파라미터 불확실성을 포함한 TS 퍼지 시스템의 강인 추종 제어
Robust Tracking Control of TS Fuzzy Systems with Parametric Uncertainties

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Abstract

In this paper, a tracking control technique of Takagi-Sugeno (TS) fuzzy systems with parametric uncertainties is developed. The uncertain TS fuzzy system is represented as an uncertain multiple linear system. The tracking problem of TS fuzzy system is converted into the regulation problem of a multiple linear system. A sufficient condition for robust tracking is obtained in terms of linear matrix inequalities (LMI). A Design example is illustrated to show the effectiveness of the proposed method.

1 Introduction

Many frameworks in real world have hard nonlinearity and uncertainty, so a lot of control techniques have been developed and the fuzzy control is one of the major nonlinear control theories. However, the main drawback of fuzzy control is that it is difficult to analyze the stability of a fuzzy system. The Takagi-Sugeno (TS) fuzzy model is widely used, since it is possible to apply the systematic linear control theory to design a controller.

Since the TS fuzzy model has been introduced, the stability issue has been extensively studied and most TS fuzzy model-based-control deal with the regulation problem. The tracking problem of TS fuzzy system is also very important in real control process and a theoretically challenging control issue. We discuss the tracking problem of TS fuzzy system with parametric uncertainties, which is the contribution of this paper.

We first develop the uncertain multiple linear system which represents the continuous-time TS fuzzy system with parametric uncertainties. Using simple mapping technique, the tracking problem of TS fuzzy system is converted into the regulation problem of the multiple linear system. The sufficient condition to robustly track arbitrary reference signals with guaranteed-cost is derived and formulated in linear matrix inequalities (LMI) framework. The advantage of the studied results in this paper are verified from the computer simulation of the chaotic Lorenz system with parametric uncertainties.

2 Preliminaries

Consider a continuous-time uncertain nonlinear system of the form:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + \Delta f(x(t)) + (g(x(t)) + \Delta g(x(t)))u(t)\,, \\ y(t) &= Cx(t)\,. \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $f(x(t))$ and $g(x(t))$ are nonlinear vector functions, $\Delta f(x(t))$ and $\Delta g(x(t))$ are uncertain nonlinear vector functions, $y(t) \in \mathbb{R}^l$ is the output vector to track some reference signals, and C is the output matrix. The uncertain nonlinear system (1) can be modeled as the following TS fuzzy system:

Plant Rule i

If $x_1(t)$ is Γ_1^i and $\cdots$ and $x_n(t)$ is Γ_n^i

THEN $\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)$ (2)

where Γ_j^i $(j = 1, \ldots, n, \quad i = 1, \ldots, q)$ is the fuzzy set, Rule i denotes the ith fuzzy inference rule. $\Delta A_i, \Delta B_i$ are time varying matrices with appropriate dimension, which represent uncertainties in the TS fuzzy system. The defuzzified output of this TS fuzzy system (2) is represented as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{q} \mu_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t))\,, \\ y(t) &= Cx(t)\,. \end{aligned} \tag{3}$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t))\,, \qquad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))}\,.$$

in which $\Gamma_i^j(x_j(t))$ is the grade of membership of $x_j(t)$ in Γ_i^j. Hereforth we assume, as usual, that the uncertain matrices ΔA_i and ΔB_i are admissibly norm-bounded and structured.

Assumption 1 *The parameter uncertainties considered here are norm-bounded, in the form*

$$\begin{bmatrix}\Delta A_i & \Delta B_i\end{bmatrix} = D_i F_i(t) \begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix},$$

where D_i, E_{1i}, and E_{2i} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^T F_i(t) \leq I$, in which I is the identity matrix of appropriate dimension.

In ith subspace divided by the fuzzy membership functions, the TS fuzzy system has much highly nonlinear interaction among the fuzzy rules, which complicates the analysis and control of the TS fuzzy system [4]. In order to get rid of these theoretical difficulties, we represent the uncertain TS fuzzy system as an uncertain multiple linear system with the following subspace [2].

$$\Theta_i = \{x(t)|\mu_i(x(t)) \geq \mu_j(x(t)), \quad j = 1, 2, \ldots, q, \quad i \neq j\} \qquad i = 1, 2, \ldots, r. \tag{4}$$

The characteristic function of Θ_i is defined by

$$\eta_i = \begin{cases} 1, & x(t) \in \Theta_i \\ 0, & x(t) \notin \Theta_i \end{cases}, \qquad \sum_{i=1}^{r} \eta_i = 1. \tag{5}$$

Then, on every subspace the fuzzy system (2) can be represented with an uncertain multiple linear system as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \eta_i(x(t)) \left((A_i + \Delta A_i + \Delta \mathbf{A}_i) x(t) \right. \\ &\quad \left. + (B_i + \Delta B_i + \Delta \mathbf{B}_i) \right) u(t), \\ y(t) &= Cx(t). \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Delta \mathbf{A}_i &= \sum_{j=1, j\neq i}^{r} \mu_j(x(t)) \Delta \mathbf{A}_{ij}, \\ \Delta \mathbf{B}_i &= \sum_{j=1, j\neq i}^{r} \mu_j(x(t)) \Delta \mathbf{B}_{ij}, \\ \Delta \mathbf{A}_{ij} &= A_j + \Delta A_j - A_i - \Delta A_j, \\ \Delta \mathbf{B}_{ij} &= B_j + \Delta B_j - B_i - \Delta B_j. \\ & \qquad i = 1, 2, \ldots, q. \end{aligned}$$

3 Problem Statement

In this section, we develop the tracking control technique of TS fuzzy system with parametric uncertainties.

The reference signal to be track is the output $r(t)$ generated by the exogenous system

$$\begin{aligned} \dot{w}(t) &= Fw(t), \\ r(t) &= Gw(t). \end{aligned} \tag{7}$$

where $w(t) \in \mathbb{R}^k$ is the state vector of exogenous system and $r(t) \in \mathbb{R}^l$ is the output vector to be tracked.

Problem 1 The objective in this paper is to design a TS fuzzy-model-based controller which stabilize the plant (2) and track the reference signal vector $r(t)$ such that the tracking error

$$e(t) := y(t) - r(t) = Cx(t) - Gw(t), \tag{8}$$

asymptotically to be zero.

In order to construct the error system, a new state vector is defined as

$$z(t) := x(t) - Tw(t), \tag{9}$$

where T is a solution to the following matrix equations:

$$\begin{bmatrix} A_i & B_i \\ C & 0 \end{bmatrix} \begin{bmatrix} T \\ L_i \end{bmatrix} = \begin{bmatrix} T & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix}. \qquad i = 1, 2, \ldots, q. \tag{10}$$

Remark 1 The matrix equalities (10) is assumed to be solvable. If a solution cannot be found to satisfy (10), then a different reference model must be chosen.

Assuming that T and L_i have been found to satisfy (10), consider the tracking control law of the following form:

$$u(t) = \sum_{i=1}^{r} \eta_i(x(t))(L_i w(t) + v(t)), \tag{11}$$

where $v(t)$ remains to be defined. Using this control law, and the matrix equations (10), the newly defined state vector $z(t)$ in (9) satisfies

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{r} \eta_i(x(t)) \left((A_i + \Delta A_i + \Delta \mathbf{A}_i) z(t) \right. \\ &\quad \left. + (B_i + \Delta B_i + \Delta \mathbf{B}_i) \right) v(t), \\ e(t) &= Cz(t). \end{aligned} \tag{12}$$

If the newly constructed system (12) is asymptotically stable, then the tracking error $e(t)$ converge to zero. Therefore Problem 1 can be converted to the following problem.

Problem 2 The objective in this paper is to design a TS fuzzy-model-based state feedback controller $v(t)$ which asymptotically stabilize the dynamic system (12).

4 Robust Tracking Controller Design

Before preceding this section, remind following Lemmas.

Lemma 1 [1] *Given constant symmetric matrices N, O, and L of appropriate dimensions, the following two inequalities are equivalent:*

(a) $\quad O > 0, \quad N + L^T O L < 0,$

(b) $\quad \begin{bmatrix} N & L^T \\ L & -O^{-1} \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -O^{-1} & L^T \\ L & N \end{bmatrix} < 0.$

Lemma 2 [1] *Given constant matrices D and E, and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:*

$$S + DFE + E^T F^T D^T < 0\,,$$

where F satisfies $F^T F \leq R$, if and only if for some $\epsilon > 0$,

$$S + \begin{bmatrix} \epsilon^{-1} E^T & \epsilon D \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \epsilon^{-1} E \\ \epsilon D^T \end{bmatrix} < 0\,.$$

The objective is to design a state feedback controller of the following form:

$$v(t) = \sum_{i=1}^{q} \eta_i(x(t)) K_i z(t)\,. \tag{13}$$

The main result on the robust tracking of the continuous-time TS fuzzy system with parametric uncertainties is summarized in the following theorem.

Theorem 1 *If there exist a symmetric positive definite matrices, P_i, a symmetric positive definite matrix, Q, and matrices, K_i such that the following LMIs are satisfied, then the TS fuzzy system (2) is asymptotically stable, and the output of TS system (2) can robustly track the reference signals $r(t)$ via TS fuzzy-model-based controller (11) in the presence of admissible parametric uncertainties with guaranteed-cost.*

$$\begin{bmatrix} \Psi_i & * & * & * & * & * & * & * \\ W & -Q^{-1} & * & * & * & * & * & * \\ E_{1i}W + E_{2i}M_j & 0 & -\epsilon_i I & * & * & * & * & * \\ \tilde{E}_{1i}W + \tilde{E}_{2i}M_i & 0 & 0 & -\epsilon_i I & * & * & * & * \\ \hat{E}_{1i}W + \hat{E}_{2i}M_i & 0 & 0 & 0 & -\epsilon_i I & * & * & * \\ D_i^T & 0 & 0 & 0 & 0 & -\epsilon_i^{-1} I & * & * \\ \tilde{D}_i^T & 0 & 0 & 0 & 0 & 0 & -\epsilon_i^{-1} I & * \\ \hat{D}_i^T & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon_i^{-1} I \end{bmatrix} < 0\,, \tag{14}$$

$$i = 1, 2, \ldots, r.$$

where

$$\tilde{D}_i = \begin{bmatrix} D_1 & D_2 & \cdots & D_q \end{bmatrix}, \; \tilde{E}_{1i} = \begin{bmatrix} E_{1i} \\ E_{1i} \\ \vdots \\ E_{1q} \end{bmatrix}, \; \tilde{E}_{2i} = \begin{bmatrix} E_{2i} \\ E_{2i} \\ \vdots \\ E_{2q} \end{bmatrix},$$

$$\hat{D}_1 = \begin{bmatrix} I & \cdots & I \end{bmatrix}, \; \hat{E}_{1i} = \begin{bmatrix} A_1 - A_i \\ A_2 - A_i \\ \vdots \\ A_q - A_i \end{bmatrix}, \; \hat{E}_{2i} = \begin{bmatrix} B_1 - B_i \\ B_2 - B_i \\ \vdots \\ B_q - B_i \end{bmatrix},$$

and

$$\Psi_i = W A_i^T + A_i W + M_i^T B_i^T + B_i M_i\,,$$

and $W = P^{-1}$, $M_i = K_i P^{-1}$, and $*$ denotes the transposed elements in the symmetric positions.

proof: The proof is omitted due to lack of space.

Remark 2 The matrices D_i, E_{1i} and E_{2i} in Theorem 1 can be arbitrarily chosen, to express parametric uncertainties. Moreover, matrices $\hat{D}_i, \hat{E}_{1i}, \hat{E}_{2i}, \tilde{D}_i, \tilde{E}_{1i}$ and $\tilde{E}_{2i}$ may have other structure. Note, however, that the choices of these matrices generally influence on the performance of the controller.

5 An Example

This section presents an example to show the effectiveness of the proposed tracking controller design technique. The chaotic Lorenz system with parametric uncertainties is adopted as a testbed. The TS fuzzy model of the Lorenz system is as follows [3].

Plant Rule 1

If $x_1(t)$ is about M_1 THEN $\dot{x}(t) = A_1 x(t)$

Plant Rule 2

If $x_1(t)$ is about M_2 THEN $\dot{x}(t) = A_2 x(t)$

where

$$A_1 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_1 \\ 0 & M_1 & -b \end{bmatrix}, \quad A_2 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_2 \\ 0 & M_2 & -b \end{bmatrix}$$

The nominal values of (σ, r, b) are $\left(10, 28, \frac{8}{3}\right)$. The membership functions are

$$\Gamma_1(x_1(t)) = \frac{-x_1(t) + M_2}{M_2 - M_1}, \quad \Gamma_2(x_1(t)) = \frac{x_1(t) - M_1}{M_2 - M_1}\,.$$

where $\Gamma_i(x_1(t))$ is positive semi-definite for $\forall x \in \begin{bmatrix} M_1 & M_2 \end{bmatrix}$. Without loss of controllibility, the input matrices B_1, B_2 and the output matrix are chosen as $B_1 = B_2 = \text{diag}\begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$ $C = \text{diag}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$.

The exogenous system which generates the reference signals is arbitrary chosen as follows:

$$\dot{w}(t) = F w(t)\,,$$
$$r(t) = G w(t)\,,$$

where

$$F = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The system parameters (σ, r, b) are assumed to be bounded with 30% variation of the nominal values. From Theorem

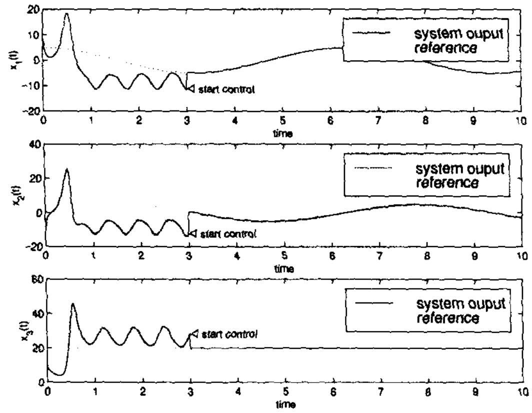

Figure 1: The controlled system response of the Lorenz system in the presence with parametric uncertainties (system parameters are varied within30% of their nominal values)

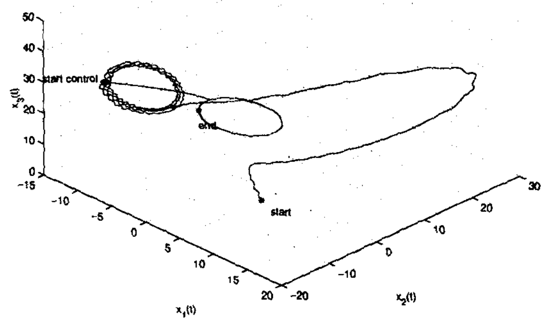

Figure 2: The phase trajectories of the chaotic Lorenz system with parametric uncertainties (system parameters are varied within 30% of their nominal values)

1, and the associated matrix equation, we get

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 5 & -5.5 & 0 \\ -27 & 1 & -30 \\ 0 & 30 & 2.67 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 5 & -5.5 & 0 \\ -27 & 1 & 30 \\ 0 & -30 & 2.67 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -990.3 & 9.9 & 0.4 \\ 62.2 & -3.8945 & -0.2 \\ 0.1 & -0.2 & -3843.5 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -2509.0 & 59.4 & -0.4 \\ 148.3 & -9.247.1 & 0.2 \\ -0.1 & 0.2 & -9136.8 \end{bmatrix}.$$

The common positive definite matrices P_1, P_2 and Q, which guarantees the asymptotic stability with guaranteed cost are found as

$$P_1 = 1.0\mathrm{e}-004 \times \begin{bmatrix} 0.0684. & -0.0026 & 0 \\ -0.0026 & 0.1477 & 0 \\ 0 & 0 & 0.1458 \end{bmatrix},$$

$$P_2 = 1.0\mathrm{e}-004 \times \begin{bmatrix} 0.1645 & -0.0057 & 0 \\ -0.0057 & 0.3377 & 0 \\ 0 & 0 & 0.3338 \end{bmatrix},$$

$$Q = 1.0\mathrm{e}+007 \times \begin{bmatrix} 9.854 & 0 & 0 \\ 0 & 9.854 & 0 \\ 0 & 0 & 9.854 \end{bmatrix}.$$

The initial value of the Lorenz system is $x(0) = [10 \ \ -10 \ \ 10]^T$, and the initial value of the exogenous system is $w(0) = [5 \ \ 0 \ \ 5]^T$. During the simulation process, all system parameters varies are varied within the bounds of 30% of the their nominal values. The simulation results are in Fig. 1, and 2. For the purpose of comparison, the control input is activated at $t = 3$ seconds. Before the control input was activated, although the system parameters have been varied, the trajectory of the Lorenz system was chaos-like. After $t = 3$ seconds, the trajectory very quickly guided to the reference trajectory. From the the simulation results, the TS fuzzy-model-based controller have not only the tracking performance but strong robustness against admissible parametric uncertainties.

6 Conclusion

In this paper, the robust tracking controller design technique for TS fuzzy system with parametric uncertainties is presented. The tracking problem of uncertain TS fuzzy system was converted into the regulation problem of the uncertain multiple linear system. The sufficient condition was formulated in LMI framework. The simulation example ensured us the feasibility of the developed design technique.

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