The definition of the power quality factors is equal in both three-phase systems and single-phase systems. The only difference occurs in the definition of the fundamental displacement factor DISF. The DISF describes the angle between the fundamental components of the voltage and current. This factor is equal to the traditional power factor cosΦ in ideal single-phase systems or ideal three-phase systems. But the DISF can be defined in any circuit conditions such as linear or nonlinear loads, balanced or unbalanced conditions in single-phase or three-phase systems.

The active power distortion factor APDF shows how much useless distorted active power P_d that gives harmful effects on rotating machines is contained in the active power P. The APDF may come from the reverse-sequence active power P_r and/or the harmonic active power P_h. The active power reverse factor APRF shows how much reverse-sequence active power P_r exists in relation to the forward-sequence active power P_f. The active power harmonic factor APHF shows how much harmonic active power P_h exists in relation to the forward-sequence active power P_f. In ideal case, these values must be zero.

The total power distortion factor TPDF evaluates all the distortions from the active power, interactive power, and the scattering power, which are all useless and only increase the apparent power S. The TPDF comes from the voltage and current that are unbalanced or distorted by harmonics. The total power unbalance factor TPUF evaluates all the power

TABLE IV
DEFINITION OF POWER QUALITY FACTORS

Definition	Description
$\text{DISF} \equiv P_f \,/\, \sqrt{P_f^2 + Q_f^2}$	Fundamental displacement factor in three-phase systems.
$\text{DISF} \equiv \dfrac{P_{1f}}{E_{p1}\cdot I_{p1}} = \cos\Phi_{p1}$	Fundamental displacement factor in single-phase systems.
$\text{APDF} \equiv P_d \,/\, P_f$	Active power distortion factor.
$\text{APRF} \equiv P_r \,/\, P_f^{*}$	Active power reverse factor.
$\text{APHF} \equiv \text{APDF - APUF}$	Active power harmonic factor.
$\text{TPDF} \equiv \sqrt{P_d^2 + Q_d^2 + T^2} \,/\, S$	Total power distortion factor.
$\text{TPUF} \equiv \sqrt{P_r^2 + Q_r^2 + T_u^2} \,/\, S^{*}$	Total power unbalance factor.
$\text{TPHF} \equiv \sqrt{P_h^2 + Q_h^2 + T_h^2} \,/\, S$	Total power harmonic factor.
$\text{FPTF} \equiv P_f \,/\, S$	Forward-seq. power transfer factor.
$\text{TPTF} \equiv P \,/\, S$	Total power transfer factor.

*: This factor cannot defined in single-phase systems.

distortions resulting from the unbalanced voltage or current. The total power harmonic factor TPHF evaluates all the power distortions resulting from the harmonics of the voltage or current. In the ideal case, all these factors TPDF, TPUF and TPHF must be zero.

The forward-sequence power transfer factor FPTF evaluates how much forward-sequence active power P_f is contained in the apparent power S. Thus, the FPTF is important in the application of rotating machines.

The total power transfer factor TPTF describes how much total active power P is contained in the apparent power S. The TPTF is equal or larger than the FPTF. A large value of the total power distortion factor TPDF means small values of both the FPTF and TPTF, since the TPDF evaluates how much useless power is contained in the apparent power.

V. SIMULATIONS

To evaluate various circuit conditions such as single-phase/three-phase systems, linear/non-linear loads, balanced/unbalanced, a simulation model as shown in Fig. 3 is used.

Three key switches are used to select the load types between linear and nonlinear loads. The linear loads are comprised with three R-L loads. There are two types of the nonlinear loads; one type with a three-phase rectifier, the other type with three single-phase rectifiers. The three-phase rectifier has a resistive load of Rna. Each of the three single-phase rectifiers has resistive loads of Rna, Rnb, and Rnc respectively. The dc-link filter in each rectifier has 1 [mH] inductance and 1000 [μF] capacitance.

With the proposed simulation circuit model, 19 different circuit conditions were simulated to evaluate the power qualities by the 7 major power quality factors defined in Table II. Simulations were performed by PSIM V4.1.

Table V and VI show the various circuit conditions and the calculated power quality factors. Table V is the case of linear loads. Table VI is the case of nonlinear loads.

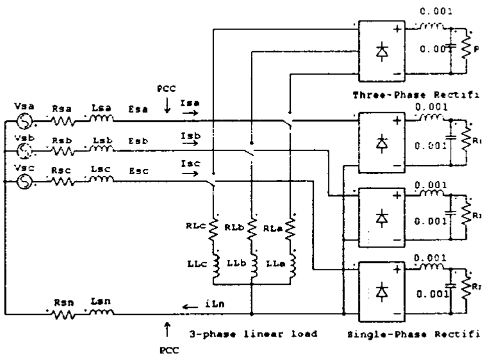

Fig. 3. Simulation circuit model for testing the defined power quality factors.

In the top row sector of each table; the 34 means 3-phase 4-wire systems, the 11 means single-phase systems. For the system voltages; the BV means balanced voltages, the UV means unbalanced voltages, the ZV means that zero-sequence components are included in the system voltages, the HV means that harmonic components are included in the system voltages.

The three phase voltages are 120 [Vrms] in 34BV. The a-phase voltage is reduced to 80 [Vrms] in 34UV that means the system voltage is unbalanced about 8.0[%]. The 3RD harmonic voltage usually becomes zero-sequence component in three-phase systems. The magnitude of the 3RD harmonic voltage is 1/3 times the fundamental voltage.

When the loads have no neutral-line such as in three-phase full-bridge rectifiers in three-phase four-wire systems, the 3RD harmonic voltage contributes to the voltage unbalance and the 2ND harmonics. In this case the unbalance factor becomes about 22.5 [%], and the 2ND harmonic becomes about 2.0 [%].

For the load sides; the BRL means balanced pure resistive loads, the BLL means balanced linear resistive and inductive loads, the ULL means unbalanced linear resistive and inductive loads, the BNL means balanced nonlinear loads, the UNL means unbalanced nonlinear loads.

The resistance and inductance in each phase is 5 [Ω] and 14 [mH] in BLL. Each phase of the load resistances changes to Ra=15 [Ω], Rb=5 [Ω], and Rc=3 [Ω] in ULL. The resistance of each phase is 5 [Ω] in BNL. Each phase of the load resistances changes to Rna=15 [Ω], Rnb=5 [Ω], and Rnc=3 [Ω] in UNL.

Although not simulated here, if the three-phase loads are balanced and pure-resistive in a balanced sinusoidal three-phase systems, all the seven power quality factors will be ideal values such as DISF=100 [%], APDF=0 [%], TPDF=0 [%], TPUF=0 [%], TPHF=0 [%], FPTF=100 [%], and TPTF=100 [%].

Compared to this, when the load is pure-resistive in sinusoidal single-phase systems, the factors DISP, APDF and TPHF are still ideal values, but the other factors are no more ideal values (eg. TPDF=70 [%] and TPTF=72 [%]), which means that the apparent power carries almost the same amount of useful active power P and useless scattering power T. This shows that three-phase systems are more efficient to transfer electrical energy than single-phase systems.

The displacement factors DISF in both 34BV-BLL and 11BV-BLL are 75 [%]. This is the same value calculated by the traditional definition of power factor in R-L loads. When the system voltages are distorted, DISF tends to decrease as can be seen in the cases of 34BV-ULL, 34UV-ULL, 34ZV-BLL, 34ZV-ULL, 34HV-BLL, and 34HV-ULL.

The active power distortion factor APDF increases from the ideal value when the system voltages are unbalanced or distorted by the harmonics. Comparing the cases between balanced linear loads (BLL) and unbalanced linear loads (ULL), the factor APDF becomes large in the case of ULL since the unbalanced loads generate more distorted active power by interacting with the distorted system voltage component.

Comparing the cases between the 34ZV and the 34HV, the factor APDF is larger in the case of the 34HV even though the 3[RD] harmonic voltage is the same in both cases. This comes from the fact that the 3[RD] harmonic voltages are converted to a very large amount of reverse-sequence voltage component among the three phase-lines in the case of the 34HV, while the 3[RD] harmonic voltages become a zero-sequence voltage in the neutral-line where relatively small amount of current flows in the case of 34ZV. If the loads are nonlinear (BNL or UNL), the factor APDF becomes larger. Although, the factor APDF usually has no significant meaning in nonlinear loads such as rectifiers.

The circuit 34BV-BLL that has balanced voltage and balanced linear R-L loads is free from unbalance or harmonics since the total power distortion factor TPDF is zero. But the circuits 11BV-BRL and 11BV-BLL that also have linear load are heavily distorted by the unbalance since the total power unbalanced factor TPUF is more than 70 [%]. Even though the source voltages are the same in the cases of 11BV-BRL and 11BV-BLL, the circuit 11BV-BLL is worse than the circuit 11BV-BRL because the R-L loads interacts with the unbalanced single-phase voltage and make the system worse. The circuits of 34BV-ULL, 34UV-BLL, 34UV-ULL, and 34HV-BLL are also distorted by pure unbalance, since the total power harmonic factors TPHF are all zeros but the total power unbalanced factors TPUF exist in those cases.

Table V
Simulation Conditions And Results; Linear Loads

Linear Loads		34BV -BLL	34BV -ULL	34UV -BLL	34UV -ULL	34ZV -BLL	34ZV -ULL	34HV -BLL	34HV -ULL	11BV -BRL	11BV -BLL
Source oltages [V]	Va 50 [Hz]	120∠0	120∠0	80∠0	80∠0	120∠0	120∠0	120∠0	120∠0	120∠0	120∠0
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠0	40∠0	0	0
	Vb 50 [Hz]	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	-	-
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠-120	40∠-120	-	-
	Vc 50 [Hz]	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	-	-
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠-240	40∠-240	-	-
Loads R & L	Ra [Ω]	5	15	5	15	5	15	5	15	5	5
	Rb [Ω]	5	5	5	5	5	5	5	5	-	-
	Rc [Ω]	5	3	5	3	5	3	5	3	-	-
	La [mH]	14	14	14	14	14	14	14	14	0	14
	Lb [mH]	14	14	14	14	14	14	14	14	-	-
	Lc [mH]	14	14	14	14	14	14	14	14	-	-
Power Quality Factor	DISF [%]	75	72	76	72	74	71	74	67	100	75
	APDF [%]	0	0	0	2	0	1	1	7	0	0
	TPDF [%]	0	55	26	62	32	59	45	65	70	79
	TPUF [%]	0	55	26	62	0	53	45	62	70	79
	TPHF [%]	0	0	0	0	32	27	0	17	0	0
	FPTF [%]	75	60	73	57	70	57	65	50	72	61
	TPTF [%]	75	60	74	58	70	57	66	54	72	61

Table VI
Simulation Conditions And Results; Nonlinear Loads

Nonlinear Loads		34BV -BNL	34BV -UNL	34UV -BNL	34UV -UNL	34ZV -BNL	34ZV -UNL	34HV -BNL	11BV -BNL	11HV -BNL
Source oltages [V]	Va 50 [Hz]	120∠0	120∠0	80∠0	80∠0	120∠0	120∠0	120∠0	120∠0	120∠0
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠0	0	40∠0
	Vb 50 [Hz]	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	120∠-120	-	-
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠-120	-	-
	Vc 50 [Hz]	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	120∠-240	-	-
	150 [Hz]	0	0	0	0	40∠0	40∠0	40∠-240	-	-
Loads Rn	Rna [Ω]	5	15	5	15	5	15	5	5	5
	Rnb [Ω]	5	5	5	5	5	5	-	-	-
	Rnc [Ω]	5	3	5	3	5	3	-	-	-
Power Quality Factor	DISF [%]	98	98	98	98	100	100	100	98	92
	APDF [%]	0	0	2	2	0	0	15	0	26
	TPDF [%]	70	76	73	75	73	80	82	89	83
	TPUF [%]	0	46	27	48	0	42	66	77	30
	TPHF [%]	70	61	68	58	73	68	48	44	78
	FPTF [%]	70	64	67	65	69	61	51	46	45
	TPTF [%]	70	64	67	65	69	61	58	46	57

The circuit 34ZV-BLL is distorted by pure harmonic with the factor of TPHF=32 [%] since the 3^{RD} harmonic component exists in the source voltages. The circuit 34ZV-ULL is distorted by both unbalance and harmonic with the factors of TPUF=53 [%] and TPHF=27 [%] since the source voltages are distorted by 3^{RD} harmonics and the linear R-L loads are unbalanced.

In the case of 34HV-BLL, the power distortion seems to result only from system unbalance with the factor of TPUF=45 [%], since the source voltage is unbalanced at about 22.5 [%] and distorted very slightly by the 2^{ND} harmonics at about 2.0 [%]. Though in the case of 34HV-ULL, the total power harmonic factor TPHF also exists since the unbalanced loads interacts with the 2^{ND} harmonic of the source voltages and generates the 4^{Th} order frequency of the power component.

The total power harmonic factor TPHF appears in all the cases of nonlinear loads, -BNL and -UNL. When the source voltages and the nonlinear loads are balanced as in the cases of 34BV-BNL and 34ZV-BNL, the factor TPHF exists while the factor TPUF is zero, which means the system is distorted by pure harmonics.

It is interesting to note that the total power transfer factor TPTF is better in the case of 11HV-BNL than in case of 11BV-BNL, 57 [%] to 46 [%]. The total power distortion factor TPDF is decreased by injecting the 3^{RD} harmonic to the source voltage in single-phase systems which have nonlinear loads.

VI. Conclusion

This paper defined on powers both in the time domain and in the frequency domain for single-phase systems and three-phase systems consistently. The powers maintain the conservation law. Power was decomposed into active, interactive and scattering power both in the time domain and in the frequency domain. The active power was decomposed into forward-sequence active power, reverse-sequence active power, and harmonic active power.

This paper showed that the conventional reactive power in sinusoidal single-phase systems is classified into the scattering power and the conventional reactive power in balanced sinusoidal three-phase systems is classified into the interactive power.

Several useful power quality factors were defined to identify and evaluate the power quality for the various circuit conditions. The simulation results verified that the proposed power quality factors evaluate and classify the various circuit conditions very clearly.

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