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시변 파라메터를 갖는 불확실 비선형 시스템의 적응 출력궤환 제어

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Adaptive Output Feedback Control of Uncertain Nonlinear Systems with Time-Varying Parameters

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Abstract - In this paper, we present an adaptive output feedback control scheme for a class of uncertain nonlinear output-feedback form with time-varying parameters to which adaptive observer backstepping technique may not be applicable directly. In observer design, with the introduction of design function, we can deal with time-varying parameters in a very effective way. By the presented scheme, estimation error can be tuned to a desired small region around the origin via the design constants. Consequently, the observer with the presented design functions and the backstepping methodology achieve a robust regulation of the output tracking error while maintaining boundedness of all the signals and states.

1. Introduction

Adaptive output feedback nonlinear control problems have been given a lot of attention in control community during the recent years. Detailed discussions in such a direction can be found in [1,2]. Under the assumption of full-states measurement, adaptive backstepping scheme can achieve a global stabilization for a class of parametric strict-feedback systems [3,4]. Several authors have developed the design methods for a wider class of nonlinear systems under full-state feedback [5,6,7].

In case of only a single output measurement, the existing works show semiglobal results for a class of systems whose nonlinearities depend on the unmeasured variables [8,9]. If the nonlinearities depend on output measurement, current works can achieve global results only for a class of parametric output-feedback systems. With adaptive observer backstepping technique [2], an adaptive output-feedback controller that guarantees asymptotic tracking of the reference signal y_r by the output while keeping all the signals bounded can be designed for a class of parametric output-feedback form. As the first step to extend a class of output-feedback nonlinear systems that can be globally stabilized, adaptive controller was constructed for a class of nonlinear systems where the unmeasured states are appearing linearly with regrad to nonlinear functions [10]. Under the assumption that the unmeasured states are generated by pre-stabilized subsystems, Freeman and Kokotovic presented a global stabilization result for a class of extended strict feedback systems. In [11], the assumption of the pre-stabilization of the subsystem represented by Freeman and Kokotovic was removed. More recently, with novel state estimation technique and adaptive backstepping, a global adaptive output feedback controller was designed for a class of nonlinear systems where the unmeasured states appear linearly and quadratically in [12]. Also, For a more extended class than output-feedback structure considered in [12], Ahn et al. presented an adaptive output feedback control scheme based on proposed novel state estimation technique [13].

In this paper, we consider a class of output feedback nonlinear systems which has time-varying parameters as one of the recent efforts to extend a class of output-feedback nonlinear systems which can be controlled to guarantee a global stabilization. Since a class of introduced systems does not constant uncertain parameters, the adaptive observer backstepping technique [2] can not be applicable directly. However, for this extended structure, we can construct an adaptive output-feedback tracking controller based on the observer with design function and the backstepping methodology. With the introduction of design function in observer design, the time-varying parameter can be manipulated very effectively. The proposed observer design technique and the backstepping scheme achieve a robust tracking of the output to the given reference signal while maintaining the global boundedness of all the signals.

The class of nonlinear systems considered in this paper is described in Section 2. In Section 3, observer design technique based on design function is proposed. With this observer design technique, the backstepping scheme is presented in Section 4. The conclusion is given in Section 5.

2. Problem Formulation

The class of nonlinear systems to be controlled in this paper is the following output-feedback form with time-varying parameters:

$$\dot{x}_i = x_{i+1} + \phi_i(x_1, \theta(t)), \quad (1 \le i \le n-1)$$
$$\dot{x}_n = u + \phi_n(x_1, \theta(t)) \quad (1)$$
$$y = x_1$$

where $y \in R$, and $u \in R$ are the control input and the output, respectively and x_1 is the measured state while $x_2, \ldots, x_n$ represent the unmeasured states. ϕ_i is known smooth function. $\theta(t) \in R^p$ is a vector of unknown time-varying parameters which takes values in the known compact set $\Omega \in R^p$. Throughout this paper, we assume that the reference signal y_r and the derivatives of y_r up to the n-th order are bounded and piecewise continuous.

The control objective in this paper is to construct an adaptive output feedback nonlinear control law so that the tracking error $y - y_r$ is driven to an arbitrarily small region around origin with exponential convergence rate while maintaining globally uniformly ultimately boundedness of all the signals and states in spite of time-varying uncertain parameters.

Remark 1. In contrast to a class of output-feedback nonlinear systems presented in [8,9,10,11,12,13], it is noted that we consider a wider class of nonlinear outptut-feedback systems with nonlinear parameterization of time-varying parameters.

3. Observer Design Technique

In this section, observer design technique based on design function is proposed. With the introduction of this design function, we can deal with time-varying parameters in a very effective way. First of all, we introduce the following observer

$$\dot{\hat{x}}_i = \widehat{x_{i+1}} + l_i(x_1) - k_i(\hat{x}_1 - x_1), \quad (1 \le i \le n-1)$$
$$\dot{\hat{x}}_n = u + l_n(x_1) - k_n(\hat{x}_1 - x_1) \quad (2)$$

where l_i is a smooth design function of the measured state x_1, k_i is a positive design constant, and $\hat{\cdot}$ represents the estimate of $\cdot$. Denoting the state estimation error by $e_i = \hat{x}_i - x_i$, we have the following error equation.

$$\dot{e}_i = e_{i+1} + l_i(x_1) - \phi_i(x_1, \theta(t)) - k_i e_1, \quad (1 \le i \le n-1)$$
$$\dot{e}_n = l_n(x_1) - \phi_n(x_1, \theta(t)) - k_n e_1 \quad (3)$$

In vector notation,

$$\dot{e} = Ae + l - \phi(x_1, \theta(t)) \quad (4)$$

where $e = [e_1, \ldots, e_n]^T$, $l = [l_1, \ldots, l_n]^T$, $\phi = [\phi_1, \ldots, \phi_n]^T$, and

$$A = \begin{bmatrix} -k_1 & 1 & 0 & \ldots & 0 \\ -k_2 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & & & \vdots \\ -k_n & 0 & 0 & \ldots & 0 \end{bmatrix} \quad (5)$$

is Hurwitz matrix. In this case, A is a stability matrix and there exists a positive definite symmetric matrix P such that

$$PA + A^T P = -I \quad (6)$$

Let us consider the candidate Lyapunov function $V = e^T P e$. It follows that

$$\dot{V} = -\|e\|^2 + 2e^T P[l - \phi]$$
$$\le -\|e\|^2 + 2\|e\| \lambda_{\max}(P)[\|l\| + \|\phi\|] \quad (7)$$

Since Ω is a known compact set, we can find a smooth function $f(x_1)$ such that

$$\|\phi(x_1, \theta(t))\| \le f(x_1) \quad (8)$$

Therefore, we have

$$\dot{V} \le -\|e\|^2 + 2\lambda_{\max}(P)\|e\|[\|l\| + f(x_1)] \quad (9)$$

Using fact that

$$\|l\| + f(x_1) \le 2\sqrt{\|l\| \cdot f(x_1)} \quad (10)$$

it can be shown that

$$\dot{V} \le -\|e\|^2 + 4\lambda_{\max}(P)\|e\|\sqrt{\|l\| \cdot f(x_1)} \quad (11)$$

If the design function l is chosen to satisfy the following equality

$$l_1^2 + \ldots + l_n^2 = \frac{1}{f(x_1)} \quad (12)$$

we have

$$\dot{V} \le -\frac{1}{2\lambda_{\max}(P)} V + 8\lambda_{\max}^2(P) \quad (13)$$

Theorem 1. If we use the obsever (2) and the design function is chosen as in (12), the state estimation error satisfies the following property.

$$\|e(t)\| \le \sqrt{\frac{16\lambda_{\max}^3(P)}{\lambda_{\min}(P)}[1 - \exp(-\frac{t}{2\lambda_{\max}(P)})]} \quad (14)$$

Remark 2. As seen in (14), if k_i is chosen appropriately, then it is possible to make the state estimation error $e(t)$ as small as desired to any prescribed accuracy.

4. Backstepping Controller Design

In this section, with the state estimation technique proposed in the section 3, we employ the backstepping scheme and design a nonlinear controller that guarantees exponential result and the boundedness of all the signals.

Let $z_i = \widehat{x}_i - \alpha_{i-1} - y_r^{(i-1)}$ ($1 \le i \le n$) and $\alpha_0 = 0$. If we select α function as

$$\alpha_i = -c_i z_i - z_{i-1} - l_i(x_1) + k_i(\widehat{x}_1 - x_1) + \dot{\alpha}_{i-1} \tag{15}$$

where c_i is a positive design constant, then we have

$$\dot{z}_i = z_{i+1} - c_i z_i - z_{i-1} \tag{16}$$

Consider the following Lyapunov Function candidate

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 \tag{17}$$

where $V_1 = \frac{1}{2} z_1^2$. Then,

$$\dot{V}_i = -\sum_{j=1}^{i} c_j z_j^2 + z_i z_{i+1} \tag{18}$$

In final step, if we select the control input u as

$$u = -c_n z_n - z_{n-1} - l_n(x_1) + k_n(\widehat{x}_1 - x_1) + \dot{\alpha}_{n-1} + y_r^{(n)} \tag{19}$$

then we have

$$\dot{z}_n = -c_n z_n - z_{n-1} \tag{20}$$

Consider the following Lyapunov Function candidate

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 \tag{21}$$

Then, the time derivative of V_n is

$$\dot{V}_n = -\sum_{j=1}^{n} c_j z_j^2 \le 0 \tag{22}$$

Therefore, z_i is globally uniformly bounded. From (16), we have $\dot{z}_i \in L_\infty$. Since $z_i \in L_\infty$, we can obtain z_i, $\dot{z}_i \in L_\infty$. From the definition of V_i and (22), we see that $z_i \in L_2$. According to Barlalat Lemma [1,2], we obtain the following main theorem.

Theorem 2. Consider the system (1). If we apply the control input (19) and the design procedure in this section, then

$$\lim_{t\to\infty} |y(t) - y_r(t)| \le \sqrt{\frac{16\lambda^3_{\max}(P)}{\lambda_{\min}(P)}[1 - \exp(-\frac{t}{2\lambda_{\max}(P)})]} \tag{23}$$

Remark 3. As seen in (23), since k_i is constant for design, we can make the tracking error as small as desired and thus achieve robust tracking to small region around the origin by an appropriate choice of design parameters.

5. Conclusion

In this paper, we presented an adaptive output feedback control scheme for a class of uncertain nonlinear output-feedback systems with time-varying parameters. As one of the recent efforts to extend a class of output-feedback nonlinear systems which can be controlled to guarantee a global stabilization, we constructed an adaptive output feedback controller based on the observer with design function and the backstepping methodology. With the introduction of design function in observer design, the time-varying parameters can be manipulated very effectively. For a class of output-feedback systems to which adaptive observer backstepping can not be applicable directly, the proposed observer design technique and backstepping scheme achieve robust tracking of the output while maintaining boundedness of all the signals and states.

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