이산시간 TS 퍼지 시스템의 스위칭모드 제어기의 설계

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Design of Switching-Type Controller for Discrete-Time TS Fuzzy Systems

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Abstract - A controller design problem for a discrete-time Takagi-Sugeno (TS) fuzzy systems is discussed. The switching-type controller is employed in this study. A switching-type fuzzy-model-based controller is constructed based on the spirit of "devide and conquer". The design condition of this controller is formulated in terms of linear matrix inequalities (LMIs), which guarantees the global stability of the controlled TS fuzzy systems. An example is included for ensuring the effecienct of the proposed control method.

1. Introduction

Recently, a switching-type fuzzy-model-based controller has been extensively considered in designing suitable controllers for a class of nonlilnear systems that can be represented as TS fuzzy systems [5,7]. However, it has been known as an important and difficult problem to design a controller which guarantees its global stability for actual systems. Furthermore, the specific design of a fuzzy logic controller has difficulties in acquisition of the expert's knowledge and relies to a great extent on empirical and heuristic knowledge and that, in many cases, may not be justified.

Takagi and Sugeno proposed a new kind of the fuzzy inference system, called Takaki-Sugeno (TS) fuzzy model in 1985 [1]. It can combine the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools into an unified framework. Since it employs linear models in the consequent parts, it is convenient to apply the conventional linear systems theory for synthesis of the controller [6]. For this reason, various kinds of TS fuzzy model based controllers have been suggested. In these methods, fuzzy sets are used to construct suitable local linear state models from which local controllers can be determined. The stability of the overall systems is then determined by a Lyapunov stability analysis and linear matrix inequalities (LMIs) approach. However, in these works, especially when the number of fuzzy rules were very large, it was not easy to find a common positive definite matrix to satisfy Lyapunov equations.

Another approach is proposed by Cao et al.[2-4] They use an uncertain linear model to analyze the stability of the fuzzy-model-based controller. However, it shows conservatism since the upper bounds which represent the interactions between fuzzy rules can not be exactly determined. The purpose of this work is to construct more relaxed stability condition of fuzzy-model-based controller with the techniques in modern control theory. The main contribution of this work can be summarized as follows. It is shown that the TS fuzzy model can be reduced to the smaller TS fuzzy models if the number of fuzzy rules in the fuzzy rule base. The stability analysis of the fuzzy-model-based controller is performed with piecewise quadratic (PQ) Lyapunov function.

This paper is organized as follows: In Section 2, discrete-time TS fuzzy system is briefly reviewed. In section 3, we will design a fuzzy-model-based controller which satisfies the stability condition. Finally, the proposed control method is verified by simulation in section 4.

2. TS Fuzzy Model in Discrete Time

Consider a discrete-time TS fuzzy system:
Plant Rule i:

$$\text{If } x_1(t) \text{ is } F_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } F_n^i \quad (1)$$

$$\text{THEN } x(t+1) = A_i x(t) + B_i u(t)$$

where F_j^i $(j=1,\cdots,n,\ i=1,\cdots,r)$ are fuzzy sets. Rule i denotes the ith fuzzy inference rule. t and $t+1$ denote the indexes of the time steps. The defuzzified output of this TS fuzzy system is represented as follows:

$$x(t+1) = \sum_{i=1}^{r} \mu_i(x(t))(A_i x(t) + B_i u(t)) \quad (2)$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} F_j^i(x_j(t)),\quad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{r}\omega_i(x(t))}$$

in which $F_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in F_j^i.

3. Design of a Switching Type Fuzzy-model-based Controller

This section deals with the design method which derived from a switching-type controller theory. Piecewise quadratic Lyapunov functions are used to analyze the stability of the switching-type fuzzy-model-based controller. before illustrating we need to know the following assumption.

Assumption 1 *The number of fuzzy rules, which are fired simultaneously for all, $t>0$ is $s<r$.*

In this case, we can define the subspace S_l ($l=1,2,\cdots,s$) in the entire input spaces as the space where s rules fired concurrently at an instant. The characteristic function of S_l is defined by

$$\eta_l = \begin{cases} 1 & x \in S_l \\ 0 & x \notin S_l \end{cases} \qquad \sum_{l=1}^{m}\eta_l = 1 \tag{3}$$

Define I_l as the set of indices of fuzzy rules in S_l. then, on every subspaces, the fuzzy system can be denoted by

$$\begin{aligned} x(t+1) &= \overline{A}_l(t)x(t) + \overline{B}_l(t)u(t) \\ &= \sum_{i\in I_l}\mu(x(t))A_i + \sum_{i\in I_l}\mu(x(t))B_i u(t) \end{aligned} \tag{4}$$

where $x(t) \in S_l$. Therefore, the global system can be represented using (3), (4) as follows:

$$x(t+1) = \sum_{l=1}^{m}\eta_l\,(\overline{A}_l x(t) + \overline{B}_l(t)u(t)) \tag{5}$$

Actually system (5) is the piecewise linear combination of the smaller fuzzy systems. The above result is summarized as the following theorem.

Theorem 1 *The fuzzy system (1) can be transformed to the piecewise linear time varying system (5), where each subsystem is the smaller fuzzy system if the number of fuzzy rules fired simultaneously for all $t>0$ is $s<r$.*

In order to use the PQ Lyapunov functions, of the Lyapunov function need to be carefully handled, since the employed piecewise quadratic Lyapunov function candidate is a class of discontinuous functions on the boundary of any two adjacent subspaces.

Assumption 2 *If lth subsystem is in the lth state space, it will stay in the lth subspace for a period of time t where*

$$x(t) \in S_l \textit{ at } t=\tau_i \textit{ and } x(t) \in S_j,\ \ j \neq l,\quad t=\tau_i+1$$

and the number of traversing tome instants among regions S_l is finite.

First, to represent a switching-type fuzzy-model-based controller, we consider the autonomous fuzzy system(u=0) as follows. We use PQ Lyapunov functions as a tool for analyzing Lyapunov stability of (5) with $u(t)=0$. Let

$$V_l(x(t)) = x(t)^T P_l\, x(t) \tag{6}$$

be a Lyapunov function for subspace S_l. Then the global Lyapunov function can be constructed as

$$V(x(t)) = \sum_{l=1}^{m}\eta_l(x(t))\ x^T(t)P_l\,x(t) \tag{7}$$

This kind of Lyapunov function widely used for the stability analysis of piecewise linear systems. [2-5]

Lemma 1 *The fuzzy system (1) (with $u=0$) is quadratically stable if there exists symmetric matrix P_l such that*

$$P_l > 0 \tag{8}$$

$$\overline{A}_l^T P_l\,\overline{A}_l - P_l < 0,\quad (l=1,2,\cdots,m) \tag{9}$$

If input exist we can obtain following theorem.

Theorem 2 *The fuzzy system (1) is quadratically stabilizable via fuzzy controller if there exists symmetric matrices P_l such that*

$$P_l > 0 \tag{10}$$

$$(\overline{A}_l - \overline{B}_l\,\overline{K}_l)^T P_l (\overline{A}_l - \overline{B}_l\,\overline{K}_l) - P_l < 0,\quad (l=1,2,\cdots,m) \tag{11}$$

where $\overline{K}_l = \sum_{i\in I_l}\mu_i\ (x(t))K_i$

From the stability condition of Theorem 2, the design problem can be defined as well. Schur complement is used in the transformation.

$$\begin{bmatrix} -X_l & (A_l X_l - B_l M_l)^T \\ A_l X_l - B_l M_l & -X_l \end{bmatrix} < 0,\quad (l=1,2,\cdots,m) \tag{12}$$

where

$$X_l = P_l^{-1},\ M_l = K_l X_l$$

4. An Example

In this section, to show the effectiveness of the proposed design method, we adopt discrete-time TS fuzzy system that comrises three fuzzy rules, whose system matrices are as follows:.

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix},\qquad A_2 = \begin{bmatrix} 0 & 1 \\ -6.75 & -0.1 \end{bmatrix}$$

$$A_3=\begin{bmatrix} 0 & 1 \\ -10 & -0.1 \end{bmatrix},$$

$$B_1=\begin{bmatrix} 1 \\ 0 \end{bmatrix},\quad B_2=\begin{bmatrix} 1 \\ 0 \end{bmatrix},\quad B_3=\begin{bmatrix} 1.1 \\ 0 \end{bmatrix}$$

First, we devide the universe of discou into two subspaces- S_1, S_2. Each subspace contains two fuzzy rules as shown in Fig.1.

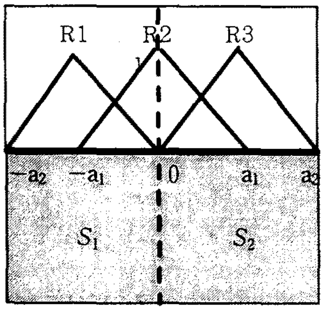

Fig 1. Partition of the system

Now, what one should do is to decide the control gain matirces K_1, K_2 such that each static state feedback control law on each subspace S_1, S_2 can stabilize all subsystems associated with its subspace of the TS fuzzy system, respectively. For instance, the control gain matrix K_1 sould be designed so as to guarantee the stability of the closed-loop consisting of subsystem (A_1, B_1) and (A_2, B_2). From Theorem 2 and solving the associated LMIs, we get the following control gain matrices:

$$K_1=[\ 0.0115\ \ 0.6608] \qquad K_2=[\ 0.0619\ \ 0.0854]$$

The global asymptotic stability of the given TS fuzzy system is checked by finding the positive definite matrices for each subspace as follows:

$$P_1=\begin{bmatrix} 19.2994 & 0.2367 \\ 0.2367 & 0.3768 \end{bmatrix} \quad P_2=\begin{bmatrix} 38.5569 & 0.4237 \\ 0.4237 & 0.3689 \end{bmatrix}$$

5. Conclusion

In this paper, we have dealed with a switching-type TS fuzzy-model-based controller design problem for the discrete-time TS fuzzy systems. To design a suitable controller that guarantees the global asymptotic stability in the sense of Lyapunov, some conditions have been derived. For certain complex nonlinear systems, a cleverheaded controller designer can split the system into several subsystems. Next he can also design a controller which satisfies the stability conditions for each subsystem. In this paper, we used this theory to design a switching-type fuzzy-model-based controller. The design of a discrete-time TS fuzzy control system was decomposed into the controller design of a set of subsystems. Each subsystem was able to be designed independently. The individual solutions can be combined to get a solution for the overall design problem. This process is often referred as a devide-and-conquer strategy. This method is suitable especially when the number of the rules in the TS fuzzy system is large. It indicates the potential of the new design method for future industrial applications.

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