퍼지연산

Fuzzy arithmetic

Se Hwa Chung

Dept. of Mathematics and information, Kyungwon University, Sungnam City, Kyunggi-do, 461-701

Abstract

Using the concept of a piecewise linear function, we present new operations for fuzzy arithmetic and then compare the operation based by the extension principle with the new operation.

1. Preliminaries

A ***fuzzy set*** is a map A on a set X to the unit interval. For any fuzzy set A on a set X and any number $\alpha \in [0, 1]$, the ***α-cut***, A^{α}, and ***support***, A^{+0}, are the crisp sets $A^{\alpha} = \{x \in X \mid A(x) \geq \alpha\}$ and $A^{+0} = \{x \in X \mid A(x) > 0\}$, respectively. $S(A)$ denotes the closure of A^{+0} in the real line.

A ***fuzzy number*** is a fuzzy set A on the set of real numbers $\mathbb{R}$ such that for each $\alpha \in (0, 1]$, A^{α} is a non-empty closed interval and A^{+0} is bounded.

A fuzzy number A is said to be **pointed** if the core of A is a one point set.

A linear function f on $[a, b]$ to $[c, d]$ is said to be:

1) ***increasing*** if for each $x \in X$, $f(x) = \dfrac{d-c}{b-a}(x-a)+c$,

2) ***decreasing*** if for each $x \in X$, $f(x) = \dfrac{c-d}{b-a}(x-a)+d$.

3) **piecewise** if there are $a_2, a_3 \in [a_1, a_4]$ and $b_2, b_3 \in [b_1, b_4]$ such that $f|_{[a_n, a_{n+1}]}^{[b_n, b_{n+1}]}: [a_n, a_{n+1}] \to [b_n, b_{n+1}]$ is a linear function for each $n \in \{1, 2, 3\}$, where $a_1 = a$, $a_4 = b$, $b_1 = c$ and $b_4 = d$.

Notation For a fuzzy number A, let $S(A) = [a, b]$ and $A^1 = [d, e]$. Then $R(A)$ and $L(A)$ denote $[a, d]$ and $[e, b]$, respectively.

Definition 1.1 Let A and B be fuzzy numbers. Then a piecewise linear funtion $f: S(A) \to S(B)$ is said to be:

1) ***increasing*** if $f|_{R(A)}^{R(B)}: R(A) \to R(B)$, $f|_{A^1}^{B^1}: A^1 \to B^1$ and $f|_{L(A)}^{L(B)}: L(A) \to L(B)$ are increasing linear functions.

2) ***decreasing*** if $f|_{R(A)}^{L(B)}: R(A) \to L(B)$, $f|_{A^1}^{B^1}: A^1 \to B^1$ and $f|_{L(A)}^{R(A)}: L(A) \to R(B)$ are decreasing linear functions.

Definition 1.2 A binary operation $*: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is said to be:

1) ***increasing (decreasing,*** resp.) if $x_1 < y_1$, $x_2 < y_2$ imply ($x_1 * x_2 > y_1 * y_2$, resp.),

2) ***hybrid*** if $y_2 < x_2$, $x_1 < y_1$ imply $x_1 * x_2 < y_1 * y_2$,

In the following, $*$ denotes the continuous binary operation on $\mathbb{R}$ and we assume that every fuzzy number is continuous.

Remark Addition +, meet $\wedge$ and join $\vee$ are continuous increasing binary operations on $\mathbb{R}$ and subtraction - is a continuous hybrid binary operation on $\mathbb{R}$.

Notation Let A and B be fuzzy numbers and $f: S(A) \to S(B)$ a piecewise linear function. Then $\iota(A * B)$ denotes the set $\{x * f(x) \mid x \in S(A)\}$.

Proposition 1.3 Let A and B be fuzzy numbers and $f: S(A) \to S(B)$ a piecewise linear function. Then one has the following:

1) $\iota(A * B)$ is a closed interval.

2) If f and $*$ are increasing, then $\iota(A*B) = \{x * y \mid x \in S(A),\ y \in S(B)\}$.

3) If f is decreasing and $*$ is hybrid, then $\iota(A*B) = \{x * y \mid x \in S(A),\ y \in S(B)\}$.

For a general theory of fuzzy sets and that of fuzzy numbers we refer to [1, 3, 6].

2. Operations on fuzzy numbers

Theorem 2.1 Let A and B be fuzzy numbers and f: $S(A) \rightarrow S(B)$ a piecewise linear function. Then, we define a fuzzy set on $\mathbb{R}$, $A \circledast B$, as follows:

$$(A \circledast B)(z) = \begin{cases} A(x) \wedge B(f(x)) & \text{if } z = x * f(x) \text{ for } \textit{some}\, x \in S(A) \\ 0 & \text{if } \quad \textit{otherwise} \end{cases}.$$

Then one has the following:

1) If f and $*$ are increasing, then $A \circledast B$ is a fuzzy number.

2) If f is decreasing and $*$ is hybrid, then $A \circledast B$ is a fuzzy number.

Proposition 2.2 Let A and B be fuzzy numbers. Then $(A \circledast B)(z) \leq (A * B)(z)$ for all $z \in \mathbb{R}$.

Definition 2.3 Let A and B be fuzzy numbers. Then a piecewise linear function f: $S(A) \rightarrow S(B)$ is said to be a ***shift*** (from A to B) if for each $x \in S(A)$, $A(x) = B(f(x))$.

Definition 2.4 1) Two fuzzy numbers A and B are said to be ***i-equipotent*** ***(d-equipotent***, resp.), symbolized as $A \sim B$ ($A \approx B$, resp.), provided that there exists an increasing(decreasing, resp.) shift from A to B.

2) Two fuzzy numbers A and B are said to be ***equipotent*** if they are i-equipotent or d-equipotent.

Proposition 2.5 The relation $\sim$ is a crisp equivalence relation on $\mathbb{F}(\mathbb{R})$, where $\mathbb{F}(\mathbb{R})$ is the set of fuzzy numbers.

Remark It is easy to show that if A and B are $L-R$ type pointed fuzzy numbers, then they are equipotent.

Theorem 2.6 Let A and B be fuzzy numbers. Then one has the following:

1) If $A \sim B$ and $*$ is increasing, then $(A * B)(z) = \bigvee_{z=x*y} A(x) \wedge B(y)$.

2) If $A \approx B$ and $*$ is hybrid, then $(A * B)(z) = \bigvee_{z=x*y} A(x) \wedge B(y)$.

Definition 2.7 A fuzzy number A is said to be ***positive***(***negative***, resp.) if $S(A) \subseteq [0, \infty)$ ($S(A) \subseteq (-\infty, 0]$, resp.).

Using the exactly same argument as that of Theorem 2.2, we have the following:

Theorem 2.8 1) Let A and B be positive(negative) fuzzy numbers such that $A \sim B$. Then $(A \boxtimes B) = A \times B$.

2) Let A be a positive fuzzy number and B a negative fuzzy number such that $A \approx B$. Then $(A \boxtimes B) = A \times B$.

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