

# Nonlinear Sensorless Control of Induction Motor by using Feedback Linearization and Current Error

Kang-Sung Seo\*, Youn-Ok Choi\*, Geum-Bae Cho\*, Hyung-Lae Baek\*, Sam-Yong Jeong\*\*

\*Dept. of Electrical Eng., Chosun University, 375, Seo-suk Dong, Dong Gu, Gwang Ju, Korea

\*\*Korea hydro & Nuclear Power Co., Korea

## ABSTRACT

This paper describes the nonlinear sensorless control of induction motor by using feedback linearization and current error; the feedback linearization technique and the current error are applied for independent between rotor flux and electric torque and for speed estimation.

The dynamic characteristics of the proposed nonlinear control algorithm involving field weakening area are verified by simulation and experiment.

## 1. INTRODUCTION

In the variable speed control system, induction motor has been widely used since field oriented control method introduced by Hasse on 1968 due to its simple structure, ruggedness, maintenance, low price and so on<sup>[1,2]</sup>.

When the rotor flux decreased below nominal value in order to limit the inverter output voltage on field weakening area, the field oriented control (FOC) is hardly ensure the exact independent control of flux and torque even though the nonlinearity is compensated through feed-forward compensator since this compensator derived by asymptotic linearization<sup>[3]</sup>.

The nonlinear control method based on the feedback linearization technique<sup>[4]</sup> is considered on high performance induction motor drive system to achieve the exact decoupling control of rotor flux and electric torque in a wide speed range. In addition to, we considered the current error on the rotary reference frame for speed estimation with help of one of the recursive parameter identification algorithm, Fixed Trace Algorithm<sup>[5]</sup>.

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## 2. SYSTEM MODELING

The squirrel-cage induction motor is described with a voltage equation on stationary reference frame as equation (1),

$$\begin{bmatrix} V_{ds}^s \\ V_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + pL_s & 0 & pL_m & 0 \\ 0 & R_s + pL_s & 0 & pL_m \\ pL_m & \omega_r L_m & R_r + pL_r & \omega_r L_r \\ -\omega_r L_m & pL_m & -\omega_r L_r & R_r + pL_r \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (1)$$

and its well known differential equation can be derived from the voltage model equation (1) and current and flux relationship as equation (2).

$$\begin{aligned} p i_{ds} &= -a_1 i_{ds} + a_3 \phi_{dr} + a_4 \omega_r \phi_{qr} + b v_{ds} \\ p i_{qs} &= -a_1 i_{qs} - a_4 \omega_r \phi_{dr} + a_3 \phi_{qr} + b v_{qs} \\ p \phi_{dr} &= a_5 i_{ds} - a_7 \phi_{dr} - a_8 \omega_r \phi_{qr} \\ p \phi_{qr} &= a_5 i_{qs} + a_8 \omega_r \phi_{dr} - a_7 \phi_{qr} \\ p \omega_r &= a_9 (i_{qs} \phi_{dr} - i_{ds} \phi_{qr}) - a_{10} T_L \end{aligned} \quad (2)$$

where,

$$\begin{aligned} a_1 &= \left( \frac{R_r M^2}{\sigma L_s L_r^2} + \frac{R_s}{\sigma L_s} \right), \quad a_3 = \frac{R_r M}{\sigma L_s L_r^2}, \quad a_4 = \frac{M}{\sigma L_s L_r}, \\ a_5 &= \frac{R_r M}{L_r}, \quad a_7 = \frac{R_r}{L_r}, \quad a_8 = 1, \quad a_9 = \frac{3N^2 M}{2J L_r}, \\ a_{10} &= \frac{N}{J}, \quad b = \frac{1}{\sigma L_s} \end{aligned}$$

The stator and rotor flux model derived from the voltage model are shown on equation (3) and (4), respectively.

$$\lambda_{dqs} = \int (V_{dqs} - R_s i_{dqs}) dt \quad (3)$$

$$\lambda_{dqr} = (L_r / L_m) \lambda_{dqs} - \sigma L_s i_{dqs} \quad (4)$$

Equation (5) shows the rotor resistance of induction motor<sup>[5]</sup>.

$$R_r = - \frac{p \lambda_{dqs} \lambda_{dqs} + p \lambda_{qrs} \lambda_{qrs}}{\lambda_{dqs} i_{dqs} + \lambda_{qrs} i_{qrs}} \quad (5)$$

### 3. INDUCTION MOTOR CONTROL

#### 3.1 Field Oriented Control

Rotor flux and electric torque can be independently controlled under field oriented control(FOC). Under FOC, the nonlinearity of induction motor is normally compensated through asymptotic linearization.

When PI controller applied on FOC, control scheme can be classified into inner loop for current control and outer loop for flux or speed control.

##### 3.1.1 Compensation of Mutual Coupling Element

The inherent nonlinearity of induction motor under FOC can be compensated through the decoupling voltage equation as shown on equation (6).

$$\begin{aligned} E_{ds}^e &= -\omega_e \sigma L_s i_{qs}^e \\ E_{qs}^e &= \omega_e \sigma L_s i_{ds}^e + \omega_r \frac{M}{L_r} \phi_{dr}^e \end{aligned} \quad (6)$$

##### 3.1.2 Vector Control by PI Controller

Applying the decoupling voltage (6) on equation (1) and (2), the stator current and rotor flux can be linearized as equation (7) and (8).

$$i_{ds}^e = \frac{V_{ds}^e}{R + p \sigma L_s} \quad (7)$$

$$\lambda_{dr}^e = \left( R_r \frac{L_m}{L_r} \right) / \left( p + \frac{R_r}{L_r} \right) i_{ds}^e \quad (8)$$

Using the above linearized current (7), flux model (8) and PI controllers, whole block diagram of the induction motor drive system can be constructed as Fig 1 based on the rotary reference frame.

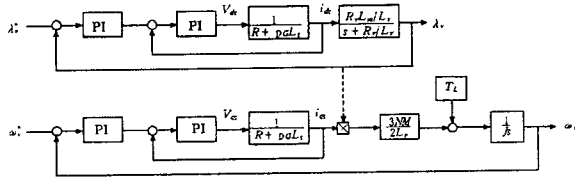


Fig. 1 Vector control of IM with the cascade PI controllers.

On Fig. 1, the DC-AC inverter is eliminated and the dotted line means the asymptotic decoupling of torque from rotor flux since the rotor flux is maintained constantly at steady state condition.

#### 3.2 Nonlinear Control Feedback Linearization

Independent control of rotor flux and torque is not maintained when rotor flux varied in field weakening

area since the electric torque is a function of rotor flux and current under FOC.

In order to derive the exact decoupling model of rotor flux and electric torque of induction motor, the feedback linearization technique is applied on equation (2).

##### 3.2.1 Feedback Linearization

Induction motor can be linearized by applying the feedback linearization method as following<sup>[6]</sup>.

If we define new variables as equation (9),

$$\begin{aligned} \phi_1(x) &= x_3^2 + x_4^2 \\ \phi_2(x) &= x_5 \end{aligned} \quad (9)$$

and new z coordinates as equation (10)

$$\begin{aligned} z_1 &= \phi_1(x) \\ z_2 &= L_f \phi_1(x) \\ z_3 &= \phi_2(x) \\ z_4 &= L_f \phi_2(x) \end{aligned} \quad (10)$$

The equation (10) can be rewritten as equation (11).

$$\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_3 \end{bmatrix} = \begin{bmatrix} L_f^2 \phi_1 \\ L_f^2 \phi_2 \end{bmatrix} + D \begin{bmatrix} u_d \\ u_q \end{bmatrix} \quad (11)$$

where,  $L_f^2 \phi_1$  and  $L_f^2 \phi_2$  are second order Lie derivatives and D is decoupling matrix as equation (12).

$$D = \begin{bmatrix} L_{g_d} L_f \phi_1 & L_{g_q} L_f \phi_1 \\ L_{g_d} L_f \phi_2 & L_{g_q} L_f \phi_2 \end{bmatrix} \quad (12)$$

From the equation (11), input u variable is expressed with a new controlled input v as equation (13)

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = D^{-1} \left( \begin{bmatrix} -L_f^2 \phi_1 \\ -L_f^2 \phi_2 \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \right) \quad (13)$$

##### 3.2.2 Applying the PI Controller on Nonlinear Control

In this paper, PI controller is applied on linearized model of induction motor instead of state feedback control as shown on Fig. 2. And the stator current is limited to pre-determined value at transient condition.

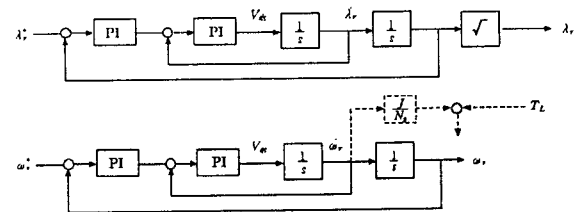


Fig. 2 Nonlinear control with PI controllers.

## 4. INDUCTION MOTOR VARIABLE ESTIMATION

### 4.1 Rotor Flux and Speed Estimation

If all parameter of induction motor is known, we can easily estimate the rotor flux from equation (2). Also, we can estimate the rotor speed from the q-axis current error between measured and estimated current. Fig. 3 shows the concept of speed estimation by using the current error.

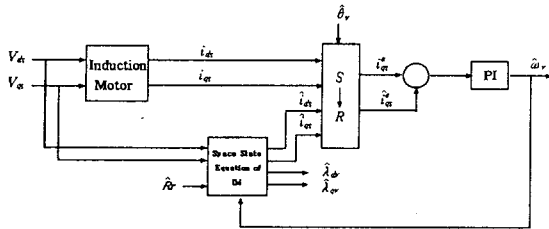


Fig. 3 Speed estimation with current error.

### 4.2 Rotor Resistance Estimation

On speed estimation by using the current error, we assume that all parameter is known. However, motor parameters can be varied according to operation condition. The rotor winding resistance, as a typical parameter varied according to operation condition, should be compensated to achieve exact decoupling control and speed and rotor flux estimation. Rotor resistance described as equation (6) can be traced by recursive parameter identification such as Fixed Trace Algorithm (FTA)<sup>[4]</sup>. The recursive algorithm of Fixed Trace Algorithm (FTA) is shown on equation (14).

$$\hat{\theta}[m] = \hat{\theta}[m-1] - P[m](\theta[m-1]x[m] - y[m]) \quad (14)$$

where,  $\hat{\theta}[m]$  is estimated parameter and  $P[m]$  is gain as follow;

$$P[m] = \frac{P[m-1]x[m]}{\lambda + P[m-1]x[m]^2} \quad (15)$$

where, the forgetting factor  $\lambda$  is set as equation (16).

$$\lambda = \frac{1}{1 + \gamma x[m]^2} \quad (16)$$

## 5. SIMULATION

The proposed nonlinear control and rotor speed estimation algorithm are verified by simulation. Table 1 shows parameters of induction motor used on simulation.

Table 1 Induction Motor Parameters

Quantity	Symbol	Value
Power	P	1.5[kW]
Phase Voltage	V	220[V]
Stator Resistance	R <sub>s</sub>	1.1806[Ω]
Rotor Resistance	R <sub>r</sub>	1.1712[Ω]
Stator Inductance	L <sub>s</sub>	94.84[mH]
Rotor Inductance	L <sub>r</sub>	94.84[mH]
Mutual Inductance	M	91.89[mH]
Rotor Inertia	J	0.007[kg.m <sup>2</sup> ]
Rated Speed	ω <sub>r</sub>	1800[rpm]

### 5.1 FOC vs Nonlinear Control

Fig. 4 shows the simulation results under speed reversal within  $\pm 3000$  [rpm] under FOC. Where, the reference signal of rotor flux is fixed at 0.4[Wb] when rotor speed is lower than rated speed  $\pm 1800$  [rpm]. When rotor speed is beyond the rated speed, the reference signal of rotor flux is inverse proportion to rotor speed and it reduced to zero at twice of rated speed. From Fig. 4, the rotor flux(b) and electric torque(d) have nonlinearity while q-axis stator current is kept at 10[A] during the transient condition owing to speed reversal. From these results, we can see that independent control of rotor flux and torque is not maintained in field weakening area.

Simulation results under nonlinear control are shown on Fig. 5. Trend of q-axis current(c) is opposed to that of rotor flux(b) during speed reversal period while torque(d) is kept at constant level. From these results, we can easily understand that the electric torque is independent on the rotor flux under the proposed nonlinear control at field weakening area.

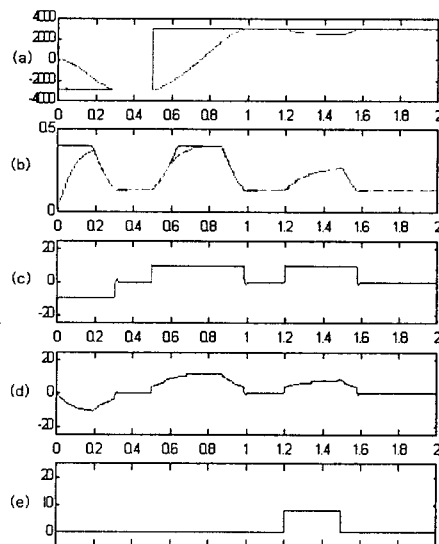


Fig. 4 Dynamics of IM under vector control

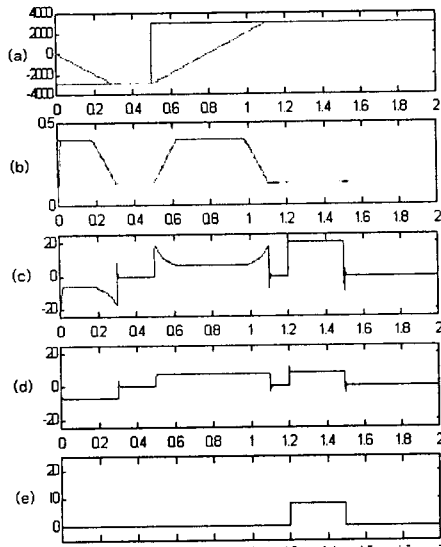


Fig. 5 Dynamics of IM under nonlinear control

### 5.2 Sensorless Control

Fig. 6 shows the simulation results under speed sensorless nonlinear control with speed and rotor resistance estimation. The rotor resistance(f) is set at nominal value of Table 1 for initial 0.4[sec] to prevent transient phenomena owing to large estimated rotor resistance error during simulation starting. Reference signal of rotor flux(c) is set at 0.3 [Wb] below rated speed. Rotor resistance(f) is step changed from 100% of nominal value to 200% at 0.7[sec]. There are some noise on the estimated speed(b) and

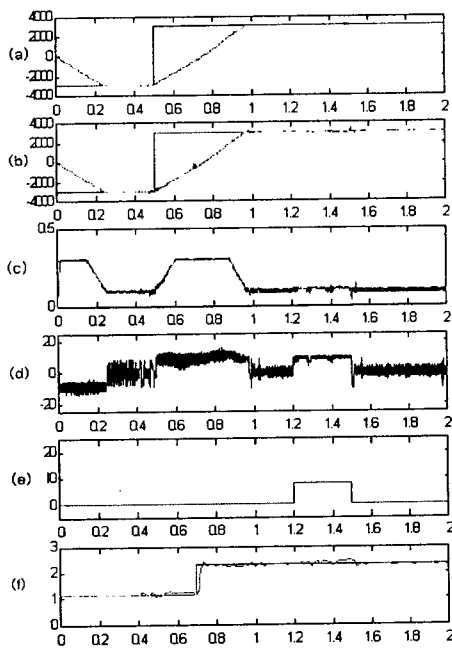


Fig. 6 Nonlinear sensorless control of induction motor

relatively high noise signal on the electrical torque(d) which caused by the estimation error of rotor resistance.

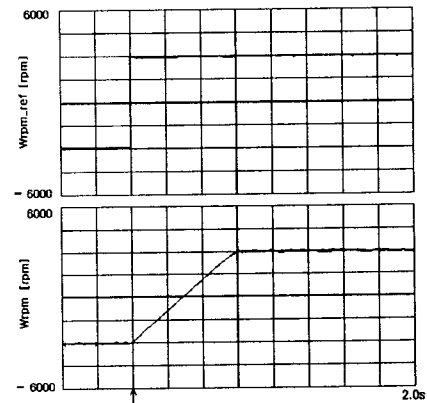
From Fig. 6, the independent control of rotor flux and torque is still maintained under the proposed estimation algorithm even though further study is needed on the noise cancellation

## 6. EXPERIMENT RESULTS

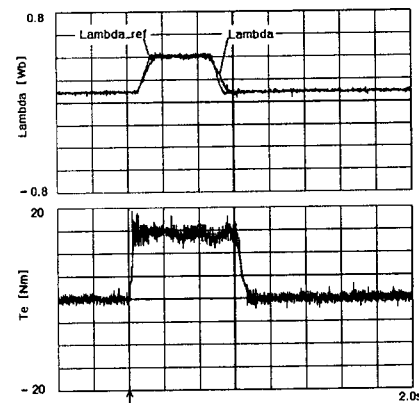
Experimental results of the proposed control with a 1.5[kW] induction motor drive system are shown on Fig. 7 and Fig. 8.

Fig. 7 shows the dynamic characteristics of induction motor at field weakening area under reversal operation within  $\pm 3000$  [rpm].

Fig. 8 shows the dynamic characteristics of induction motor at field weakening area under reversal operation within  $\pm 3000$  [rpm] with speed estimation.

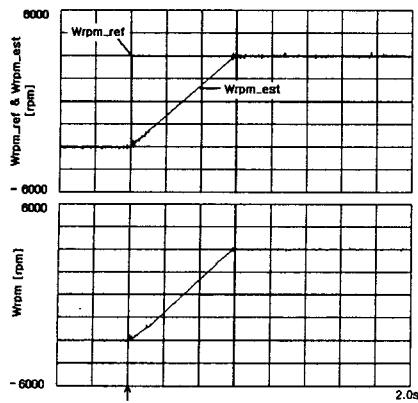


(a) Speed reference and rotor speed

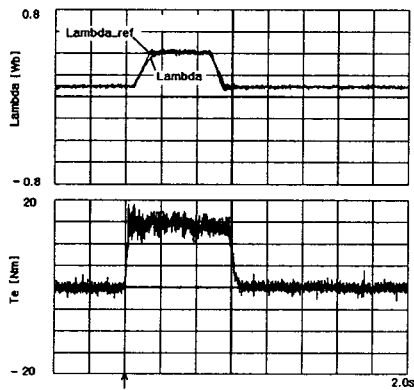


(b) Rotor flux and electrical torque

Fig. 7 Dynamic response of IM under reversal operation with  $\pm 3000$ [rpm].



(a) Speed reference & rotor speed



(b) Rotor flux and electrical torque

Fig. 8 Sensorless control of IM under reversal operation with  $\pm 3000$ [rpm].

Form Fig. 7 and Fig. 8, we can see that independent control of rotor flux and electric torque is maintained under the proposed sensorless nonlinear control even though the noise on the estimated speed and electric torque are increased on Fig. 8.

## 7. CONCLUSION

In this paper, the nonlinear control of induction motor by feedback linearization method and simple rotor speed estimation algorithm based on the current error of space state equation of induction motor are introduced.

In comparison with simulation results under FOC and the proposed nonlinear control at field weakening area, independent control of rotor flux and motor torque under nonlinear control is maintained while FOC does not.

On this paper, a simple speed estimation algorithm based on the estimated current error and parameter identification algorithm is also introduced for speed

sensorless control. Further study is expected on noise cancellation involved on the estimated speed and motor torque.

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