Multiple Path Based Vehicle Routing in Dynamic and Stochastic Transportation Networks

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ABSTRACT

In route guidance systems fastest-path routing has typically been adopted because of its simplicity. However, empirical studies on route choice behavior have shown that drivers use numerous criteria in choosing a route. The objective of this study is to develop computationally efficient algorithms for identifying a manageable subset of the nondominated (i.e. Pareto optimal) paths for real-time vehicle routing which reflect the drivers' preferences and route choice behaviors. We propose two pruning algorithms that reduce the search area based on a context-dependent linear utility function and thus reduce the computation time. The basic notion of the proposed approach is that i) enumerating all nondominated paths is computationally too expensive, ii) obtaining a stable mathematical representation of the drivers' utility function is theoretically difficult and impractical, and iii) obtaining optimal path given a nonlinear utility function is a NP-hard problem. Consequently, a heuristic two-stage strategy which identifies multiple routes and then select the near-optimal path may be effective and practical. As the first stage, we utilize the relaxation based pruning technique based on an entropy model to recognize and discard most of the nondominated paths that do not reflect the drivers' preference and/or the context-dependency of the preference. In addition, to make sure that paths identified are dissimilar in terms of links used, the number of shared links between routes is limited. We test the proposed algorithms in a large real-life traffic network and show that the algorithms reduce CPU time significantly compared with conventional multi-criteria shortest path algorithms while the attributes of the routes identified reflect drivers' preferences and generic route choice behaviors well.

I. INTRODUCTION

Fastest-path routing has typically been adopted in Route Guidance System (RGS) applications because of its simplicity. However, empirical studies on route choice behavior have shown that drivers use numerous criteria in choosing a route including, but not limited to, travel time, number of signals, travel time reliability, safety, familiarity, and congestion (Khattak et al., 1992; Abdel_Aty et al., 1995; Schofer et al., 1997; Wohlschlaeger, 1997). For this purpose, we need to solve the so called multi-criteria (or attributes) shortest path problem (MCSPP). This problem also is referred to in the literature as the multiobjective and vector weights shortest path problem (Sancho, 1988; Current et al., 1990).

There is a rich literature on the MCSPP in the operations research and management science areas, which is known as an NP-hard problem unless we have a known linear utility function. However, existing approaches are limited for application in real-time RGS because they may i) generate a set of the nondominated paths which usually are unreasonably large, ii) focus on the computationally efficient approximation of the set of the nondominated paths without considering the utility function or preference of the driver, iii) require an explicit utility function which may be impractical for real-time in-vehicle routing, and/or iv) need direct interaction with the decision maker (DM).

The central hypothesis of this paper is that there is a need to identify multiple reasonable alternative paths for the real-time in-vehicle routing if criteria other than the minimum travel time (or any other single criterion) are to be used. The basic notion of the proposed approach is that i) enumerating all nondominated paths is computationally too expensive, ii) obtaining a stable mathematical representation of the driver's utility function is theoretically difficult and impractical, iii) obtaining optimal path given a nonlinear utility function is a NP-hard problem, and iv) link costs are stochastic rather than deterministic. Consequently, a heuristic two-stage strategy which identifies reasonable multiple routes by considering deterministic components of the link costs and then select the "near-optimal path" by considering random components of the link costs as well may be effective and practical. As the first stage, we utilize the relaxation based pruning technique based on an entropy model to recognize and discard most of the nondominated paths that do not reflect the driver's generic preference and the context-dependent preference. That is, we develop computationally efficient algorithms for identifying a manageable subset of the nondominated (i.e. Pareto optimal) paths for real-time vehicle routing. We propose two pruning algorithms that reduce the search area based on a context-dependent linear utility function and thus reduce the computation time. In addition, to make sure that paths identified are dissimilar in terms of links used, the number of shared links between routes is limited. The ultimate route choice will be supported by a fuzzy logic-based decision support system (Rilett and Park, 1999) or will be made by the driver by direct judgement.

The balance of the paper is organized as follows. In Section II we mathematically formulate the problem. In Section III, We discuss the modeling assumptions which form the framework of the proposed approaches. In Section IV we discuss the conceptual framework of the context-dependent weighting based pruning approach. We then present two algorithms in Section V. In Section VI, we test the two algorithms on a traffic network from Austin, Texas under various traffic conditions, and analyze the results. A concluding discussion follows in Section VII.

II. NOTATIONS AND PROBLEM FORMULATION

Consider a graph, $G=(N, A)$, composed of a finite set of nodes N and a set of links $A \in N \times N$. Denote the origin node O and the destination node D in the network and let P be the set of all feasible paths in the network from O to D. Note that we are interested in only acyclic (i.e. loopless) paths because a cyclic path is not reasonable within the driver route choice context. A path p from the origin node O to the destination node D is a sequence of links: $p=\{(i_O, i_1), (i_1, i_2), \ldots, (i_{D-1}, i_D)\}$, where the initial node of each link is the same as the terminal node of the preceding link in the sequence, and $i_O, \ldots i_D$ are distinct nodes. With each link (i, j) of the network, associate a vector of m attributes $c_{ij} = (c_{ij}^1, \ldots, c_{ij}^m)^T$.

The vector attributes corresponding to path p, z(p), is given by $z(p) = \sum_{(i,j)\in p} c_{ij}$, and the value of a path p with respect to i-th attribute is given by $z^i(p) = \sum_{(i,j)\in p} c^i_{ij}$. Then

$z(p) = \left[z^1(p),....,z^m(p)\right]$. Let Z be the set of all attribute vectors z for all the paths in P and H(Z) be the convex hull of Z. Assume that "less is better" with respect to all objectives, so that path p_j is at least as preferred as path p_k if and only if $z(p_j) \leq z(p_k)$. We also assume that all components of link cost vectors are positive.

Definition 1. A path $p_i \in P$ is a "nondominated path" if there does not exist any other path $p_j \in P$ such that $z^j \leq z^i$, with the inequality being strict for at least one component of z. Note that the literature also refers to nondominated paths as efficient or Pareto optimal paths.

Using Definition 1, we now formulate the MCSPP without a utility function as (Wiecek and Hadavas, 1995):

$$P1: \quad \text{v-min } z \quad \text{s.t. } z \in Z \tag{1}$$

Let Z_N be the solution set of problem P1. That is, P_N is the set of all nondominated paths in P and Z_N is the set of corresponding nondominated attribute vectors.

Definition 2. A path $p \in P_N$ is an "extreme nondominated path" if its attribute vector z is on the boundary of H(Z) (i.e. an extreme point of H).

For example, in the case of two attributes, a path $p_i \in P$ is a extreme nondominated path with respect to $\alpha \in (0,1)$ if $\alpha v^1(p_i)+(1-\alpha)v^2(p_i) < \alpha v^1(p)+(1-\alpha)v^2(p)$ for all $p \in P$ such that $(v^1(p),v^2(p)) \neq (v^1(p_i),v^2(p_i))$. In other words, the extreme nondominated paths minimize convex combinations of the objectives (i.e. attributes in this paper). Obtaining a path in this set is computationally equivalent to identifying a shortest path with linear generalized cost or utility function because each extreme point of H is in some supporting hyperplane to H (Henig, 1985a). In general, identifying the set of extreme nondominated paths is faster than finding the set of nondominated paths. Denote P_{EN} as the set of all extreme nondominated paths and Z_{EN} as the set of corresponding extreme nondominated attribute vectors. Then we obtain following relationships: $P_{EN} \subseteq P_N \subseteq P$ and $Z_{EN} \subseteq Z_N \subseteq Z$.

Alternatively, the MCSPP formulation of P1 can be formulated as the multiple objective linear programming problem listed below:

$$P2: \quad \text{v-min } f(x) = [f_1(x),....,f_m(x)] \quad \text{s.t } x \in X, \tag{2}$$

where $f_k(x) = \sum_{(i,j)\in A} c_{ij}^k x_{ij}, \ k = 1,\ldots,m$

X = the set of flows x_{ij} along the links in the network satisfying Equation 3:

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & for\ all\ i\in N-\{O,D\} \\ -1, & i=D \end{cases} \qquad (3)$$
$$x_{ij} = 0\ or\ 1 \quad for\ all\ (i,j)\in A$$

where x_{ij} =1 indicates that link (i, j) lies on the solution path and x_{ij}=0 otherwise. Observe that the set Z is the image of the feasible set X under vector-valued mapping $[f_1(x),\ldots,f_m(x)]$ and we refer to Z as the attribute or objective space and X is referred to as the decision space of P1 and P2.

Let U: $\mathbb{R}^m \Rightarrow \mathbb{R}$ be a real-valued utility function for evaluating each path in the set of P such that z is preferred to z′ if and only if U(z)>U(z′). Then the utility associated with path p is U(z) and the MCSPP with a multiattribute utility function is formulated as the following utility function program:

$$P3: \quad \max \; U(z) \quad s.t. \; z \in Z \qquad (4)$$

III. MODELING CONSIDERATIONS AND ASSUMPTIONS

1. Expected Utility Function for the Real-Time In-vehicle Routing Problem

A Multiple Attribute Utility Function (MAUF) (a generalized single-criterion function to maximize utility or expected utility) has been adopted to represent the DM's evaluation of the alternatives in the context of Multiple Criteria Decision Making (MCDM). We present two propositions about the nature of the utility function in the context of the real-time in-vehicle routing problem.

Proposition 1. Utility functions in the context of real-time in-vehicle routing are inherently "nonlinear".

We assume that the marginal rate of substitution is not constant over the entire range of attribute values. For example, when one minute of route travel time savings may be worth nothing, 10 minutes of savings equals to 1 dollar, and an hour worth of 10 dollars (See also Mirchadani and Wiecek, 1993; Gabriel and Berstein, 1997; Scott and Bernstein,1998).

Consider the following two definitions on the shape of the utility function.

Definition 3. A utility function U: $R^m \Rightarrow R$ is "quasiconcave" iff for all $z(p_1), z(p_2) \in R^m$ such that $U(z(p_1)) \le U(z(p_2))$, $U(z(p_1)) \le U[\lambda z(p_1)+(1-\lambda)z(p_2)]$, for all $\lambda \in [0,1]$.

Definition 4. A utility function U: $R^m \Rightarrow R$ is "quasiconvex" iff for all $z(p_1), z(p_2) \in R^m$ such that $U(z(p_1)) \ge U(z(p_2))$, $U(z(p_1)) \ge U[\lambda z(p_1)+(1-\lambda)z(p_2)]$, for all $\lambda \in [0,1]$.

That is, for the quasiconcave utility function $U[\lambda z(p_1)+(1-\lambda)z(p_2)] \ge$ minimum $\{U(z(p_1)),U(z(p_2))\}$, and similarly for the quasiconvex utility function $U[\lambda z(p_1)+(1-\lambda)z(p_2)] \le$ maximum

$\{U(z(p_1)), U(z(p_2))\}$ for each $\lambda \in [0,1]$. Now consider Proposition 2.

Proposition 2. A utility function in the context of real-time vehicle routing is either "quasiconcave" or "quasiconvex".

The marginal rate of substitution is not constant by Proposition 1. Moreover, the marginal rate of substitution is not strictly increasing or decreasing but could change as the absolute value of an attribute changes. Therefore, it appears that the quasiconcave or quasiconvex form is well suited to represent the utility function for the real-time vehicle routing problem. Finally, we obtain the following two theorems and can conclude that all nondominated paths should be considered as possibilities for the optimal path. In other words, a non-extreme nondominated path may be an optimal path which makes the problem NP-hard.

Theorem 1. The optimal solution (i.e. optimal path in this paper) exists within the set of the nondominated paths if a utility function is non-increasing.
Proof: See the theorem 6.11 in Steuer (1986).□

Theorem 2. The optimal solution does not necessarily exist within the set of extreme nondominated paths if a utility function is quasiconcave .
Proof: See the proofs of Henig (1985a).□

2. Difficulty of Assessing Utility Function

MAUT assumes the existence of a consistent, absolute utility function. An important step in the process of identifying the utility function is to verify the existence of relevant independence assumptions. These assumptions are then used to identify the appropriate form of the utility function such as (weighted) additive, multiplicative, quasiadditive, multilinear, etc. The two major assumptions are the "preferential independence (PI)" and "utility independence (UI)" of attributes. In real-time in-vehicle routing problem, however, it is easy to envision situations which violate these two assumptions (see Park(1998) for Example).

From a practical perspective, it has been argued that the additive and multiplicative utility functions are considered as simple and robust approximations of the MAUF when there are more than four attributes even if they require the independence assumptions (Zeleny, 1982). In fact because proving the independence assumptions is extremely difficult to perform in practice, many analysts simply assume that a particular utility decomposition is correct for a given situation. For example, a number of studies have utilized additive linear utility functions in solving MCSPP or MCDM problems (White, 1982; Henig, 1985b; Steuer, 1986; Malakooti, 1989). Alternatively, incorporating direct interaction with the DM in the MCDM or MCSPP have been utilized to alleviate the difficulty in eliciting the explicit utility function (Zeleny, 1982; Henig, 1985b/1994; Malakooti, 1989).

3. Relaxing Uniqueness of Utility Function with Context-Dependency of Preference

Another potential pitfall in application of MAUT for the real-time in-vehicle routing is its basic axiom stating that a DM has unique and stable preferences over various situations. Weber

(1987) argues that the information requirement for the DM describing the decision situation which consists of given set of alternatives, a set of objectives (attributes), and the DM's stable preference structure is much too strict in most practical applications. This is because the DM cannot realistically decide on attribute importance until the set of alternatives is completely defined. He states that MAUT treats the DM's stable and unstable preference structures alike without inferring the DM's uncertainty over the relevance of product attributes from their stated preferences. Zeleny (1982) pointed out a similar deficiency of MAUT. Motivated by the restrictions of the assumption of unique utility function, Malakooti (1989) relaxed the assumption of existence of unique global weights of the additive utility function and proposed a quasiconcave (and quasiconvex) MAUF which is a generalization of additive MAUFs. The algorithm uses a combination of local unique or range weights determined by direct interaction with the DM and linear programming. The algorithm implicitly accounts for the context-dependency of weights by assuming that different alternatives may have different attributes weights.

<Example 1> illustrates the case under which unique weights of the utility function may not be assessed with certainty even if the PI assumption holds, and therefore the assumption of unique and stable preference of the MAUF may not be applicable for real world in-vehicle routing problems.

Example 1. Assume that three attributes, travel time (TT), distance (DIS), and V/C ratio (VC), are preferentially independent, and that the utility function of a driver is approximated by $U = -0.5TT^2 - 0.3DIS^{1.5} - 0.2VC^2$. The maximum values of travel time, distance, and V/C ratio for this choice are assumed to be 70 minutes, 50 km, and 2.0, respectively, and TT, DIS, and VC values in the utility function are normalized values based on the maximum values (i.e. anchor values) of each attribute. Assume three nondominated routes whose attributes and normalized values are summarized as follows:

- Route A: (60 min., 42 km, 1.5) and (0.86, 0.84, 0.75)
- Route B: (62 min., 44 km, 1.2) and (0.89, 0.88, 0.60)
- Route C: (64 min., 47 km, 0.9) and (0.91, 0.94, 0.45)

The utility values of the three routes A, B, and C are -0.71, -0.72, and -0.73, respectively, and accordingly route A is preferred over routes B and C. In contrast to applying the approximate utility function directly, assume that the driver is confronted with a choice between routes A and B only. In this case, because the relative difference of all attributes are not significantly different from each other, she may reveal a very similar preference to the given utility function and would prefer route A over route B. Similarly she may prefer route B over route C if she is given a choice of only routes B and C. However, if she is confronted with routes A and C only or routes A, B, and C simultaneously, and she feels the relative difference in V/C ratios among routes compared to those of other attributes is significant, she may put more weight than previously on the V/C ratios. Accordingly she may prefer route C over route A. That is, in this case, the so called "intransitivity of preference" of paired comparisons may occur because the following preference relationships have been revealed: $R^A \succ R^B$, $R^B \succ R^C$, and $R^C \succ R^A$.

The result of Example 1 may be attributed to the "anchor dependency of the preferences" or "context-dependency of the information or preferences". That is, if the situation changes from which the initial utility measurement was based, then the preference may also change. In human decision making and assessment of intensities of preference, reference objects are not selected arbitrarily but are characterized by distinct desirable properties. In this sense, anchored scales are important because people usually express their preferences only with respect to a given reference point (or points)(Zeleny, 1982). In Example 1, the travel time may not transmit information to the driver. There is no sense in claiming that the travel time is important if a particular traveler does not find it useful as a decision making tool in a given situation. This occurs because the concept of attribute importance is relative, not absolute, and it depends on the particular set of alternatives being considered at any one time (Beckwith and Lehmann, 1973; Roy and Vincke, 1981; Giuliano, 1985). More importantly, the distribution of attributes of all alternatives may affect the weights of all alternatives. In other words, the choice of appropriate anchors will influence the intensity or even rank order of preferences, which can violate the MAUT's basic assumptions that i) weights of attribute importance are somehow fixed (at least temporarily) in the DM's head, and ii) the weights are independent of the actual decision situation.

In summary there are essentially two weights for attributes in a given decision situation: "primary weights" and "secondary weights". The primary weight is the subjective assessment of the attributes' importance, reflecting the DM's cultural, empirical, psychological, and environmental history. This is a generic preference which may be directly obtained from a DM for a particular situation (i.e. journey to work). The secondary weight is the average intrinsic information generated by the given set of feasible alternatives (Zeleny, 1976/1982). If decision making is considered as an information-processing activity, most *a priori* declared weights (i.e. primary weight) are independent of the actual alternatives and their associated attributes. Consequently, it turns out that the conventional MAUT may not be the most reliable tool to represent the drivers route choice behavior. For this reason, we present Proposition 3 which relaxes the uniqueness of the utility function.

Proposition 3. The utility function within the context of real-time in-vehicle routing is not consistent or certain but context-or situation-dependent.

Formally,

$$U = f\,(\,PR_B,\, PR_{CD}) \tag{5}$$

where PR_B and PR_{CD} are DM's basic (or primary) and context-dependent preferences or utility functions. Alternatively,

$$U = f\left(\, x_1^1, x_1^2, \ldots, x_1^n, x_2^1, \ldots\ldots, x_m^n \right) \tag{6}$$

where x_i^j is the i-th attribute value of the j-th alternative, and m and n are the numbers of attributes and alternatives, respectively, for a given situation.

4. Combining Primary and Context-Dependent Preferences

In MAUT, the additive utility function for m attributes which is independent of a given situation is formalized as follows:

$$U = \sum_{i=1}^{m} w_i u_i(x_i) \tag{7}$$

where m = the number of attributes
$u_i(x_i)$ = individual one-attribute utility function of the i-th attribute
w_i = *a priori* (primary) weight for i-th attribute

Denote w_i^c as the context-dependent weight or secondary weight. Then, based on Proposition 3, by including the context-dependent preferences (i.e. PR_{CD}) into the utility function, the utility function would be

$$U = \sum_{i=1}^{m} f\left(w_i, w_i^c\right) u_i(x_i) \tag{8}$$

That is, the optimal weight is a combination of the basic and context-dependent weight. However, a driver would rely on the following method if she confronts with all alternatives and their associate attributes for a given trip.

$$U = \sum_{i=1}^{m} w_i^* u_i(x_i) \tag{9}$$

where w_i^* is an optimal weight of the i-th attribute. Therefore, if it were possible to directly obtain the optimal weight, the problem would be solved easily. However, note that to accomplish this, the entire set of nondominated alternatives (i.e. routes) and their associated attributes would be required. In the real-time in-vehicle routing problem, there may be hundreds of nondominated routes between an origin and destination and attribute values may be the function of O-D distance, level of congestion, time of day, etc. In addition, drivers may have different basic preferences or utility functions for different trips.

In this context, the former (i.e. Equation 8) appears to be more practical. To solve the problem using Equation 8, we adopt a two-step process. The first step measures the context-dependent weight, w^c, as will be detailed later and the second step combines the prior and context-dependent weights into an "optimal" weight. For the second step, rather than explicitly combining those two weights, we present Proposition 4 which defines the possible range of the optimal weights.

Proposition 4. The optimal weight for each attribute, w_i^* , lies between *a priori* weight w_i and context-dependent weight (or *posterior* weight), w_i^c.

Mathematically, we can formulate Proposition 4 as follow:

$$MIN\left[w_i,\ w_i^c\right] \leq w_i^* \leq MAX\left[w_i,\ w_i^c\right], \quad \forall i \tag{10}$$

For instance, in Example 1 a driver would decrease the weights for travel time and distance and increase the weight for V/C ratio. Based on Proposition 4, rather than identifying a single optimal path, we consider any nondominated path between those two weights as a ″context-dependent nondominated path (CDND)″.

5. Treatment of Diversity in Utility Function: Modularization

Intuitively, a utility function of a driver within the context of route choice could vary for different trip purposes, departure times, time limitations (or time windows), etc. Therefore, in contrast to problems which deal with a single situation (e.g. buying a new car), the real-time in-vehicle routing problem may be extended or interpreted as multiple sub-problems. In this sense, by including the generic diversity of utility function for the in-vehicle routing problem, we can reformulate Equations 8 and 9 as follows:

$$U = \sum_{i=1}^{m} \left(w_i^* | S\right) \left(u_i(x_i) | S\right) = \sum_{i=1}^{m} \left(f\left(w_p w_i^c\right) | S\right) \left(u_i(x_i) | S\right) \quad (11)$$

where S is a given situation for each trip (e.g. trip purpose, departure time, time window, etc.). Together with the theoretical difficulty in defining a utility function as shown in the preceding sections, Equation 11 illustrates the difficulty in defining a utility function of a driver for every situation.

There may be two ways to extract the basic utility function or preference of a driver for each trip. One strategy may be to use a single universal utility function which can accommodate all possible trip situations. The other is to employ an utility function for each situation. In this study we base on the second strategy, and accordingly we assume that a driver can express her basic preference for each trip given a situation.

IV. CONTEXT-DEPENDENT WEIGHTING-BASED PRUNING TECHNIQUE

1. Entropy-Based Context-Dependent Weighting

Once the set or a subset of feasible alternatives is identified, we can measure the context-dependent (or *posterior*) weights (i.e. W^c in Equation 8). In this paper we make use of the entropy-based weighting which is based on the traditional entropy measure proposed by Zeleny (1976). Assume that a normalized vector $\overline{Z}^i = \left(\overline{z}^i(p_1), \ldots\ldots, \overline{z}^i(p_n)\right)$ characterizes the set Z of paths in terms of the *i*th attributes, and is defined as:

$$\overline{Z}^i = \sum_{j=1}^{n} \overline{z}^i(p_j), \quad \forall i = 1, m \quad (12)$$

where m is the number of attributes and n is the number of alternatives. In Equation 12, the normalized attribute values are used for comparative purposes although the technique will work if non-normalized values are used. Note that the scaling factors are chosen so that $0 < \overline{z}^i(p_i) \leq 1$. The entropy measure of the *i*th attribute ″contrast intensity″ is:

$$e(\overline{Z}^i) = -K \sum_{j=1}^{n} \frac{\overline{z}^i(p_j)}{\overline{Z}^i} \ln \frac{\overline{z}^i(p_j)}{\overline{Z}^i}, \quad \forall i = 1, m \quad (13)$$

In Equation 13, $e(\overline{Z}^i) > 0$. If all $\bar{z}^i(p_j)$ are identical for a given attribute i, then $\bar{z}^i(p_j)/\overline{Z}^i = 1/n$, and $e(\overline{Z}^i)$ assumes its maximum value of ln*n* (e_{max}=ln*n*). Therefore, we set K to 1/e_{max} so the $e(\overline{Z}^i) \leq 1$ for all i. Accordingly, we define Equation 13 as:

$$e(\overline{Z}^i) = -\frac{1}{\ln n}\sum_{j=1}^{n}\frac{\bar{z}^i(p_j)}{\overline{Z}^i}\ln\frac{\bar{z}^i(p_j)}{\overline{Z}^i}, \quad \forall i = 1, m \tag{14}$$

We then define the total entropy of Z as Equation 15.

$$E = \sum_{i=1}^{m} e(\overline{Z}^i) \tag{15}$$

It can be seen that the context-dependent (or *posterior*) weights w_i^c, are inversely related to $e(\overline{Z}^i)$ and the summation of the new weights, w_i^c is equal to 1. We then use Equation 16 to obtain the context-dependent weights.

$$w_i^c = \frac{1-e(\overline{Z}^i)}{[1-e(\overline{Z}^1)]+[1-e(\overline{Z}^2)]+\ldots+[1-e(\overline{Z}^m)]} = \frac{1-e(\overline{Z}^i)}{m-E}, \quad \forall i=1, m \tag{16}$$

In summary, as the normalized attribute values for attribute i become more distinct and differentiated with respect to other attributes, the corresponding contrast intensity for the attribute increases. Consequently, the amount of "decision information" contained in and transmitted by attribute i also increases and this decision information is represented by the context-dependent weight of attribute i. While the entropy model explicitly considers the relative dispersion of an attribute across all alternatives, the measure of central tendency of the attributes are also factored into the weight calculation (see Park (1998) for details).

2. Context-Dependent Weighting-Based Pruning

The proposed approach first identifies all single attribute optimal paths (SAOP) by solving a series of shortest path problems denoted as P4 below. We define a SAOP as a path which minimizes the cost function with respect to each attribute.

$$P4: \quad \min \sum_{(ij)\in A} c_{ij}^i x_{ijp} \qquad \forall i \tag{17}$$

s.t.

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & for\ all\ i\in N-\{O,D\} \\ -1, & i=D \end{cases}$$
$$x_{ij} = 0\ or\ 1 \quad for\ all\ (i,j)\in A$$

where i = attribute

Subsequently, we identify the first temporary optimal path (TOP1) based on the primary weight vector obtained from the driver by solving P5.

$$P5: \quad \min \sum_{(i,j)\in A} \left(w_1 c_{ij}^1 + w_2 c_{ij}^2 + \ldots\ldots + w_m c_{ij}^m\right) x_{ij} \tag{18}$$

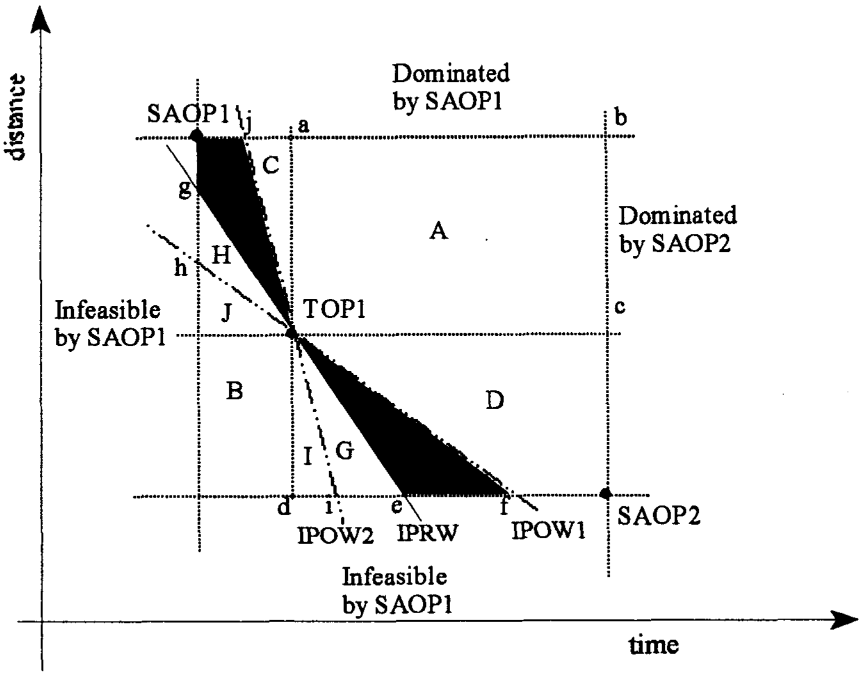
s.t.

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & for\ all\ i\in N-\{O,D\} \\ -1, & i=D \end{cases}$$
$$x_{ij} = 0\ or\ 1 \quad for\ all\ (i,j)\in A$$

where

$$w_i \in W, \forall i$$
$$\sum_{i=1}^{m} w_i = 1$$

We then apply the entropy-based technique to obtain a context-dependent (or *posterior*) weights using the SAOPs' as anchor points. <Figure 1> shows how the proposed approach prunes the search area based on the *a priori* weights and *posterior* weights. The isoquant based on the *a priori* weight (IPRW) (i.e. utility contour line or preference indifference line) reflects the *a priori* weight obtained from a driver. Observe that because we use an additive linear utility function to approximate the nonlinear utility function, the IPRW is linear. As discussed above, a primary or *a priori* weight for an attribute may increase or decrease. If the weight of the primary attribute (i.e. time) decreases in the *posterior* situation, an "isoquant", referred to here as the isoquant with the *posterior* weights (IPOW1), forms a triangular area E (TOP1, e and f) in which any nondominated path would be a proper alternative path according to the drivers' preference. Note that triangle H (TOP1, g, and h) is an infeasible region because if a path were to exist in this area it should have been identified as a TOP1 instead of the current TOP1 by P5.



<Figure 1> Context-Dependent Feasible Region

If the weight of the primary attribute increases, we may define an isoquant, referred to as IPOW2, where any nondominated path within the trapezoid F (SAOP1, j, TOP1, and g) may be considered as a viable alternative path. Triangle G (TOP1, i, and e) is an infeasible region. Conversely, we consider the triangle C (a, SAOP1 and TOP1) and Trapezoid D (TOP1, c, SAOP2, and f) to be context-dependent dominated regions because any nondominated path within these regions are dominated not only by the IPRW but also by the IPOW1(2). Of course, a path within rectangle A is dominated by TOP1. To summarize, by introducing the context-dependent domination concept, we can explicitly reflect the context-dependent preference and basic preference and reduce the search area. Based on this approach, we introduce a new type of dominance concept.

Definition 5. Denote the utility value of path p with respect to weight W as $U_w(z(p))$. A path p is a "context-dependent dominated path (CDDP)" if and only if it is dominated by TOP1 with respect to not only the *a priori* weighting (W), i.e. $U_w(z(p))>U_w(z(TOP1))$, but also the *posterior* weighting (W^c), i.e. $U_{W^c}(z(p)) > U_{W^c}(z(TOP1))$. Conversely, a path p is a "context-dependent nondominated path (CDNP)" if it is not dominated by TOP1 with respect to the *a priori* and the *posterior* weights simultaneously.

Denote P_{CN} as the set of all CDNP. If p_1 is context-dependently dominated by p_2, denote that $p_2 \succ_c p_1$. Then we can say that the set of CDNP, P_{CN} , is a subset of the set of the nondominated paths, P_N. In <Figure 1>, the feasible regions in which a nondominated path with respect to the TOP1 with *a priori* weight exists includes regions of C, D, E, and F. These regions clearly include the feasible regions, E and F, in which CDNP path may exist. In extreme cases when the *posterior* weights provide emphasis on only the first or second attributes, a CDNP can exist within D and E, and C and F, which is still a subset of the feasible regions of the nondominated paths. Therefore, the following relationships are obtained: $P_{CN} \subseteq P_N \subseteq P$ and $Z_{CN} \subseteq Z_N \subseteq Z$. Note that now we can can identify non-extreme nondominated paths as alternative paths (i.e. CDNP) by incorporating context-dependent dominance concept, which is impossible if only a linear utility function is used.

Based on Definition 5, we can formulate the following constraint which prunes the context-dependent infeasible region and results in a reduced search area.

$$\sum_{(i,j)\in A} \left(w_1^c c_{ij}^1 + w_2^c c_{ij}^2 + \ldots\ldots + w_m^c c_{ij}^m\right) x_{ij} \le U_{W^c}\left(z(TOP1)\right) \qquad (19)$$

where

$$w_i^c \in W^c, \ \forall i$$
$$\sum_{i=1}^{m} w_i^c = 1$$

The next step, therefore, is to identify other CDNP by solving problem P6 until there are no more feasible paths in the context-dependent feasible region.

$$P6: \quad \min \sum_{(i,j)\in A} \left(w_1 c_{ij}^1 + w_2 c_{ij}^2 + \ldots + w_m c_{ij}^m\right) x_{ij} \qquad (20)$$

s.t.

$$\sum_{(i,j)\in A} \left(w_1^c c_{ij}^1 + w_2^c c_{ij}^2 + \ldots\ldots + w_m^c c_{ij}^m\right) x_{ij} \le U_{W^c}\left(z(TOP1)\right)$$

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & \text{for all } i\in N-\{O,D\} \\ -1, & i=D \end{cases}$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for all } (i,j) \in A$$

where

$$w_i \in W, \quad \forall i$$
$$w_i^c \in W^c, \quad \forall i$$
$$\sum_{i=1}^{m} w_i = 1$$
$$\sum_{i=1}^{m} w_i^c = 1$$

3. Unique and Reasonable Alternative Paths

We introduce an important concept of "route uniqueness and reasonableness" in the context of real-time in-vehicle routing. The point is that the drivers may not consider a route as an viable or reasonable alternative route over the previous one if an alternative path is too "similar" to a previously identified path in terms of the links used, or if an alternative path has an unreasonable route attribute. In this sense we propose Definitions 6 and 7.

Definition 6. A path is a *unique alternative path* if it is dissimilar in terms of links used with respect to other such paths.

Definition 7. A path is a *reasonable alternative path* if it not only has "acceptable attribute value(s)" but also is "unique" in terms of links used with respect to other such paths.

Identifying reasonable alternative path is a *NP-Complete* problem. Intuitively identifying a path which has acceptable attributes value(s) and is unique in terms of links used from others depends on the number of paths required, the characteristics of the network (i.e. level of congestion), and particular O-D pair, etc. Therefore, we may not know *a priori* how much an alternative path should be different from paths previously found. Therefore the most reliable approach would be to i) identify all feasible paths, ii) examine their associated attribute value(s) and uniqueness, and iii) obtain a compromise solution which satisfies the two requirements simultaneously. Therefore, the computation time therefore grows exponentially with the number of nodes, which indicates that the problem is NP-complete (Warburton, 1983).

In this study we assume that CDNP have acceptable attribute values. We thus propose following definition.

Definition 8. A path is a *reasonable context-dependent nondominated path (RCDNP)* if and only if a path is a CDNP and unique with respect to those of other such paths.

We then treat RCDNP as the "reasonable alternative path" in this study.

To identify unique path, we apply the concept of "δ-similar path" originally proposed by Scott et al. (1997) and formulate the following constraint. Instead of a number of shared links, we use weighted average of shared links by shared link lengths. That is, the following constraint limits the maximum ratio of the lengths of the shared links to the total route length between the k-th and l-th paths. This notion would be more acceptable that Scott et al.&s in that links may have different lengths in transportation networks.

$$\frac{[X^k]^T X^l L}{L^k} = \frac{\sum_{(i,j)\in A} x_{ij}^k x_{ij}^l l_{ij}}{L^k} \leq \delta_{k,l} \tag{21}$$

where X^l = set of links consisting of *l-th* path
x_{ij}^l = 1 if link (i, j) is on the *l-th* path, otherwise 0.
L^k = route length of *k-th* path
l_{ij} = length of link (i,j)

Consequently, we now reformulate the proposed problem for identifying RCDNP as Equation 22. Note that we solve P7 to identify the k-th RCDNP and repeat this procedure to identify new such paths.

$$P7: \quad \min \sum_{(i,j)\in A} \left(w_1 c_{ij}^1 + w_2 c_{ij}^2 + \ldots + w_m c_{ij}^m\right) x_{ij} \tag{22}$$

s.t.

$$\sum_{(i,j)\in A} \left(w_1^c c_{ij}^1 + w_2^c c_{ij}^2 + \ldots + w_m^c c_{ij}^m\right) x_{ij} \leq U_{W^c}\ (z(TOP1))$$

$$\frac{\sum_{(i,j)\in A} x_{ij}^k x_{ij}^l l_{ij}}{L^k} \leq \delta, \qquad \forall l,\ l \neq k$$

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & \text{for all } i\in N-\{O,D\} \\ -1, & i=D \end{cases}$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for all } (i,j) \in A$$

where

$$w_i \in W, \quad \forall i$$
$$w_i^c \in W^c, \quad \forall i$$
$$\sum_{i=1}^{m} w_i = 1$$
$$\sum_{i=1}^{m} w_i^c = 1$$

V. SOLUTION ALGORITHMS

1. Algorithm 1: Labeling-Based Branch-and-Bound Approach

To solve P7, we need to make use of a CSPP solution algorithm. The existing CSPP algorithms can be broadly classified into two groups: 1) a Lagrangian relaxation approach (Handler and Zang, 1980; Scott et al., 1997) and a branch-and-bound approach (Morin and Marsten, 1976; Aneja et al., 1983). The Lagrangian relaxation approach brings the constraints into the objective function with scalar multipliers, and identifies lower and upper bounds of the Lagrangian multipliers by solving a series of SPP. The best multipliers are identified by using a k-shortest path algorithm which satisfies the constraints (i.e. closing the duality gap). Assume that we have found out the "k-1"-th RCDNP and we are interested in the k-th such a path. If we employ the Lagrangian relaxation approach for the algorithm, P7 would be reformulated into P8 by incorporating Lagrangian multipliers.

$$P8{:}\min \sum_{(i,j)\in A}\left(w_1 c_{ij}^1 + w_2 c_{ij}^2 + \dots + w_m c_{ij}^m\right)x_{ij} + \lambda_1\left(\sum_{(i,j)\in A}\left(w_1^c c_{ij}^1 + w_2^c c_{ij}^2 + \dots + w_m^c c_{ij}^m\right)x_{ij} - U_{W^c}(z(TOP1))\right)$$
$$+\lambda_2\left(\frac{\sum_{(i,j)\in A} x_{ij}^1 x_{ij}^k l_{ij}}{L^1} - \delta\right) + \lambda_3\left(\frac{\sum_{(i,j)\in A} x_{ij}^2 x_{ij}^k l_{ij}}{L^2} - \delta\right) + \dots\dots + \lambda_k\left(\frac{\sum_{(i,j)\in A} x_{ij}^{k-1} x_{ij}^k l_{ij}}{L^{k-1}} - \delta\right) \tag{23}$$

s.t.

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i=O \\ 0, & \text{for all } i\in N-\{O,D\} \\ -1, & i=D \end{cases}$$
$$x_{ij} = 0 \text{ or } 1 \quad \text{for all } (i,j)\in A$$

where

$$w_i \in W, \quad \forall i$$
$$w_i^c \in W^c, \quad \forall i$$
$$\sum_{i=1}^{m} w_i = 1$$
$$\sum_{i=1}^{m} w_i^c = 1$$

The additional terms in the objective function serve to penalize violations of the constraints and for any particular λ's, the solution to P8 serves as a lower bound to P7. Observe that when we identify the first RCDNP (i.e. except TOP1), we are confronted with two constraints: one for context-dependent nondominance (i.e. Inequality 19) and the other for route uniqueness (i.e. Inequality 21). However, as we identify more alternative paths, the number of constraints also increases due to the requirement that all of RCDNP be dissimilar with respect to each other. As shown in Equation 23, when we are interested in identifying the k-th path we need to find out "k" Lagrangian multipliers. In short to identify "k" RCDNP, we need "$(k^2+k-2)/2$" Lagrangian multipliers (i.e. 2+3+....+k). In addition, due to the difficulty of finding the δ value, we may have to repeat the same procedures with different δ values.

In the first step, rather than solving P8 directly, Algorithm 1 identifies all CDNP. Assume that we need to identify as many as "Γ" RCDNP and we have identified "k" such paths by solving P8. If k is greater than Γ, we choose k number of RCDNP from the set of CDNP by solving subproblem P9. If k is less than "Γ", we don't solve P9. That is, we relax the definition of reasonable alternative path so that we do not concern about the uniqueness requirement in case if the number of alternative paths which satisfy the reasonable attribute value requirement is less than or equal to the number of required reasonable alternative paths. Note that Algorithm 1 does not discard the TOP1 by keeping it as an anchor path when solving P9.

$$P9: \quad \mathit{argmax}_{\delta}\ \Delta = [\delta_{1,2}, \delta_{1,3}, \ldots, \delta_{1,k}, \ldots\ldots, \delta_{k-1,k}] \qquad (24)$$
$$s.t.\ \Pi = \Gamma$$

where

$$\Pi = \pi_{1,2} + \pi_{1,3} + \ ,\ldots, + \pi_{k-1,k}$$
$$\pi_{k,l} = \begin{Bmatrix} 1 & \text{if } \delta_{k,l} \geq \Delta \\ 0 & \text{otherwise} \end{Bmatrix} \quad \forall\ k,l$$
$$\delta_{k,l} = \frac{\sum_{(i,j)\in A} x_{ij}^{k} x_{ij}^{l} l_{ij}}{L^{k}}$$

Instead of solving a series of the CSPP's, Algorithm 1 identifies nodes which do not satisfy constraint (19), and then ignores them in the labeling procedures rather than identifying all nondominated paths. As will be shown later, branch-and-bound techniques fathom a significant portion of the nodes or links searched. Assume that there are "n" number of paths from the origin to node j and from node j to destination. Denote $\Omega(j)$ and $\Phi(j)$ be the set of all paths leading from the origin to node j and those leading from node j to the destination, respectively. Let $\omega_k(j)\in\Omega(j)$ be the k-th path from the origin to node j and $\phi_k(j)\in\Phi(j)$ be a path from node j to the destination, and denote $z(\omega_k(j))$ be vector attributes of $\omega_k(j)$and similarly $z(\phi_k(j))$ the vector attributes of $\phi(j)$. For each node j, denote that $\omega_{W^c}(j)$ and $\phi_{W^c}(j)$ be the path which has the maximum utility value with respect to *posterior* weight vector, W^c, from the origin to node j and from node j to the destination, respectively, as shown in Equations 25 and 26.

$$\omega_{W^c}(j) = \mathit{argmax}\ [U_{W^c}(z(\omega_1(j))),\ U_{W^c}(z(\omega_2(j))),\ \ldots\ ,U_{W^c}(z(\omega_n(j)))],\quad \omega_i(j)\in\Omega(j) \qquad (25)$$

$$\phi_{W^c}(j) = \mathit{argmax}\ [U_{W^c}(z(\phi_1(j))),\ U_{W^c}(z(\phi_2(j))),\ \ldots\ ,U_{W^c}(z(\phi_n(j)))]\quad \phi_i(j)\in\Phi(j) \qquad (26)$$

Note that Equations 25 and 26 can be solved easily using conventional shortest path algorithm with *posterior* weights, similar to P5. We then obtain Inequality 27 to check the context-dependent dominance of the best path using node j with respect to TOP1; if Inequality 27 holds, any path from the origin to node j should be deleted from the vector labeling because any entire path including the k-th path from the origin to the destination will be context-dependent dominated. Thus only "context-dependent nondominated" solutions arriving at a node would be kept until the final stage of calculations and all subpaths that satisfy Inequality 27 are discarded.

$$U_{W^c}\,[z(TOP1)] < U_{W^c}[z(\omega_{W^c}(j))+z(\phi_{W^c}(j))], \quad \omega_{W^c}(j)\in\Omega(j), \ \forall j \qquad (27)$$

Denote P_{RCN} to be a set of RCDNP, and k is number of RCDNP. The proposed algorithm 1 is outlined as follows. Hansen (1980)'s algorithm is utilized for solving MCSPP in Step 5.

STEP 1. Initialization
- $P_{RCN} = \emptyset$, and k=0.
- Obtain m attributes to be considered and *a priori* weight, $W = (w_1, w_2, \ldots, w_m)$, from a driver.

STEP 2. Identifying SAOP
- Identify single attribute optimal paths, SAOPs, by solving P4 for all attributes.

STEP 3. Identifying the First RCDNP (TOP1) by *a Priori* Weight
- 3.1 k=k+1. Identify the first TOP1 by solving P5.
- 3.2 Insert TOP1 into P_{RCN}.

STEP 4. Identifying Context-Dependent Feasible Region and Pruning
- 4.1 Obtain *posterior* weight using entropy model.
- 4.2 Identify context-dependent feasible region by applying SPP with *Posterior* weight.
- 4.3 Prune nodes that are not within the feasible region

STEP 5. Identifying CDNP using Conventional Labeling-based MCSPP Algorithm
- 5.1 Extend labeling within feasible region and obtain nondominated subpaths for each node using Hansen (1980)'s MCSPP algorithm.
- 5.2 If current subpath is context-dependent dominated by TOP1, delete it. If not, keep it.
- 5.3 If destination is reached, GOTO Step 6. Otherwise, GOTO Step 5.1

STEP 6. Identifying RCDNP
- 6.1 If the number of CDNP is less than or equal to Γ, then k=k+1 and insert all CDNP into P_{RCN} and STOP.
- 6.2 If not, identify the number of required paths by solving P9, and insert all RCDNP into P_{RCN} and STOP.

2. Algorithm 2: Algorithm 1 with Hierarchical Search

Algorithm 2 uses a hierarchical search method combined with Algorithm 1. The basic idea of the hierarchical search is to concentrate the search on those links that have a higher functional classification, before examining links with a lower classification (see Park and Rilett, 1997 for details).

VI. COMPUTATIONAL EXPERIENCE: CASE STUDIES

1. Experimental Design

We tested the proposed two MCSPP algorithms using the Austin, Texas network. The Austin network consists of 4,463 nodes and 7760 links. One thousand O-D pairs were randomly chosen for this analysis. Travel time, distance, hierarchy, congestion, and travel time reliability, were used as route attributes. We coded the two algorithms in FORTRAN. Due to the difficulty in obtaining real-time travel time information, different volume to capacity ratios (V/C) were assumed for different functional classifications. We assumed three congestion levels (i.e. three scenarios),I(low), II(middle), and III(high) to capture the effect of the level of congestion.

To measure congestion, we used "relative congestion" which is estimated by the difference between acceptable speed and actual speed to the acceptable speed. The actual speed was defined as the speed in the level of service C. The travel time reliability was based on empirical results from a previous link travel time forecasting studies (Park and Rilett, 1998; Park et al., 1999). We summarized the attributes and their associated *a priori* weights in <Table 1>. Hereafter, unless mentioned otherwise, congestion level III is assumed.

<Table 2> Attributes Considered and Their Associated *A Priori* Weights

Number of Attributes	Travel Time	Distance	Hierarch	Congestion	Travel Time Reliability
Two	Yes (0.70)*	Yes (0.30)	No	No	No
Three	Yes (0.60)	Yes (0.20)	Yes (0.20)	No	No
Four	Yes (0.55)	Yes (0.15)	Yes (0.15)	Yes (0.15)	No
Five	Yes (0.45)	Yes (0.15)	Yes (0.15)	Yes (0.15)	Yes (0.15)

* *A priori* weight value for the corresponding attribute

2. Analysis of Results

1) Nondominated Paths versus Context-Dependent Nondominated Paths

As expected, the number of nondominated paths was found to increase rapidly as the number of attributes increases. The average number of nondominated paths with two, three, four, and five attributes were 11.9, 43.6, 129.8, and 231.5, respectively. It may be also seen that the number of nondominated paths increases as the O-D distance increases.

The average number of CDNP of Algorithm 1 is significantly smaller than the number of nondominated paths from the conventional exact algorithm. The average number of CDNP with the attributes of two, three, four, and five were 2.0, 9.5, 24.6, and 46.2, respectively. These results correspond to a relative 83.2, 78.2, 81.0, and 80.0 percent reduction in alternative paths when compared with the number of nondominated paths from the conventional labeling-based MCSPP algorithm with two, three, four, and five attributes, respectively. It can be seen that as the number of attributes increases, the relative reduction in the number of paths also increases.

The average number of CDNP from Algorithm 2 were smaller than those from the conventional MCSPP algorithm and Algorithm 1. The average number of CDNP from Algorithm 2 for situations including two, three, four, and five attributes were 1.6, 3.9, 8.2, and 12.1, respectively. These results correspond to a relative 20.0, 58.9, 66.7, and 73.9 percent reduction compared with Algorithm 1 with two, three, four, and five attributes, respectively.

It is also important to examine the computational complexity of the proposed algorithms compared to the conventional exact MCSPP algorithm. The average number of nodes label-set by the conventional exact algorithm with two, three, four, and five attributes are 1312.2, 1568.2, 1848.5, and 1941.7, respectively. However, Algorithm 1 labeled only 147.3, 185.4, 314.9, and 452.7 nodes when using two, three, four, and five attributes, respectively. That is, Algorithm 1 only label-set 11.2 % (with two attributes) and 23.3 % (with five attributes) as many nodes as the conventional algorithm. As was expected, it was found that Algorithm 2 searched approximately 20~30 % of the nodes searched by Algorithm 1.

2) Effect of Congestion Level

To examine the effect of congestion level on the number of CDNP, we implemented Algorithm 2 under three different levels of congestion, I, II, and III. It was found that the average number of CDNP increases significantly as the level of congestion changes from II to III, while the changes are very marginal between I and II. The average number of CDNP under congestions I, II, and III are 2.9, 4.5, and 12.1, respectively. These results may be attributed to the fact that when the congestion is light, the likelihood of a particular path's domination over other paths in terms of all attributes increases.

3) Reasonable Context-Dependent Nondominated Paths from Algorithm 2

It is also useful to examine the route and attributes characteristics from the set of RCDNP. To solve P9, we set "Γ" to five. The effect of "uniqueness" requirement can be observed from the O-D pairs which have more than five CDNP. The average route similarity from those O-D pairs is 0.58, which is significantly less than those of i) the CDNP (i.e. 0.72), ii) the RCDNP (i.e. 0.68), and iii) the CDNP from the O-D pairs which have less than or equal to five CDNP (i.e. 0.79). The COV values of the travel time, distance, link type, congestion, and travel time reliability of the RCDNP are 0.08, 0.07, 0.12, 0.25, and 0.15, respectively. The tradeoff between route uniqueness and COV values of the attributes indicates that as a cost of having more unique routes, multiple alternative routes have wider variations in attribute values.

The reasonableness of Algorithm 2 can be recognized by comparing the route similarity of the RCDNP and that of the paths from the traditional k-shortest path algorithm. That is, the route similarity of the RCDNP is 0.68 while that of the traditional k-shortest path algorithm is 0.77 (Park and Rilett, 1997). Note that the traditional k-shortest path algorithm can consider only one attribute, and thereby significant portion of the paths from it may be dominated paths that cannot be considered as reasonable alternative paths. That is, RCDNP are more unique in terms of route similarity and better in terms of route attributes than multiple paths from the conventional k-shortest path algorithm. We also found that the fuzzy logic-based decision support system which was applied to the RCDNP from Algorithm 2 identified other RCDNP rather than TOP1 as the best path 43.1 percent of time. In that case the travel times of the best paths (i.e. non-TOP1) were very similar to those of the TOP1's (Park, 1998; Rilett & Park, 1999). To summarize, the results from proposed algorithm appear to be promising in that i) they can reduce significantly the search area and significantly reduce the computational complexity while taking into account the driver's generic (or basic) and context-dependency of the preferences simultaneously, and ii) they can identify a manageable number of reasonable alternative paths.

VII. CONCLUDING REMARKS

When the utility function of a driver is quasiconcave or when the link costs are nonadditive, complete enumeration of nondominated paths is the only procedure that can guarantee a global optimum in the vector weights traffic networks. In addition, to explicit driver's utility function in the context of real-time in-vehicle routing is impractical and difficult from both a theoretical and practical perspective. As a practical alternative, we developed two heuristic algorithms which identify manageable number of reasonable alternative paths. We designed the algorithms to reflect not only drivers generic preferences but also the context-dependency of the preference which is measured by entropy model. The key is in the use of additive utility function and relaxation based pruning techniques. This greatly reduces the size of search area of the network while measurably increasing computational efficiency. To make sure that paths are dissimilar in terms of links used, we limited the route similarity.

From the large real world test problems, we found that the proposed algorithms were acceptable in terms of the computational complexity and they showed a significant reduction in computational complexity compared with the conventional labeling-based multi-criteria shortest path algorithm. The proposed algorithms would also be advantageous in that i) they can identify non-extreme nondominated paths as alternative routes for selection which is impossible with an weighted linear utility function or additive weighting method, ii) we don't need to obtain the exact nonlinear utility function, and iii) standard labeling based exact MCSPP algorithm can be used without solving a series of multiple constrained shortest path problems.

While the results of the proposed approaches are promising, a number of issues still need to be resolved before the algorithms can be implemented. This study did not take into account turn penalty as a link/route cost. Obviously, it is required that the algorithms be enhanced such that they can include it in identifying reasonable alternative paths. It would be also important to update the proposed approaches such that they can apply the first and second stochastic dominance concepts in order to combine travel time and travel time reliability rather than treating them as independent attributes.

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