# Reduction of Torque Ripple of PMSM Using Iterative Flux Estimation

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**Abstract** - PMSM drives are widely used in the industrial and residential applications because of high efficiency, high power density and high performance. For better performance of PMSM, however, torque ripples should be reduced.

This paper investigates a reduction of torque ripple due to the unsinusoidal flux linkage produced by the shapes of stator slot and magnetic pole. To minimize torque ripple, a simple flux estimator is proposed. This method interatively compensates the distributed flux linkage from an error between the measured and estimated currents. The proposed algorithm is verified through simulation.

**Key Words** PM synchronous motor, torque ripple, state observer, iterative flux estimator.

#### 1. INTRODUCTION

PM synchronous motor drives are widely used in the industrial and residential applications because of high efficiency, high power density and high performance. Especially, in the servo drive system and home appliance, demand of PM synchronous motor drives is much increasing recently.

The problems of PM synchronous motor drives are the use of expensive position sensor and torque ripple. Sensorless control of PM synchronous is much studied for eliminating position sensor[1-2]. However, torque ripple of PM synchronous motor drives due to linkage flux harmonics, cogging and current harmonics is an important research subject, yet.

In the normal speed region, it is not necessary to consider this problem because the torque ripple is filtered out by the inertia of the motor and load. However, torque ripple of PM synchronous motor largely affects drive performance in the low-speed and direct-drive applications[3-5].

This paper investigates a reduction of the torque ripple of the PM synchronous motor produced from

unsinusoidal flux distribution. In order to minimize torque ripple, the iterative linkage flux compensation is proposed in this paper. The proposed linkage flux compensation is obtained from current error between reference and adaptive model. The proposed algorithm for reduction of the torque ripple is very simple. This method is verified through simulation results.

#### 2. MATHEMATICAL MODELING OF PMSM

The PM synchronous motor considered in this paper has a surface-mounted permanent magnet rotor and three-phase stator winding. Fig. 1 shows the equivalent model of PM synchronous motor expressed in the 2-phase reference coordinates. In the figure,  $L_s$  and  $R_s$  indicates the inductance and resistance of the stator winding respectively. N and S explain the pole of the magnetic rotor.

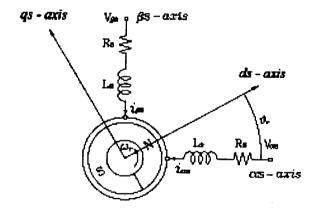


Fig. 1 The equivalent model of PM synchronous motor in the 2-phase reference frame

With assumption of cylindrical rotor and sinusoidal flux distribution, The stator voltage equation in the

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rotor reference frame can be expressed as follows[1].

$$v_{ds} = R_s i_{ds} + p L_s i_{ds} - \omega_r L_s i_{qs}$$

$$v_{qs} = R_s i_{qs} + p L_s i_{qs} + \omega_r L_s i_{ds} + \omega_r \Psi_m$$
(1)

where,  $\omega_r$  is the angular speed of the motor.  $\Psi_m$  is the linkage flux and p is differential operator.  $v_{d,qs}$  and  $i_{d,qs}$  are the voltage and current of the motor in the rotor reference frame respectively.

In (1), flux of rotor is expressed as a constant value with the assumption of sinusoidal flux distribution. However, the actual flux distribution of PMSM motor has periodic flux harmonics because of slot effect and unsinusoidal flux distribution.

The voltage equation of PMSM with unsinusoidal flux distribution can be described as follows.

$$v_{ds} = R_s i_{ds} + p L_s i_{ds} - \omega_r L_s i_{qs} + \omega_r \psi_{qm}(\theta_r)$$

$$v_{qs} = R_s i_{qs} + p L_s i_{qs} + \omega_r L_s i_{ds} + \omega_r \psi_{dm}(\theta_r)$$
(2)

where,  $\psi_{dm}(\theta_r)$ ,  $\psi_{qm}(\theta_r)$  are the  $d_s$ - $q_s$  fluxes and can be described as a function of rotor position. Considering the non-saliency in surface mounted PM synchronous motor, the stator inductances do not vary with rotor position. Hence, the  $d_s$ - $q_s$  axis inductance can be taken as  $L_{ds} = L_{qs} = L_s$ .

From mutual relation between unsinusoidal flux and stator currents, the output torque is obtained as follows.

$$T_e = \psi_{dm}(\theta_r) i_{qs} + \psi_{qm}(\theta_r) i_{ds}$$
 (3)

From (3), the torque of PM synchronous motor produces the additional torque ripple with constant torque current and unsinusoidal flux distribution. For the constant torque, the current must be corrected according to the actual fluxes.

### 3. ITERATIVE FLUX ESTIMATION

In this paper, iterative flux estimator is proposed to minimize torque ripple of the PM synchronous motor. The actual linkage flux has the periodic harmonic components and can be described as a function of rotor position. In order to estimate the actual flux distribution, flux estimator with iterative learning technique is designed.

State equations of the PM synchronous motor with unsinusoidal flux distribution as follows.

$$\dot{x} = A x + B u + D \psi$$

$$\dot{i}_s = C x$$
(4)

where, 
$$\mathbf{x} = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}^T$$
,  $\mathbf{u} = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T$ ,  $\mathbf{i}_s = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}^T$ 

$$\mathbf{\psi} = \begin{bmatrix} \psi_{dm}(\theta_r) & \psi_{qm}(\theta_r) \end{bmatrix}^T$$

$$\mathbf{\dot{x}} = \begin{bmatrix} pi_{ds} & pi_{qs} \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -\frac{R_s}{L_s} & \omega_r \\ -\omega_r & -\frac{R_s}{L_s} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} 0 & -\frac{\omega_r}{L_s} \\ -\frac{\omega_r}{L_s} & 0 \end{bmatrix}$$

The adaptive observer can be expressed from the reference model as follows.

$$\hat{x} = A \hat{x} + B u + D \hat{\phi} + L e \qquad (5)$$

where  $e = x - \hat{x}$ , L: gain matrix

The *e* matrix denotes the error of reference and adaptive model and is used to estimate the actual flux. With assumption of stator resistance and inductance are equal to the actual model, the error term is produced from ideal flux and actual flux distribution. The proposed iterative flux estimation can tracking the actual value. The propose flux estimator as follows.

$$\Delta \widehat{\phi_{dqm}}(\theta_r)_k = k_{fp}e + k_{fi} \int e dt$$
 (6)

$$\widehat{\phi_{dqm}}(\theta_r)_k = \widehat{\phi_{dqm}}(\theta_r)_{k-1} + \Delta \widehat{\phi_{dqm}}(\theta_r)_k \qquad (7)$$

where,  $k_{fp}$ ,  $k_{fi}$  are the proportional and integral gain of flux estimator respectively. The actual flux may be obtained from iterative flux estimation.

The reference current to minimize torque ripple from unsinusoidal flux distribution can be determined as follows.

$$i_{qs}^* = \frac{T_e^* - i_{ds} \cdot \widehat{\psi}_{qm}(\theta_r)}{\widehat{\psi}_{dm}(\theta_r)}$$

$$i_{ds}^* = 0$$
(8)

where,  $T_e^*$  is reference torque, and  $\widehat{\psi}_{dm}(\theta_r)$  and  $\widehat{\psi}_{qm}(\theta_r)$  are the estimated fluxes.

Fig. 2 describes a bolck diagram of state observer with the proposed flux estimation.

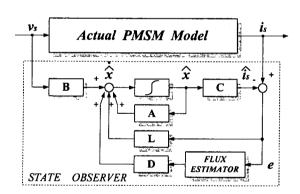


Fig. 2 The bolck diagram of adaptive observer and flux estimator

# 4. SIMULATION AND DISCUSSION

To verify the proposed algorithm for torque ripple minimization, simulation has been performed. Table 1 shows the specificaion of the prototype PMSM.

From the FEM analysis, flux of  $d_s - q_s$  axis according to the rotor position can be obtained.

Table 1 The specification of PM synchronous motor

The number of rotor pole	8 pole
Rated output	600 W
Rated current	5.8 A
E.M.F constant	0.175 Vsec/rad
Resistance of stator winding	0.85 Ω
Inductance of stator winding	3.5 mH
Winding connection	Y connection
Rotor inertia	0.0243 Kgm <sup>2</sup>

For the analysis of flux distribution, Flux-2D FEM analysis tool is used. Fig. 3 shows the linkage flux distribution in the motor of table 1. The motor has 24 stator slots.

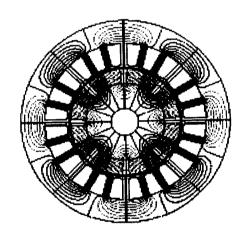


Fig. 3. The distribution of linkage flux

Fig. 4 shows the flux  $\Phi_{dm}(\theta_r)$ ,  $\Phi_{qm}(\theta_r)$  of  $d_s$  -  $q_s$  axis according to the rotor position.

From the results,  $\Phi_{dm}(\theta_r)$  and  $\Phi_{qm}(\theta_r)$  are not constant value and has periodic harmonics due to the shape of magnetic pole and slot effects. The major harmonics of the flux distribution are the 6th and 12th harmonics which is proportional to the slot numbers per pole.

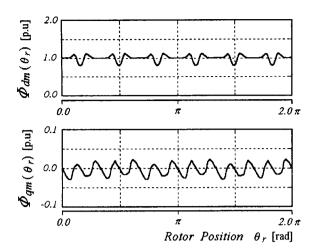


Fig. 4. The flux waveform with rotor position

In order to verify the effectiveness of the proposed algorithm, the sampling period of the current controller is set as  $100\mu$ s and 2ms for speed controller. The reference speed is 100rpm that is 0.1 p.u. of the motor and load torque is 2Nm.

In the simulation, the proposed stator observer with iterative flux estimator operates after 1.0 second.

Fig 5, fig 6 and fig 7 show the simulation results.

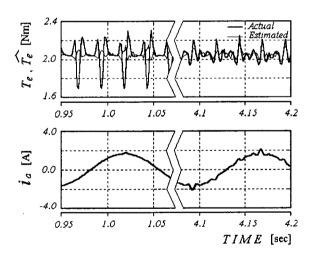


Fig. 5. Torque and phase current waveforms

In the fig. 5, the actual torque and phase current waveform are explained. In the figure, the subscript '^' denotes the estimated values.

With the mutual reaction of the sinusoidal phase current and unsinusoidal flux, actual torque has periodic torque ripple. In order to reduce the periodic torque ripple, phase current has periodic harmonics of 6th and 12th as same as the estimated flux.

With the proposed algorithm, the torque ripple is reduced and current contains the harmonic components to produce constant torque.

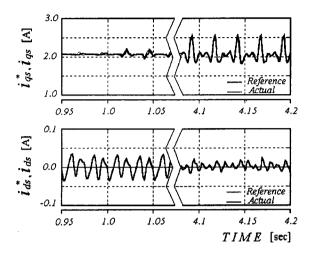


Fig. 6. The reference and actual currents

Fig. 6 shows the  $d_s$ - $q_s$  reference currents and actual ones. The constant torque current  $i_{qs}$  produces the torque ripple with unsinusoidal flux distribution. In the proposed algorithm, the torque current is distorted to produce constant torque. In addition, with the estimated flux the current error can be decreased.

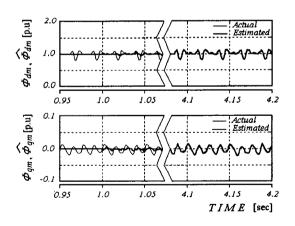


Fig. 7. The actual and estimated linkage flux

Fig. 7 shows the estimated and actual flux waveform as a simulation results.

As stated, estimated flux is well tracked as a actual one with the use of iterative flux estimator.

# 5. CONCLUSION

This paper investigates minimization of torque ripple of PM synchronous motor. In order to decrease the torque ripple due to unsinusoidal flux distribution of actual motor, iterative flux estimation is proposed. The proposed linkage flux compensation is obtained from current error between reference and adaptive model.

The actual flux distribution is obtained from iterative flux estimator and is utilized to reduce the torque ripple. The reference current is determined from the estimated flux distribution with period harmonics. As the result, reference current has additional current ripple, and the electromagnetic torque ripple can be reduced according to the mutual action with unsinusoidal flux and harmonic currents.

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