Sliding Mode Controller with Enhanced Performance Using Time-Varying Surface and Fuzzy Logic

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Abstract

In variable structure control algorithm, sliding mode makes the closed loop system insensitive to modelling uncertainties and external disturbances. However due to imperfections in switching, the system trajectory chatters, which is very undesirable. And the insensitivity property of a sliding mode controller is present only when the system is in the sliding mode. To overcome these shortcomings, in this paper, new sliding mode control algorithm using time-varying sliding surface and fuzzy PI structure is proposed.

1. Introduction

Sliding mode controller makes the system insensitive to uncertainties and disturbances and so the system has the robustness. Despite the benefits of the sliding mode controller, it has two shortcomings. First, state trajectory in reaching mode remains sensitive to parameter perturbation and external disturbance. It makes the reaching time to the equilibrium point much longer. The second shortcoming is control chattering. In the design of sliding mode controller, it is assumed that the control can be switched from one structure to another infinitely fast. However, in practice it is impossible. To reduce the chattering, boundary layer with the tracking error was proposed[1].

In this paper, to control the robot manipulator with the robust and fast-accurate performance, we propose new sliding mode control algorithm with time-varying sliding surface and fuzzy PI structure. Using time-varying sliding surface, the system have the robustness in reaching mode and the much faster response. Using the control input with fuzzy PI structure instead of boundary layer, we can not only remove chattering but also reduce steady state tracking error.

2. Tracking control of Robot manipulator

In this section, modelling of the robot manipulator is constructed and the time-varying sliding surface and the control input are designed.

2.1 Time-varying sliding surface

Consider the typical robot dynamics with n links described by

$$\mathbf{M}(\boldsymbol{\theta})\,\ddot{\boldsymbol{\theta}} + \mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{u}(t) + \mathbf{d}(t) \qquad (1)$$

Parameter uncertainties of the system exists as follows.

$$\mathbf{M}(\Theta) = \hat{\mathbf{M}}(\Theta) + \Delta\mathbf{M}(\Theta) \qquad (2)$$

$$\mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \Delta\mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \qquad (3)$$

Magnitude of the modelling error and the disturbance is limited as follows.

$$|\Delta M_{ij}(\Theta)| \le M_{ij}^{m}(\Theta), \quad (4)$$

$$|\Delta h_i(\Theta, \dot{\Theta})| \le h_i^{m}(\Theta, \dot{\Theta}) \quad (5)$$

$$|d_i(t)| \le d_i^{m}(t) \quad (6)$$

The magnitude of the desired angular velocity is limited as follows.

$$|(\Delta\mathbf{M}(\boldsymbol{\Theta})\ \ddot{\boldsymbol{\Theta}}_d)_i| \le v_i^{m}(t) \quad (7)$$

If $\boldsymbol{\Theta}_d$ is the desired state trajectory, the tracking error is defined as

$$\mathbf{e} = \boldsymbol{\Theta} - \boldsymbol{\Theta}_d = [e_1, e_2, \cdots, e_n]^T \quad (8)$$

$$\dot{\mathbf{e}} = \dot{\boldsymbol{\Theta}} - \dot{\boldsymbol{\Theta}}_d = [\dot{e}_1, \dot{e}_2, \cdots, \dot{e}_n]^T \quad (9)$$

In this n-link robot system, time varying sliding surface is defined as

$$\begin{aligned} \mathbf{s(t)} &= [s_1(t), s_2(t), \cdots, s_n(t)]^T \\ &= \boldsymbol{\Lambda}(t)\mathbf{e}(t) + \dot{\mathbf{e}}(t) - \mathbf{A}(t) \end{aligned} \quad (10)$$

where,

$$\boldsymbol{\Lambda}(t) = \mathrm{diag}(\lambda_1(t), \lambda_2(t), \cdots, \lambda_n(t)),\ \lambda_i(t) > 0 \quad (11)$$

$$\mathbf{A(t)} = [\alpha_1(t), \alpha_2(t), \cdots, \alpha_n(t)]^T \quad (12)$$

Time-varying sliding surface includes the initial error states, and moved to the predetermined surface. The movement can be executed by rotating or shifting.

type 1: If the initial states lie in 2,4 quadrant of the error state plane

$$\alpha_i(t) = 0, \lambda_{i0} = -\dot{e}_i(0)/e_i(0)$$

$$\begin{aligned} s_i(t) &= \lambda_i(t)e_i(t) + \dot{e}_i(t) - \alpha_i(t) \\ &= (\lambda_{i0} - \Delta\lambda_i[t/\Delta t_i])e_i(t) + \dot{e}_i(t) \\ &\text{for } \lambda_{i0} > \lambda_{ip} \end{aligned} \quad (13)$$

$$\begin{aligned} s_i(t) &= \lambda_i(t)e_i(t) + \dot{e}_i(t) - \alpha_i(t) \\ &= (\lambda_{i0} + \Delta\lambda_i[t/\Delta t_i])e_i(t) + \dot{e}_i(t) \\ &\text{for } \lambda_{i0} \le \lambda_{ip}. \end{aligned} \quad (14)$$

$[t/\Delta t_i]$ is the maximum integer which is not greater than $t/\Delta t_i$. Δt_i is time interval, λ_{ip} is predetermined slope, λ_{i0} is initial slope of the surface. When $\lambda_i(t)$ is equals to λ_{ip}, $\lambda_i(t)$ is fixed to λ_{ip}.

type 2: If the initial states lie in 1,3 quadrant of the error state plane

$$\lambda_i(t) = \lambda_{ip}, \alpha_{i0} = \lambda_{ip}e_i(0) + \dot{e}_i(0)$$

$$\begin{aligned} s_i(t) &= \lambda_i(t)e_i(t) + \dot{e}_i(t) - \alpha_i(t) \\ &= \lambda_{ip}e_i(t) + \dot{e}_i(t) - (\alpha_{i0} + \Delta\alpha_i[t/\Delta t_i]) \\ &\text{for } \alpha_{i0} < 0 \end{aligned} \quad (15)$$

$$\begin{aligned} s_i(t) &= \lambda_i(t)e_i(t) + \dot{e}_i(t) - \alpha_i(t) \\ &= \lambda_{ip}e_i(t) + \dot{e}_i(t) - (\alpha_{i0} - \Delta\alpha_i[t/\Delta t_i]) \\ &\text{for } \alpha_{i0} \ge 0 \end{aligned} \quad (16)$$

α_{i0} is initial intercept of the surface. When $\alpha_i(t)$ is equals to 0, $\alpha_i(t)$ is fixed to zero.

2.2 Design of Control input

Let the Lyapunov function candidate be defined as

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{M}(\Theta)\mathbf{s}(t)$$

Let the control input be

$$\begin{aligned} \mathbf{u(t)} = &-\hat{\mathbf{M}}[\boldsymbol{\Lambda}\dot{\mathbf{e}} - \ddot{\boldsymbol{\Theta}}_d] \\ &+ \hat{\boldsymbol{h}} - \boldsymbol{P}(t)\mathbf{s} - \mathbf{Q(t)}\mathrm{sgn}(\mathbf{s}) \end{aligned} \quad (17)$$

where, $\mathbf{P}(t) = \mathrm{diag}(p_1(t) \ldots p_n(t))$

$\mathbf{Q}(t) = \mathrm{diag}(q_1(t) \ldots q_n(t))$

Differentiating the function with (17) yields (18).

$$\begin{aligned} \dot{\mathbf{V}} = &-\mathbf{s}^T[\mathbf{P} - \dot{\mathbf{M}}/2]\mathbf{s} \\ &+ \mathbf{s}^T[-\mathbf{Q}\mathrm{sgn}(\mathbf{s}) + \Delta\mathbf{M}\boldsymbol{\Lambda}\dot{\mathbf{e}} - \Delta\mathbf{h} + \mathbf{d} - \Delta\mathbf{M}\ddot{\boldsymbol{\Theta}}_d] \end{aligned} \quad (18)$$

If we let $q_i(t)$ be

$$q_i(t) = \sum_{j=1}^{n}(\mathbf{M}^m\boldsymbol{\Lambda})_{ij}|\dot{e}_j| + h_i^m + d_i^m + v_i^m \quad (19)$$

then Eq.(20) is satisfied as follows.

$$\mathbf{s}^T[-\mathbf{Q}\mathrm{sgn}(\mathbf{s}) + \Delta\mathbf{M}\boldsymbol{\Lambda}\dot{\mathbf{e}} - \Delta\mathbf{h} + \Delta\mathbf{M}\ddot{\boldsymbol{\Theta}}_d] \le 0 \quad (20)$$

If we let $p_i(t)$ be

$$p_i(t) = \sum_{j=1}^{n}\dot{M}_{ij}^m/2 + k_i,\quad k_i > 0 \quad (21)$$

then following equation is satisfied.

$$\mathbf{P} - \dot{\mathbf{M}}/2 = \frac{1}{2}\begin{bmatrix} \sum_{j=1}^{n}\dot{M}_{1j}^m - \dot{M}_{11} & -\dot{M}_{12} & \cdots & -\dot{M}_{1n} \\ -\dot{M}_{21} & \sum_{j=1}^{n}\dot{M}_{2j}^m - \dot{M}_{22} & \cdots & -\dot{M}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\dot{M}_{n1} & -\dot{M}_{n2} & \cdots & \sum_{j=1}^{n}\dot{M}_{nj}^m - \dot{M}_{nn} \end{bmatrix} + \mathrm{diag}(k_1, k_2, \cdots, k_n) \quad (22)$$

As the matrix of eq.(22) is positive

definite, eq.(23) is satisfied as follows.

$$-\mathbf{s}^T[\mathbf{P}-\dot{\mathbf{M}}/2]\mathbf{s} \le -\mathbf{s}^T \mathrm{diag}(k_1, k_2, \cdots, k_n)\mathbf{s} \quad (23)$$

We can know that $\dot{V}$ is negative definite from Eq.(20), (23). and the control input(17) satisfies the Lyapunov stability criterion.

To remove chattering and reduce steady state error in response, the control input (17) is transformed into (24).

$$\mathbf{u(t)} = -\widehat{\mathbf{M}}[\Lambda\dot{\mathbf{e}} - \ddot{\boldsymbol{\theta}}_d] + \hat{\boldsymbol{h}} - \boldsymbol{P}(t)\mathbf{s} + \mathbf{Q}(t)\mathbf{F(s)} \quad (24)$$

where, **F(s)** is inferred by fuzzy reasoning The inputs for this fuzzy reasoning are s(t), $\int s(t)$ and its output determine the gain of control input, **F(s)**. The fuzzy rule base is designed as follows.

s / ∫s(t)	NB	NS	ZE	PS	PB
PB	PS	PS	ZE	NS	NS
PS	PB	PS	ZE	NS	NB
ZE	PB	PB	ZE	NB	NB
NS	PB	PS	ZE	NS	NB
NB	PS	PS	ZE	NS	NS

Table 1: Fuzzy rule base

The Fuzzy reasoning is performed using the singleton fuzzifier, product inference engine, center average defuzzifier.

3. Design example

To manifest some features of the proposed controller, we consider a 2-link robot manipulator system.

The elements of the external disturbance vector **d(t)** are given by

$d_1(t) = d_2(t) = 0.5\sin(3\pi t)$

The physical specification of the robot system is assumed as follows.

$r_1 = 1.0\text{m}$, $r_2 = 1.5\text{m}$ (length of each link)

$J_1 = 5.0\text{Kg.m}^2$, $J_2 = 5.0\text{Kg.m}^2$ (moment inertia)

$m_1 = 0.5\text{Kg}$, $m_2 = 0.5\text{Kg}$ (mass)

The desired state trajectory and initial conditions are chosen by

$\theta_{d1} = \cos(2t)$, $\theta_{d2} = \sin(2t)$

$\boldsymbol{\theta} = [0.5, 2]$, $\dot{\boldsymbol{\theta}} = [0.5, 3]$

The design parameters for each sliding surface are determined as follows.

	θ_1	θ_2
λ_D	10	10
λ_0, α_0	$\lambda_{10}=1$	$\alpha_{20}=21$
Δt	0.002 sec	0.002 sec
$\Delta\lambda, \Delta\alpha$	$\Delta\lambda_1=0.05$	$\Delta\alpha_2=0.005$

Table 2: Design parameters for sliding surface

The membership functions of the inputs and output for the fuzzy reasoning to determine the control input are as following figures.

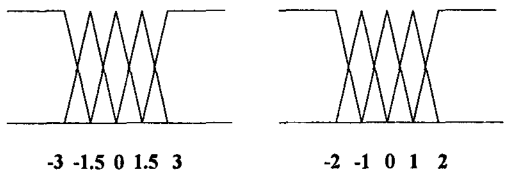

Fig. 1: Membership function of s(t), $\int s(t)$

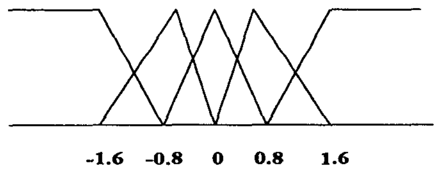

Fig. 2: Membership function of F(s)

Then we can define the time varying sliding surface like eq.(25), (26).

$$s_1(t) = (1 - 0.05[t/0.002])e_1 + \dot{e}_1 = 0 \quad (25)$$

for θ_1

$$s_2(t) = 10e_2 + \dot{e}_2 - (21 - 0.005[t/0.002]) \quad (26)$$

for θ_2

(dash-dot : desired, solid : tracking state)

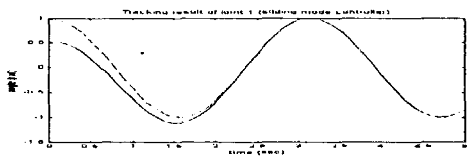

Fig. 3-1: control response of θ_1

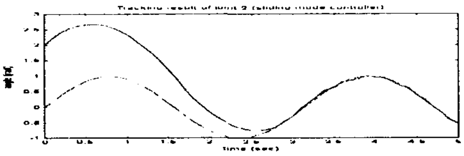

Fig. 3-2: control response of θ_2

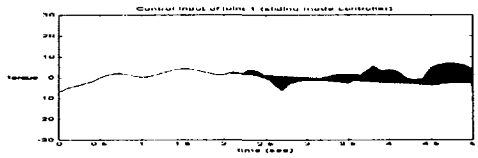

Fig. 3-3: control input of θ_1

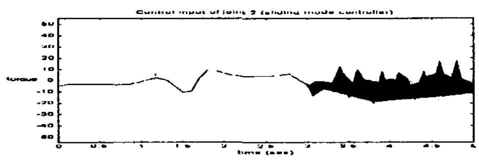

Fig. 3-4: control input of θ_2

Fig. 3: Control using conventional sliding mode controller

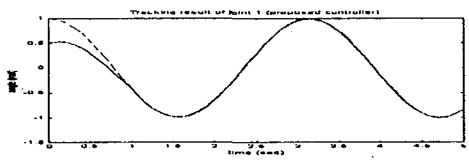

Fig. 4-1: control response of θ_1

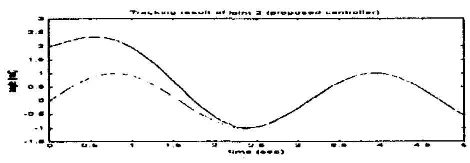

Fig. 4-2: control response of θ_2

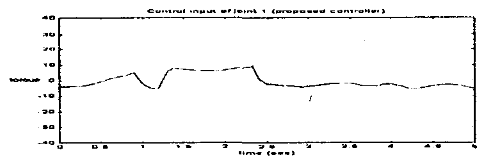

Fig. 4-3: control input of θ_1

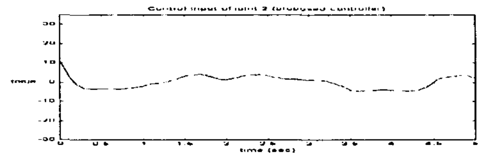

Fig. 4-4: control input of θ_2

Fig. 4: Control using the proposed controller

Figure 3 presents control response and control input for each angle using the conventional sliding mode controller. Figure 4. presents the control response and the control input using proposed controller. It is clearly observed that the response time is faster than the conventional one and the chattering is eliminated. Unlike the controller using the boundary layer, the steady state error is reduced and so the system performs more precise response.

4. Conclusions

The new design algorithm of the sliding mode controller to improve the tracking behavior has been proposed. Employing the time-varying sliding surface, it was possible to lessen the tracking time. Furthermore, the tracking accuracy is improved and the chattering was eliminated with the help of applying fuzzy PI structure to the control input.

5. References

[1] Young, K.-K. D., "Controller design of a manipulator using theory of variable structure systems," *IEEE Trans. on Sys. Man and Cyb.*, vol. 8, no. 12, pp. 101-109, 1977.

[2] S. Choi, C. Cheon and D. Park, "Moving switching surface for robust control of second order variable structure systems," *International Journal of Control*, vol. 58, no.1, pp. 229-245, 1993.

※ Acknowledgment

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