퍼지 유사-반연속성과 퍼지 유사-반연결성

FUZZY QUASI-SEMICONTINUITY AND FUZZY QUASI-SEMICONNECTEDNESS

박 진 한 · 박 진 근 · 박 성 준

Jin Han Park, Jin Keun Park and Seong Jun Park

Division of Mathematical Science Pukyong National University

ABSTRACT

The aim of this paper is to study and find characterizations of fuzzy quasi-semicontinuous and fuzzy quasi-semiopen mappings between fuzzy bitopological spaces. The notion of fuzzy quasi-semiopen sets is used to define fuzzy quasi-semicontinuous and fuzzy quasi-semiopen mappings. Finally, fuzzy quasi-semiconnectedness is introduced and studied to some extent.

1. Introduction and Preliminaries

Chang[2] used the concept of fuzzy sets to introduce fuzzy topological spaces and several authors continued the investigation of such spaces. From the fact that there are some non-symmetric fuzzy topological structures, Kubiak[5] first introduced and studied the notion of fuzzy bitopological spaces (A triple (X, τ_1, τ_2)) where X is a non-empty set and τ_1 and τ_2 are fuzzy topologies on X is called a fuzzy bitopological space (shortly, fbts)), as a natural generalization of fuzzy topological space, and initiated the bitopological aspects due to Kelly[4] in the theory of fuzzy topological spaces. Since then several authors[3,5,6,8] have contributed to the subsequent development of various fuzzy bitopological properties. In this paper, using the notion of fuzzy quasi-semiopen(shortly, fqso) introduced by Park, which is weaker form than fuzzy quasi-open set[9], we introduce the concept of fuzzy quasi-semi continuous (shortly, fq-sc) mappings and study its basic properties Finally, we introduce and investigate some extent fuzzy quasi-connectedness in fuzzy bitopological setting. For definitions and results not explained in this paper, we refer to the papers [2,8-10] assuming them to be well known. A fuzzy point in X with support $x \in X$ and value α $(0 < \alpha \leq 1)$ is denoted by x_α. For a fuzzy set A of X, $1-A$ will stand for the complement of A. By 0_X and 1_X we will mean respectively the constant fuzzy sets taking on the values 0 and 1 on X. A fuzzy set A of fbts (X, τ_1, τ_2) is called τ_i-fo (resp. τ_i-fc) if $A \in \tau_i$ (resp. $1-A \in \tau_i$). A fuzzy set A of fbts (X, τ_1, τ_2) is called τ_i-Q-nbd(resp. τ_i-nbd) of x_α if there exists a τ_i-fo set U such that $x_\alpha q U \leq A$ (resp. $x_\alpha \in U \leq A$). For a fuzzy set A of fbts (X, τ_1, τ_2), τ_i-int(A) (resp. τ_i-cl(A)) means respectively the interior and closure of A with respect to the fuzzy topologies τ_i and τ_j, where indices i and j take values {1, 2} and $i \neq j$.

Definition 1.1[9]. Let (X, τ_1, τ_2) be a fbts and A be any fuzzy set of X. Then A is called fuzzy quasi-open[9] (briefly, fqo) if for each fuzzy point $x_\alpha \in A$ there exists either a τ_1-fo set U such that $x_\alpha q U \leq A$ or a τ_2-fo set V such that $x_\alpha q V \leq A$. A fuzzy set A is fuzzy quasi-closed (briefly, fqc) if the complement $1-A$ is a fqo set.

Definition 1.2[9]. Let (X, τ_1, τ_2) be a fbts and A be any fuzzy set of X. Then A is called quasi-nbd (resp. quasi-Q-nbd) of a fuzzy point x_α if there exists a fqo set U such that $x_\alpha \in U \leq A$ (resp. $x_\alpha q U \leq A$).

Definition 1.3[9]. Let A be a fuzzy set of a fbts X.
(a) The quasi-closure of A, denoted by qcl(A), defined by

$$\mathrm{qcl}(A)= \bigcup\{B: A\leq B, B \text{ is fqc}\}$$

(b) The quasi-interior of A, denoted by qcl(A), defined by

$$\mathrm{qint}(A)= \bigcap\{B: B\leq A, B \text{ is fqo}\}$$

For a fuzzy set A of a fbts X, qint(1-A)=1-qint(A) and qcl(A) (resp.qint(A)) is fqc (resp. fqo).

2. Fuzzy Quasi-Semiopen Sets

Definition 2.1[10]. Let (X, τ_1, τ_2) be a fbts and A be any fuzzy set of X. Then A is called fuzzy quasi-semiopen (briefly, fqso) if there exists a fqo set B such that $B\leq A \leq \mathrm{qcl}(B)$. A fuzzy set A is fuzzy quasi -closed (briefly, fqsc) if the complement 1-A is a fqo set.

Every fqo set is fqso set and every τ_i-fso set is fqso set but the converses may not be true.(see Example 1.3(a) in [9] and Example 2.2)

Example 2.2. Let X={a,b,c}, τ_1={ $1_X, 0_X, A$} and τ_2={ $1_X, 0_X, B$} where A and B are fuzzy sets of X given by A(a)=0.7, A(b)=0.4, A(c)=0.7; B(a)=0.6, B(b)=0.7, B(c)=0.3. We consider a fuzzy set C of X defined by C(a)=C(b)=C(c)=0.7. Then C is a fqso set but neither a τ_1-fso set nor a τ_2-fso set.

Theorem 2.3. A fuzzy set A of a fbts (X, τ_1, τ_2) is fqso set if and only if it is the union of a τ_1-fso set and a τ_2-fso set.

Theorem 2.4[10]. (a) Any union of fqso sets is fqso ;
(b) Any intersection of fqsc sets is fqsc.

Remark 2.5. The intersection (resp. union) of fqso (resp. fqsc) sets need not be a fqso (resp. fqsc) set (In Example 3.2).
However we have the following properties.

Theorem 2.6. Let A and B be fuzzy sets of fbts (X, τ_1, τ_2).
(a) If A is a τ_1-fso and τ_2-fso set and B is a fqso set, then $A \cap B$ is a fqso set.
(b) If A is a τ_1-fsc and τ_2-fsc set and B is a fqsc set, then $A \cup B$ is a fqsc set.

The following Example shows that the product of fqso sets need not be a fqso set.

Example 2.7. Let X={a,b,c} and A_k(k=1, 2,3,4)be fuzzy sets of X defined as following:

$$A_1(a)=0.7,\ A_1(b)=0.4,\ A_1(c)=0.7;$$
$$A_2(a)=0.3,\ A_2(b)=0.4,\ A_2(c)=0.7;$$
$$A_3(a)=0.6,\ A_3(b)=0.7,\ A_3(c)=0.3;$$
$$A_4(a)=0.7,\ A_4(b)=0.7,\ A_4(c)=0.3.$$

Let $\tau_1=\{1_X, 0_X, A_1\}$, $\tau_2=\{1_X, 0_X, A_2\}$, $\sigma_1=\{1_X, 0_X, A_3\}$ and $\sigma_2=\{1_X, 0_X, A_4\}$ be fuzzy topologies on X. Then $A= A_1 \cup A_2$ is fqso in (X, τ_1, τ_2) and $B= A_3 \cup A_4$ is fqso in (X, σ_1, σ_2). But $A \times B$ is not fqso in $(X \times X, \tau_1 \times \tau_2, \sigma_1 \times \sigma_2)$.

Definition 2.8. A fuzzy set A of a fbts (X, τ_1, τ_2) is called a quasi semi-Q-nbd (resp. quasi semi-nbd) of a fuzzy point x_α $qU \leq A$ (resp. $x_\alpha \in U \leq A$).

Theorem 2.9. Let A be a fuzzy set of a fbts X. A is a fqso set if and only if it is a quasi semi-nbd of every fuzzy point $x_\alpha \in A$.

Definition 2.10[10]. Let A be a fuzzy set of a fbts X.
(a) qscl(A)= $\bigcap\{F: F$ is fqsc set and $A \leq F\}$is called quasi semi-closure of A.
(b) qscl(A)= $\bigcup\{U: U$ is fqso set and $U \leq A\}$is called quasi semi-interior of A.

For a fuzzy set A of a fbts X, qscl(1-A)=1-qsint(A) and qscl(A) (resp. qsint(A)) is fqsc (resp. fqso) set.

Theorem 2.11. Let A be any fuzzy set of a fbts X. Then $x_\alpha \in$qscl(A) if and only if for each fqso quasi semi-Q-nbd U of x_α, UqA.

Theorem 2.12. If A is any fuzzy set and B is a fqso set of fbts X with $A \not{q} B$, then qscl(A) $\not q$ B.

3. Fuzzy Quasi-Semicontinuous and Fuzzy Quasi-Semiopen(Semiclosed) Mappings

In this section, we introduce the concepts of fuzzy quasi-semicontinuous, fuzzy quasi-semi open (semiclosed) mappings by usings fqso and fqsc sets and study some of their basic properties. Several characterizations of these mappings are obtained.

Definition 3.1. Let $f: (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ be a mapping. Then f is called:
(a) fuzzy quasi-semicontinuous (fq-sc) if $f^{-1}(A)$ is fqso in X for each τ_i-fo set A of Y;
(b) fuzzy quasi-semiopen (fqs-open) if $f(A)$ is fqso in Y for each τ_i-fo set A of X;
(c) fuzzy quasi-semiclosed (fqs-closed) if $f(A)$ is fqsc in Y for each τ_i-fc set A of X.

Definition 3.2. A mapping $f : (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ is called a fuzzy pairwise semi continuous(resp. fuzzy pairwise semiopen, fuzzy pairwise semiclosed), briefly, fp-sc(resp, fps-open, fps-closed) if the induced mappinfs $f :(X, \tau_k) \to (Y, \sigma_k)$ are fuzzy semicontinuous[6] (resp. fso, fsc[11])(k=1,2).

Remark 3.3. It is clear that every fp-sc(resp. fps-open, fps-closed) mapping is fq-sc(resp. fqs-open, fqs-closed). That the converse need not be true is shown by the following examples.

Example 3.4. Let $X=\{a,b,c\}$ and A_k be fuzzy sets of X defined as follows:

$$A_1(a)=0.7,\ A_1(b)=0.4,\ A_1(c)=0.7;$$
$$A_2(a)=0.6,\ A_2(b)=0.7,\ A_2(c)=0.3;$$
$$A_3(a)=0.3,\ A_3(b)=0.7,\ A_3(c)=0.6;$$
$$A_4(a)=0.7,\ A_4(b)=0.7,\ A_4(c)=0.7.$$

Let $\tau_1=\{1_X, 0_X, A_1\}$, $\tau_2=\{1_X, 0_X, A_2\}$, $\sigma_1=\{1_X, 0_X, A_3\}$ and $\sigma_2=\{1_X, 0_X, A_4\}$ be fuzzy topologies on X.
(a) If $f: (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ is mapping defined by $f(a)=c$, $f(b)=b$, $f(c)=a$, then f is fq-sc mapping but not fp-sc.
(b) If $g: (X, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is mapping defined by $g(a)=c$, $g(b)=b$, $g(c)=a$, then g is fqs-open mapping but not fps-open.

Now we shall discuss the characteristic properties of fq-sc, fqs-open, fqs-closed mappings in fbts's.

Theorem 3.5. For a mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following are equivalent:
(a) f is fq-sc ;
(b) For each fuzzy point x_α in X and each σ_i-fo nbd of V of $f(x_\alpha)$, there exists a fqso quasi semi-nbd U of x_α such that $f(U) \leq V$;
(c) For each fuzzy point x_α in X and each σ_i-fo Q-nbd of V of $f(x_\alpha)$, there exists a quasi semi-Q-nbd $U$of x_α such that $f(U) \leq V$;
(d) For each fuzzy set A of X, $f(\text{qscl}(A)) \leq \sigma_i\text{-cl}(f(A))$;
(e) For each fuzzy set B of Y, qscl($f^{-1}(B)$) $\leq f^{-1}(\sigma_i\text{-cl}(B))$.

Theorem 3.6. For a mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:
(i)-(a) f is fqs-open;
(b) $f(\tau_i\text{-int}(A)) \leq \text{qsint}(f(A))$ for each fuzzy set A of X;
(c) $\tau_i\text{-int}(f^{-1}(B)) \leq f^{-1}(\text{qint}(B))$ for each fuzzy set B of Y.
(ii)-(a) f is fqs-closed;
(b) $f(\tau_i\text{-cl}(A)) \leq \text{qscl}(f(A))$for each fuzzy set A of X.

Theorem 3.7. Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \to (Z, \delta_1, \delta_2)$ be mappings
(a) If f is fq-sc and $g : (Y, \sigma_i) \to (Z, \delta_i)$ is fuzzy continuous (i=1,2), then $g \circ f$ is fq-sc.
(b) If $f :(X, \tau_i) \to (Y, \sigma_i)$ is fuzzy continuous (i=1,2) and g is fq-sc then $f \circ g$ is fq-sc.
(c) If $f :(X, \tau_i) \to (Y, \sigma_i)$ is fuzzy open(resp. fuzzy closed) (i=1,2) and g is fqs-open(resp. fqs-closed), then $g \circ f$ is fqs-open(resp. fqs-closed).
(d) If f is fqs-open(resp. fqs-closed) and $g : (Y, \sigma_i) \to (Z, \delta_i)$ is fuzzy open(resp. fuzzy closed) (i=1,2), then $f \circ g$ is fqs-open(resp. fqs-closed).

4. Fuzzy Quasi-Semiconnected Sets

To introduce fuzzy quasi-semiconnected sets, we first define the concept of fuzzy quasi-semiseparatedness in terms of quasi-semi closure operator as follows.

Definition 4.1. Two non-null fuzzy sets A and B of a fbts (X, τ_1, τ_2) (i.e. neither A nor B is 0_X) is called fuzzy quasi-semiseparated if qscl(A) $\not q B$ and qscl(B) $\not q A$.

Theorem 4.2. Let A and B be non-null fuzzy sets of a fbts (X, τ_1, τ_2).
(a) If A and B are fuzzy quasi-semi separated, and A_1 and B_1 are non-null fuzzy sets such that $A_1 \leq A$ and $B_1 \leq B$, then A_1 and B_1 also fuzzy quasi-semiseparated.
(b) If $A \not q B$ and either both fqso or both fqsc, then A and B are fuzzy quasi-semi separated.
(c) If A and B are either both fqso or both fqsc, and if $C_A(B)=A \cap (1-B)$ and $C_B(A)=B \cap (1-A)$, then $C_A(B)$ and $C_B(A)$ are fuzzy quasi-semiseparated.

Theorem 4.3. Two non-null fuzzy sets A and B are fuzzy quasi-semiseparated if and only if there exist two fqso sets U and V such that $A \leq U$, $B \leq V$, $A \not q V$ and $B \not q U$.

Definition 4.4. A fuzzy set which can not be expressed as the union of two fuzzy quasi-semiseparated sets is called a fuzzy quasi-semiconnected set.

Theorem 4.5. Let A be a non-null fuzzy quasi-semiconnected set of a fbts (X, τ_1, τ_2). If A is contained in the union of two fuzzy quasi-semiseparated sets B and C, then exactly one of the following conditions (a) and (b) hold:
(a) $A \leq B$ and $A \cap C = 0_X$;
(b) $A \leq C$ and $A \cap B = 0_X$.

Theorem 4.6. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy quasi-semiconnected set of a fbts X. If there exists $\beta \in \Lambda$ such that $A_\alpha \cap A_\beta \neq 0_X$ for each $\alpha \in \Lambda$, then $A = \bigcup \{A_\alpha : \alpha \in \Lambda\}$ is fuzzy quasi-semiconnected.

Theorem 4.7. Let A be a fuzzy set of a fbts (X, τ_1, τ_2) such that there exists at least one point $x \in X$ with $A(x) > 1/2$. Then A is fuzzy quasi-semiconnected if and only if any two fuzzy points of A are contained in a fuzzy quasi-semiconnected set contained in A.

Theorem 4.8. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a fq-sc surjection. If A is fuzzy quasi-semiconnected, then $f(A)$ is fuzzy semiconnected in (Y, σ_i).

References

[1]. K. K. Azad, *On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity,* J. Math. Anal. Appl. 82(1981) 14-32.
[2]. C. L. Chang, *Fuzzy topological space,* J. Math. Anal. Appl. 24(1968) 182-190.
[3]. N. R. Das and D. C. Baishya, *On fuzzy open maps, closed maps and fuzzy continuous maps in a fuzzy bitopological sapces*(Communicated).
[4]. J. C. Kelly, *Bitopological spaces,* Proc. London Math. Soc. 13(1963) 71-89.
[5]. T. Kubiak, *Fuzzy bitopological spaces and quasi-fuzzy proximities,* Proc. Polish sym. Interval and Fuzzy Math. Poznan, August (1983) 26-29.
[6]. S. S. Kumar, *Semi-open sets, semi-continuity and semi-open mappings in fuzzy bitopological spaces,* Fuzzy sets and Systems 64(1994) 421-426.
[7]. S. Nanda, *On fuzzy topological spaces,* Fuzzy Sets and Systems 19(1986) 193-197.
[8]. J. H. Park, *On fuzzy pairwise semi-pre-continuity,* Fuzzy sets and Systems 93(1998) 375-379.
[9]. J. H. Park, J. K. Park and S. Y. Shin, *Quasi-fuzzy continuity and Quasi-fuzzy separation axioms,* Proc. of KFIS Fall Conf. '98 8(2)(1998) 70-76.
[10]. Y. B. Park, M. J. Son and B. Y. Lee, *Quasi-fuzzy extremally disconnected spaces,* Proc. of KFIS Fall Conf. '98 8(2)(1998) 77-82.
[11] S. S. Thakur and R. Malviya, *Semi-open sets and semi-continuity in fuzzy bitopological spaces,* Fuzzy sets and Systems 79(1996) 251-256.
[12]. C. K. Wong, *Product and quotient theorems,* J. Math. Anal. Appl. 45(1974) 512-521.
[13]. H. T. Yalvac, *Fuzzy sets and functions on fuzzy spaces,* J. Math. Anal. Appl. 126 (1987) 409-423.