비선형 시스템의 안정을 위한 HRIV 방법의 제안

Hybrid Rule-Interval Variation(HRIV) Method for Stabilization a Class of Nonlinear Systems

Hwan-Chun Myung, Z.Zenn Bien

mhc@ctrsys.kaist.ac.kr, bien@ee.kaist.ac.kr

Dept. of Electrical Engineering and Computer Science, KAIST,

373-1 Gusong-dong Yusong-gu Taejon 305-701

Abstract

HRIV(Hybrid Rule-Interval Variation) method is presented to stabilize a class of nonlinear systems, where SMC(Sliding Mode Control) and ADC (ADaptive Control) schemes are incorporated to overcome the unstable characteristics of a conventional FLC(Fuzzy Logic Control). HRIV method consists of two modes: I-mode (Integral Sliding Mode FLC) and R-mode(RIV method). In I-mode, SMC is used to compensate for MAE(Minimum Approximation Error) caused by the heuristic characteristics of FLC. In R-mode, RIV method reduces interval lengths of rules as states converge to an equilibrium point, which makes the defined Lyapunov function candidate negative semi-definite without considering MAE, and the new uncertain parameters generated in R-mode are compensated by SMC. In RIV method, the over-contraction problem that the states are out of a rule-table can happen by the excessive reduction of rule-intervals, which is solved with a dynamic modification of rule-intervals and a transition to I-mode. Especially, HRIV method has advantages to use the analytic upper bound of MAE and to reduce its effect in the control input, compared with the previous researches. Finally, the proposed method is applied to stabilize a simple nonlinear system and a modified inverted pendulum system in simulation experiments

1. Introduction

FLC(Fuzzy Logic Control) fits well when systems to be controlled is only partly known, difficult to describe by a white box model, and few measurements are available, or the systems are highly nonlinear [1]. Especially, with a linguistic prior knowledge, FLC can play a role of man-machine interface, which makes systems controlled easily. But, at the same time, the heuristic characteristics caused by a linguistic knowledge deteriorate FLC to be analyzed for a stability. Since a stability is the first and last concern for any system design and a fundamental issue in every control system [2], many researches have been done in applying robust and nonlinear control theories [3]-[7], and intuitive methods [8]-[10] to a stability problem of FLC. Among them, there have been some researches to consider MAE(Minimum Approximation Error) [11] for a stability of FLC. In 1993, L.X.Wang [11] proposed to use an adaptive law to tune membership functions automatically with a supervisory controller appended to stabilize the total system. In [11], the concept of MAE is used to formulate a stability problem of FLC. B.S.Chen [14] presented a modified adaptive law to advance the performance of the controller proposed by [11], using Riccati-like equation. C.Y.Su [12] proposed a design method to insert a sliding mode into the controller and the modification renders the possibility to prove a stability of FLC. The method presented by Y.S.Lu [13] is to use FLC and SMC(Sliding Mode Control) simultaneously in an additive form, where MAE is compensated by SMC. But, before applying the method to a system, MAE is only assumed very small value and no analytical upper bound of MAE is considered. Such a disadvantage makes it difficult to apply the method even when a system is perfectly known. In addition, an additional feedback control input is necessary for the convergence of a sliding surface and there is no error bound considering even the case that the switching condition [18] is not guaranteed.

In this paper, a new method, called HRIV(Hybrid Rule-Interval Variation) method, is proposed to stabilize a class of nonlinear systems with FLC taking into account MAE. HRIV method consists of two modes: I-mode(Integral Sliding Mode FLC) and R-mode(RIV method). In I-mode, SMC and ADC construct a basic structure to show an asymptotic stability of FLC compensating for the analytic upper bound of MAE caused by the heuristic characteristics of FLC. Especially, a new sliding surface is designed including an integral part, which makes no additional feedback part included in a control input and gives the upper bound of the state performance. In R-mode, RIV method reduces interval lengths of rules as states converge to an equilibrium point, which makes the defined Lyapunov function candidate negative semi-definite without considering MAE, and the new uncertain parameters generated in R-mode are compensated by SMC. Moreover, since the newly generated parameters are alleviated by increasing a learning rate and decreasing RIV laws, RIV method can reduce an effect of uncertain parameters in a control input. But, in RIV method, the over-contraction problem that the states are out of a rule-table can happen by the excessive reduction of rule-intervals. To solve the problem, RIV method is complemented with the dynamic modification of rule-intervals including its increase and changed into I-mode.

Section 2 formulates a problem of DAFLC(Direct Adaptive FLC), giving some related definitions. In Section 3, Integral Sliding Mode FLC is presented and RIV method is cooperated with the Integral Sliding Mode FLC in Section 4. Section 5 completes HRIV method.

2. Problem Formulation

Consider the following nonlinear system

$$x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}) + bu, \ y = x, \qquad (1)$$

where $f : R^n \rightarrow R$ is an unknown continuous function, b is an unknown constant, and $u \in R$ and $y \in R$ are the input and the output of the system, respectively. If the function f and the constant b are known exactly, then an ideal control input u_D^* is obtained by

$$u_D^* = [-f(\underline{x}) + y_m^{(n)} + \underline{k}^T \underline{e}]/b, \qquad (2)$$

where $\underline{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T \in S_{M_x}$ is a state vector, $y_m \in R$ is a reference input, $\underline{e} = [y_m - x, \dot{y}_m - \dot{x}, \cdots, y_m^{(n-1)} - x^{(n-1)}]^T \in R^n$ is an error vector, and $\underline{k} = [k_1, k_2, \cdots, k_n]^T \in R^n$ is a design parameter vector. Substitution of (2) in (1) yields

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0, \qquad (3)$$

which can be stabilized exponentially under $k_i > 0$ for $i = 1, \cdots, n$. A proposed controller

$$u_D = u_D(\underline{x} \mid \theta_D) + K_D \operatorname{sgn}(s), \qquad (4)$$

where $\underline{\theta}_D = [\theta_{D1}, \theta_{D2}, \cdots, \theta_{Dm}]^T \in R^m$ is a center-value vector of consequent membership functions, $K_D \in R$ is a positive constant, and $s \in R$ stands for a sliding surface specified in Section 3, can be considered as an integration of FLC(Fuzzy Logic Control) and SMC (Sliding Mode Control). Especially, FLC is given by

$$u_D(\underline{x}|\underline{\theta}_D)=\underline{\theta}_D^T\underline{\xi}_D(\underline{x}), \quad (5)$$

and

$$\xi_{Dj}(\underline{x})=[\prod_{i=1}^{n}\mu_{Fij}(x_i)]/[\sum_{j=1}^{m}(\prod_{i=1}^{n}\mu_{Fij}(x_i))] \quad ,(6)$$

$$(i=1,\cdots,n, \quad j=1,\cdots,m),$$

is defined as FBF, where μ_{Fij} is MF(Membership Function), n is considered as a number of states, and m as a number of fully generated rules. Using (2), (4), and (5), MAE is defined as

$$\omega_D \equiv u_D(\underline{x}|\underline{\theta}_D^*)-u_D^*, \quad (7)$$

where

$$\underline{\theta}_D^* = \arg\min_{|\underline{\theta}_D|_2\le M_{\theta_D}}[\sup_{|\underline{x}|_2\le M_x}|u_D(\underline{x}|\underline{\theta}_D)-u_D^*|], \quad (8)$$

and M_{θ_D}, M_x are the design parameters based on practical constraints. The equation (7) means that, although best rules are obtained by a heuristic method, there always exists MAE except some special cases explained in Section 3, which leads to analytic difficulty in the following error equation. Two control laws (2) and (4) applied to (1) results in

$$e^{(n)}=-\underline{k}^T\underline{e}+b[u_D^*-u_D(\underline{x}|\underline{\theta}_D)-K_D\,\mathrm{sgn}(s)], (9)$$

or, equivalently,

$$\underline{\dot{e}}=A\underline{e}+\underline{b}_D[u_D^*-u_D(\underline{x}|\underline{\theta}_D)-K_D\,\mathrm{sgn}(s)] \quad (10)$$

$$=A\underline{e}+\underline{b}_D\underline{\phi}_D^T\underline{\xi}_D(\underline{x})-\underline{b}_DK_D\,\mathrm{sgn}(s)-\underline{b}_D\omega_D$$

(1 1)

where

$$\underline{b}_D=[0,\cdots,0,b]^T\in R^n,\ \underline{\phi}_D=\underline{\theta}_D^*-\underline{\theta}_D\in R^m,$$

and

$$A=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&1\\-k_n&-k_{n-1}&-k_{n-2}&\cdots&-k_1\end{bmatrix}\in R^{n\times n}.$$

3. Integral Sliding Mode FLC

In [19], it is proved that the relation between MAE and rule-intervals, which means the difference between the adjoining pseudo-trapezoid MFs, is given by

$$\omega\le|u^*(\underline{x})-u_c(\underline{x}|\underline{\theta})|_\infty\le\sum_{i=1}^{n}\left|\frac{\partial u^*(\underline{x})}{\partial x_i}\right|_\infty\eta_i, (12)$$

where $\eta_i>0$ is the length of rule-interval in *ith* state and n is the dimension of states. According to (12), the less the length of the rule-interval, the less MAE. Moreover, if the triangular MFs are used, (12) changes into

$$\omega\le|u^*(\underline{x})-u_c(\underline{x}|\underline{\theta})|_\infty\le\frac{1}{8}\sum_{i=1}^{n}\left|\frac{\partial^2u^*(\underline{x})}{\partial x_i^2}\right|_\infty\eta_i^2, (13)$$

By (13), the special type of the two-dimensional ideal controllers

$$u^*(\underline{x})=\sum_{l_1=0}^{1}\sum_{l_2=0}^{1}a_{l_1}a_{l_2}x_1^{l_1}x_2^{l_2}, \quad (14)$$

can be approximated perfectly by FLC, which means that MAE is zero.

For DAFLC, it is assumed that

- the sign of b is known,
- $|\omega_D|_\infty<\infty$ is known, and
- $|\underline{x}(t)|_2\le M_x, |\underline{\theta}_D^*(t)|_2\le M_{\theta_D}$ for $\forall t\ge0$.

Consider a SMC gain

$$K_D\ge|\omega_D|_\infty, \quad (15)$$

and a sliding surface s as

$$s=\underline{p}_n^TP[(I+P^{-1})\underline{e}-A\int_0^t\underline{e}(\tau)d\tau]\mathrm{sgn}(b) \quad (16)$$

$$=\underline{p}_n^TP\underline{h}\,\mathrm{sgn}(b), \quad (17)$$

where

$$\underline{h} = (I + P^{-1})\underline{e} - A\int_0^t \underline{e}(\tau)d\tau \in R^n , \quad (18)$$

$P = [\underline{p}_1, \underline{p}_2, \cdots, \underline{p}_n] \in R^{n\times n}$ is a symmetric positive definite matrix satisfying the Lyapunov equation

$$A^T P + PA = -Q , \quad (19)$$

where $Q \in R^{n\times n}$ is a positive definite matrix, $I \in R^{n\times n}$ is an identity matrix, $diag\,[1,1,\cdots,1]$, and $\int_0^t \underline{e}(\tau)d\tau = [\int_0^t e_1(\tau)d\tau,\ \int_0^t e_2(\tau)d\tau, \cdots, \int_0^t e_n(\tau)d\tau]^T$. Especially, the matrix A is designed to ensure that the sliding action of s is stable. To prove the stability of FLC with the presented controller, (4), in (11), define the Lyapunov function candidate

$$V = \frac{1}{2}\underline{e}^T P \underline{e} + \frac{b}{2\gamma_D}\underline{\phi}_D^T \underline{\phi}_D + \frac{1}{2}\underline{\psi}^T \underline{\psi} , \quad (20)$$

where $\underline{\psi} = -\underline{e} + P\underline{h} \in R^n, \gamma_D > 0$ is a constant, and b is assumed to be positive($b > 0$). The derivative of V is given by

$$\dot{V} = \frac{1}{2}\underline{\dot{e}}^T P \underline{e} + \frac{1}{2}\underline{e}^T P \underline{\dot{e}} + \frac{b}{\gamma_D}\underline{\phi}_D^T \underline{\dot{\phi}}_D + \underline{\psi}^T \underline{\dot{\psi}} , \quad (21)$$

$$= -\frac{1}{2}\underline{e}^T Q \underline{e} + \frac{b}{\gamma_D}\underline{\phi}_D^T (\gamma_D \underline{e}^T \underline{p}_n \underline{\xi}_D(\underline{x}) + \underline{\dot{\phi}}_D)$$

$$+ (-\underline{e}^T + \underline{h}^T P)(-\underline{\dot{e}} + P((I + P^{-1})\underline{\dot{e}} - A\underline{e}))$$

$$- \underline{e}^T P \underline{b}_D (K_D \operatorname{sgn}(s) + \omega_D) . \quad (22)$$

In (22), the matrix Q comes from the Lyapunov equation and, since $\underline{p}_n \in R^n$ indicates the last column of the matrix P , the relation

$$\underline{e}^T P \underline{b}_D = \underline{e}^T \underline{p}_n b , \quad (23)$$

is valid. Applying (11) to (22) gives

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q \underline{e} + \frac{b}{\gamma_D}\underline{\phi}_D^T (\gamma_D \underline{e}^T \underline{p}_n \underline{\xi}_D(\underline{x}) + \underline{\dot{\phi}}_D)$$

$$+ (-\underline{e}^T + \underline{h}^T P) P \underline{b}_D (\underline{\phi}_D^T \underline{\xi}_D(\underline{x}) - (K_D \operatorname{sgn}(s) + \omega_D))$$

$$- \underline{e}^T P \underline{b}_D (K_D \operatorname{sgn}(s) + \omega_D) \quad (24)$$

$$= -\frac{1}{2}\underline{e}^T Q \underline{e} - \underline{h}^T P^2 \underline{b}_D (K_D \operatorname{sgn}(s) + \omega_D)$$

$$+ \frac{b}{\gamma_D}\underline{\phi}_D^T (\gamma_D \underline{e}^T \underline{p}_n \underline{\xi}_D(\underline{x}) + \gamma_D(-\underline{e}^T \underline{p}_n + \underline{h}^T P \underline{p}_n)\underline{\xi}_D(\underline{x}) + \underline{\dot{\phi}}_D) \cdot$$

$$(25)$$

Since $\underline{\dot{\theta}}_D^*$ is assumed to be a constant vector

$$\underline{\dot{\phi}}_D = -\underline{\dot{\theta}}_D . \quad (26)$$

Therefore, an adaptive law to tune the center-value vector of consequent MFs

$$\underline{\dot{\theta}}_D = \gamma_D \underline{h}^T P \underline{p}_n \underline{\xi}_D(\underline{x}) \quad (27)$$

$$= \gamma_D s \underline{\xi}_D(\underline{x}) , \quad (28)$$

is obtained in (25). By the second assumption,

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q \underline{e} < 0 , \quad (29)$$

will be obtained, which means that the Lyapunov function is negative semi-definite. Since it is difficult to say about an asymptotic stability because of the negative semi-definite characteristics of the Lyapunov function. Barbalat's lemma [16][17] can be used to check a convergence of an error function.

4. Rule-Interval Variation Method

The RIV method comes from the relation between MAE and rule-intervals in (12) and is limited only to a regulation problem in this paper. In the case of a regulation problem, FLC can be transformed to use errors as input variables instead of states as follows;

$$u_c(\underline{e} \mid \underline{\theta}) = u_c(\underline{\tilde{y}}_m - \underline{x} \mid \underline{\theta}) , \quad (30)$$

where $\underline{\tilde{y}}_m = [y_m, 0, \cdots, 0]^T \in R^n$. Moreover, with (30), the upper bound of MAE in (12),

$$\omega \le \left| u^*(\underline{x}) - u_c(\underline{e}|\underline{\theta}) \right|_\infty \le \sum_{i=1}^{n} \left| \frac{\partial u^*(\underline{x})}{\partial x_i} \right|_\infty \eta_i \text{ ,(31)}$$

is still valid without any change. The RIV method means in short that rule-intervals are decreased to make the defined Lyapunov function negative semi-definite without considering MAE. Generally, since $\underline{\theta}^*$ is a constant vector,

$$\dot{\underline{\phi}} = -\dot{\underline{\theta}} , \quad (32)$$

is valid. But $\underline{\theta}^*$ becomes a time-varying vector in RIV method because the decrease of rule-intervals makes the ideal center-values of the consequent MFs move sequentially. Accordingly, (32) is rewritten as

$$\dot{\underline{\phi}} = \dot{\underline{\theta}}^* - \dot{\underline{\theta}} . \quad (33)$$

Since $\dot{\eta}_i = 0$ yields $\dot{\theta}_i^* = 0$, there exists a constant $\varphi_j > 0$ that satisfies the relation

$$\dot{\theta}_j^* \le \varphi_j \sum_{i=1}^{n} |\dot{\eta}_i| , \quad (34)$$

$$(i = 1, \cdots, n , \quad j = 1, \cdots, m) ,$$

For DAFLC, it is assumed that

- the sign of b is known: $b > 0$ is considered for convenience,
- $|\underline{\varphi}_D|_2 < \infty$ is known, where $\underline{\varphi}_D = [\varphi_{D1}, \varphi_{D2}, \cdots, \varphi_{Dm}]^T \in R^m$, and
- $|\underline{e}(t)|_2 \le M_e, |\underline{\theta}_D^*(t)|_2 \le M_{\theta_D}$ for $\forall t \ge 0$.

Consider the Lyapunov function candidate

$$V = \frac{1}{2}\underline{e}^T P \underline{e} + \frac{b}{2\gamma_D}\underline{\phi}_D^T \underline{\phi}_D + \frac{1}{2}\underline{\psi}^T \underline{\psi} + \sum_{i=1}^{n} \frac{b}{\alpha_{Di}}\eta_{Di} , \quad (35)$$

where $\alpha_{Di}(t)$ and $\eta_{Di}(t)$ are positive functions for $\forall t > 0$. The derivative of V has the relation

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q \underline{e} - bs(K_D \operatorname{sgn}(s) + |\omega_D|_\infty) + \frac{b}{\gamma_D}\underline{\phi}_D^T \dot{\underline{\theta}}_D^*$$

$$+ \sum_{i=1}^{n} \left(\frac{b}{\alpha_{Di}}\dot{\eta}_{Di} - \frac{b\dot{\alpha}_{Di}}{\alpha_{Di}^2}\eta_{Di} \right) , \quad (36)$$

where (27), the adaptive law, is used under the same conditions. By (12), (34), and the third assumption,

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q \underline{e} - b|s| K_D + \frac{2b}{\gamma_D} M_{\theta_D} |\underline{\varphi}_D|_2 \sum_{i=1}^{n} |\dot{\eta}_{Di}|$$

$$+ \sum_{i=1}^{n} \frac{b}{\alpha_{Di}} \left(\dot{\eta}_{Di} - \frac{\dot{\alpha}_{Di}}{\alpha_{Di}}\eta_{Di} + \alpha_{Di} \left| \frac{\partial u_D^*}{\partial x_i} \right|_\infty |s| \eta_{Di} \right) ,$$

(37)

Define a RIV rate and a RIV law as

$$\alpha'_{Di}(t) = \alpha_{Di}(t) \left| \frac{\partial u_D^*}{\partial x_i} \right|_\infty > 0 , \quad \forall t \ge 0 , \quad (38)$$

and

$$\dot{\eta}_{Di}(t) = \left(\frac{\dot{\alpha}'_{Di}(t)}{\alpha'_{Di}(t)} - \alpha'_{Di}(t)|s(t)| \right)\eta_{Di}(t) , \quad (39)$$

respectively. Setting the SMC gain

$$K_D \ge \frac{2}{\gamma_D} M_{\theta_D} |\underline{\varphi}_D|_2 \sum_{i=1}^{n} \left| \frac{\dot{\alpha}'_{Di}(t)}{\alpha'_{Di}(t)} - \alpha'_{Di}|s(t)|)\eta_{Di}(t) \right| , \quad (40)$$

results in

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q \underline{e} < 0 . \quad (41)$$

5. Construction of HRIV Method

In R-mode, RIV method is used reducing the rule-intervals and, in I-mode, Integral Sliding Mode FLC is applied to compensate for MAE without the reduction of the rule-intervals. Such a transition makes Integral Sliding Mode FLC and RIV method play a complementary role for each disadvantage. If the appropriate adaptation is not performed by a excessive and fast reduction of the rule-intervals, or the other reasons, R-mode is changed into I-mode. By the transition, RIV method does not need to satisfy the condition that all the elements of $\underline{\theta}$ converges to the same value any more. Conversely, by the transition, Integral Sliding Mode FLC has an advantage to use the reduced rule-interval for designing the SMC gain, which can alleviate the disadvantage in DAFLC.

For the stability of HRIV method in DAFLC, all the conditions in Section 3 and 4 are assumed to be valid. Define the control input and the SMC gain

$$u_{DH} = u_D(\underline{e} \mid \underline{\theta}_D) + K_{DH}\,\mathrm{sgn}(s), \qquad (42)$$

$$K_{DH} = (1-\delta)K_{DI} + \delta K_{DR}, \qquad (43)$$

where δ is 1 for R-mode and 0 for I-mode, K_{DI} and K_{DR} are defined as K_D in (15) and (40), respectively. Consider the same Lyapunov function in RIV method. By redefining

$$\dot{\alpha}_{DHi} = \delta\dot{\alpha}_{Di}, \qquad \dot{\eta}_{DHi} = \delta\dot{\eta}_{Di}, \qquad (44)$$

and applying the same adaptive law,

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q \underline{e} - b|s|(K_{DH} - \frac{2}{\gamma_D} M_{\theta_D} |\underline{\varphi}_D|_2 \sum_{i=1}^{n} |\dot{\eta}_{DHi}|$$

$$+(1-\delta)\omega_D) \le 0, \qquad (45)$$

is obtained.

6. Conclusions

Integral Sliding Mode FLC is presented to compensate for the analytic bound of MAE, to formulate the error bound, and to show the asymptotic stability without the additional feedback gain. To reduce the large bound of MAE, RIV method is combined with Integral Sliding Mode FLC, where it compensates for the new uncertain parameters generated by RIV method and alleviated by the learning rate and the RIV laws. In the case of RIV method, over-contraction problem can happen by the fast reduction of rule-intervals and as times goes to infinity. To handle the problem, HRIV method is developed, where the disadvantages caused by Integral Sliding Mode FLC and RIV method are reciprocally complemented with two modes; R-mode and I-mode.

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