CONFIDENCE LIMITS FOR STEADY STATE AVAILABILITY OF A REDUNDANT SYSTEM

Sang-Wook Shin[1], Jae-Hak Lim[2], Dong Ho Park[1]

Abstract

In this paper, we consider confidence limits for steady state availability of a redundant structure with the function of switchover processing. The system considered in this paper consists of three units which are an active unit, a standby unit and a switchover device. A control module does not affect the performance of the system while the active unit is operating but causes the system failure if the active unit fails at the failure of the control module. The effect of failure of control module is included in our reliability model of the simple redundant structure. The availability of the system is obtained by using the state space method. An example is given to illustrate our results.

1. INTRODUCTION

Redundancy is defined as the use of additional components or units for satisfactory operation of a system. The standby redundant structures such as electric power generator, UPS and airplane jet engines are widely used to improve the system reliability. In a two-unit repairable standby redundant system, the standby unit starts operating immediately once the active unit is detected to fail and when the failed active unit is repaired, it assumes the position of the standby unit. Then, these two units alternate their positions either active or standby whenever the failure or repair occurs. There are three types of standby units which are cold standby unit, hot standby unit and warm standby unit. The cold standby unit does not fail when it is in stand by. Hence the failure rate of the cold standby unit becomes zero. The failure rate of the hot standby unit has the same failure rate of the active unit. And the warm standby unit has a smaller failure rate than the active unit but is greater than zero. More details are in Elsayed [6]. Kumar and Agarwal [9] give excellent summaries for the cold redundant structure. Various techniques for reliability modeling of a system are discussed in Endrenyi [7].

It is common that the performance of the system is analyzed with respect to its reliability characteristics such as reliability function, availability, MTBF, mean residual life function and so on. One of the most important performance criterion of repairable systems is the measure of availability which is defined as the probability that a system is operating satisfactorily when it is required to perform the given mission. Because of the importance of the availability to evaluate the performance of the system, many researchers have worked on these subjects quite extensively. A great deal of works are focused on constructing the confidence interval for steady state availability of the system. Thompson [17] develop some techniques for constructing $100(1-\gamma)\%$ confidence limits on system availability ratio and for testing hypotheses for system availability ratio when both the time to failure and the time to repair are independent and exponentially distributed variables. Gray and Lewis [8] have derived a method for establishing an exact

1 Dept. of Statistics, Hallym University, Chunchon, Korea

2 Division of Business and Economics, Taejon Nat'l University of Technology, Taejon, Korea

confidence interval for system availability ratio under the assumption that the times between failures of the system and the repair times are independent and are exponential and lognormal random variables, respectively.And the variance of lognormal distribution is assumed to be known. Masters and Lewis [12] consider the case of gamma operation time and lognormal repair time and Masters, Lewis and Kolarik [13] assume that the operation time has the Weibull distribution and the repair time has the lognormal distribution when the variance of lognormal distribution and all the parameters of Weibull distribution are known. Chandrasekhar, Natarajan and Sujatha[4] investigate the case of lognormal operation time and lognormal repair time. They also consider the inverse Gaussian operation time and lognormal repair time with assumption that the variance of lognormal distribution and all the parameters for inverse Gaussian distribution are known. Butterworth and Nikolaisen [2] give bounds on the availability function for the case of exponential failure time and general repair distributions. Mohammad, Nuhan and Mohammed [14] obtain 100(1-γ)% confidence limits for the steady state availability of a two unit parallel system, under the assumption that a unit waiting in line for the repair must go to a second stage repair and an operable unit will not fail while the other unit is in the second stage of repair. Chandrasekhar and Natarajan [3] find 100(1-γ)% confidence limits for the steady state availability of two units cold standby and n unit parallel system with an additional assumption that an operable unit can also fail while the other unit is in the second stage of repair.

Lim [10] and Lim and Koh [11] consider a redundant systemwith a function of switchover processing which consists of three units ; an active unit, a standby unit, a switchover device. Figure 1 shows a reference model of a redundant system with a function of switchover processing. In Lim [10] and Lim and Koh [11], it is assumed that the switchover processing causes the increase of failure rate of the system since the failure of the control module can yield the failure of the system. In order to develop a reliability model, they assume that this increment of the failure rate is distributed to each unit of the system in such a way that the failure rate of each unit increases by $\lambda_\alpha = \alpha\lambda$, where $0 \leq \alpha \leq 1$. Note that $\alpha=0$ implies no failure of the control module.

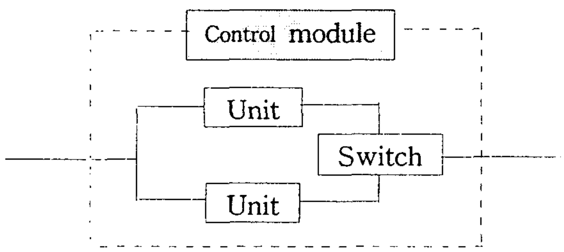


Figure 1. A reference Model of Redundant Structure with a Function of Switchover Processing

In this paper, we consider the redundant system with switchover processing and adopt the reliability modeling technique in Lim [10] and Lim and Koh [11] to develop 100(1-γ)% confidence limit for steady state availability of the system. In Section 2, we derive availability of the redundant system by using state space method and develop the confidence limits for availability. In Section 3, we consider a set of failure data in order to illustrate our results.

Throughout this paper, we assume the followings:

(i) All units are independent and have exponential life distributions, each unit having a mean of $1/\lambda$.

(ii) Repairs occur one at a time (sequential repair) and the repair times of unit and switch are exponentially distributed with a mean of $1/\mu$.

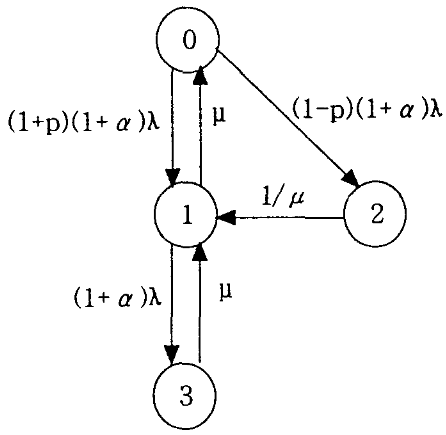
(iii) The probability of successful switchover operation is assumed to be equal to p.

(iv) The type of standby unit in the redundant system is a hot standby unit.

2. CONFIDENCE LIMITS FOR AVAILABILITY OF HSRS

In this section, we construct a $100(1-\gamma)\%$ confidence limit on steady state availability for the hot standby redundant system (hereafter, HSRS). To this end, we define four states of the system and draw the state transition diagram(STD) as shown in Figure 2. The states 2 and 3 represent the failure of the system. The state 2, which represents uncoverage outage, is caused by the failure of the active unit while control module is in the failure state and the state 3 is due to the failure of both units.

We calculate the availability of the HSRS by using the state space method. Define P_i as the probability of the system being in state i, $i=0, 1, 2, 3$. Then P_i must satisfy $P_i \sum_{j\neq i} \lambda_{ij} = \sum_{j\neq i} \lambda_{ji} P_j$ subject to the restraint $\sum_{i=0}^{3} P_i = 1$, where λ_{ij} is the transition rate from state i to state j. It implies that the flow rate out of the state equals the flow rate into the state for any state.



State	Description
0	Duplex
1	Simplex
2	Uncoverage Outage
3	Failure

Figure 2. State Transition Diagram of HSRS

On the basis of the state transition diagram in Figure 2, the flow rate equation can be established as follows:

$$
\begin{aligned}
&2(1+\alpha)\lambda P_0 = \mu P_1 \\
&[\mu + (1+\alpha)\lambda]P_1 = (1+p)(1+\alpha)\lambda P_0 + \mu P_2 + \mu P_3 \\
&\mu P_2 = (1-p)(1+\alpha)\lambda P_0 \\
&\mu P_3 = (1+\alpha)\lambda P_1 \\
&P_0 + P_1 + P_2 + P_3 = 1
\end{aligned}
\tag{2.1}
$$

Solving the equation of (2.1) for P_0, P_1 and P_2 and $\theta=\frac{\lambda}{\mu}$, we obtain the following steady state probability.

$$\begin{aligned} P_0 &= \frac{1}{1+(3-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2} \\ P_1 &= \frac{2(1+\alpha)\theta}{1+(3-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2} \\ P_2 &= \frac{(1-p)(1+\alpha)\theta}{1+(3-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2} \\ P_3 &= \frac{2(1+\alpha)^2\theta^2}{1+(3-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2} \end{aligned} \tag{2.2}$$

Since the state 2 and 3 correspond to the failure of the system, the steady state availability of the system is given by

$$\begin{aligned} A_H &= 1-P_2-P_3 \\ &= 1-\frac{(1-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2}{1+(3-p)(1+\alpha)\theta+2(1+\alpha)^2\theta^2} \end{aligned} \tag{2.3}$$

Let $X_1, X_2, \cdots, X_n$ be a random sample of times to failure with the p.d.f. given by

$$f(x)=\lambda e^{-\lambda x}, \quad x\geq 0, \quad \lambda>0 \tag{2.4}$$

and let $Y_1, Y_2, \cdots, Y_m$ be a random sample of times to repair with the p.d.f.

$$f(y)=\mu e^{-\mu y}, \quad y\geq 0, \quad \mu>0. \tag{2.5}$$

Let $\overline{X}$ and $\overline{Y}$ denote the sample means of times to failure and times to repair, respectively, where $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $\overline{Y}=\frac{1}{m}\sum_{i=1}^{m}Y_i$. It is well known that $E(\overline{X})=\frac{1}{\lambda}$ and $E(\overline{Y})=\frac{1}{\mu}$. It is also well known that $\overline{X}$ and $\overline{Y}$ are the maximum likelihood estimators(MLEs) of $\frac{1}{\lambda}$ and $\frac{1}{\mu}$, respectively and thus, the MLE of $\theta=\frac{\lambda}{\mu}$ is obtained as

$$\hat{\theta}=\frac{\overline{Y}}{\overline{X}}. \tag{2.6}$$

The MLE of the steady state availability A_H is obtained by replacing θ by its MLE, $\hat{\theta}$ of (2.6) in the expression for A_H given in (2.3). Thus, we have

$$\widehat{A_H}=1-\frac{(1-p)(1+\alpha)\hat{\theta}+2(1+\alpha)^2\hat{\theta}^2}{1+(3-p)(1+\alpha)\hat{\theta}+2(1+\alpha)^2\hat{\theta}^2} \tag{2.7}$$

In order to derive the sampling distribution of $\hat{\theta}$, we use the well known fact that $2n\lambda\overline{X}$ and

$2m\mu\overline{Y}$ are distributed according to the chi-square distributions with $2n$ and $2m$ degrees of freedom, respectively. Since the times to failure and the times to repair are independent, it follows that $\overline{X}$ and $\overline{Y}$ are independent. Thus, the distribution of random variable F^{*}, defined by

$$F^{*}=\frac{2n\lambda\overline{X}}{2n}\Big/\frac{2m\mu\overline{Y}}{2m}=\frac{\theta}{\hat{\theta}}, \tag{2.8}$$

has F-distribution with 2n and 2m degrees of freedom. Let $F_{\gamma}(2n,2m)$ be the γ-percentile of $F(2n,2m)$. Then, a 100(1-γ)% upper confidence limit(UCL) for steady state availability of the system is constructed as follows:

$$\begin{aligned}1-\gamma/2&=P[F^{*}\geq F_{\gamma/2}(2n,2m)]\\&=P[\frac{\theta}{\hat{\theta}}\geq F_{\gamma/2}(2n,2m)]\\&=P[\theta\geq\hat{\theta}F_{\gamma/2}(2n,2m)]\\&=P[A_{H}\leq 1-\frac{(1-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2n,2m)+2(1+\alpha)\hat{\theta}^{2}F^{2}_{\gamma/2}(2n,2m)}{1+(3-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2n,2m)+2(1+\alpha)^{2}\hat{\theta}^{2}F^{2}_{\gamma/2}(2n,2m)}]\end{aligned}$$

Note that the last equality holds since A_H is a decreasing function of θ. Thus, a 100(1-γ)% upper confidence limit for A_H is given by

$$\begin{aligned}UCL&=1-\frac{(1-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2n,2m)+2(1+\alpha)\hat{\theta}^{2}}{1+(3-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2n,2m)+2(1+\alpha)^{2}\hat{\theta}^{2}F^{2}_{\gamma/2}(2n,2m)}\\&=1-\frac{(1-p)(1+\alpha)\hat{\theta}F_{1-\gamma/2}(2m,2n)+2(1+\alpha)\hat{\theta}^{2}}{F^{2}_{1-\gamma/2}(2m,2n)+(3-p)(1+\alpha)\hat{\theta}F_{1-\gamma/2}(2m,2n)+(1+\alpha)^{2}\hat{\theta}^{2}}\end{aligned} \tag{2.9}$$

Using the fact that $\gamma/2=P[F^{*}\geq F_{1-\gamma/2}(2n,2m)]$ and $F_{1-\gamma/2}(2n,2m)=1/F_{\gamma/2}(2m,2n)$, we can derive a 100(1-$\gamma$)% lower confidence limit for A_H as follows:

$$LCL=1-\frac{(1-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2m,2n)+2(1+\alpha)\hat{\theta}^{2}}{F^{2}_{\gamma/2}(2m,2n)+(3-p)(1+\alpha)\hat{\theta}F_{\gamma/2}(2m,2n)+(1+\alpha)^{2}\hat{\theta}^{2}} \tag{2.10}$$

Combining (2.9) and (2.10), we obtain a two sided 100(1-γ)% confidence limit for A_H.

3. EXAMPLE

The data for times to failure of a unit and for times to repair of a unit are collected in order to estimate the availability of the system and to construct a 100(1-γ)% confidence limit for the availability of the system.

The data for times to failure of a unit which is supposed to follow an exponential distribution from the past experience are observed (in days) as follows:

95.7, 98.5, 93.4, 97.2, 100.5

The data for times to repair of a unit which is supposed to follow an exponential distribution from the past experience are also observed (in days) as follows:

6.2, 3.8, 7.9, 7.3, 5.4

The average time to failure($\overline{X}$) and the average time to repair($\overline{Y}$) were 97.06 days and 6.12 days, respectively, and the maximum likelihood estimator of θ is given by 0.063054. To accomodate the possibility of the failure of the control module, we consider several values of α for the purpose of illustration. The values of α we consider are 0, 0.3, 0.6, 1.0.

Using the failure and repair data, we compute the maximum likelihood estimate of availability and construct 100(1- γ)% confidence limits for the availability of the system for various values of p and α and for γ = 0.05. The results are given in Tables 1 and 2.

Table 1. MLE and 95% Confidence Limits for A_H when $\alpha = 0$ and 0.3

p	α =0			α =0.3		
	MLE of A_H	LCL	UCL	MLE of A_H	LCL	UCL
0.0	0.940686	0.841911	0.979266	0.924240	0.803790	0.973212
0.1	0.945667	0.851699	0.981218	0.930295	0.814529	0.975689
0.2	0.950701	0.861718	0.983177	0.936430	0.825559	0.978179
0.3	0.955789	0.871975	0.985145	0.942647	0.836891	0.980682
0.4	0.960932	0.882480	0.987120	0.948947	0.848539	0.983198
0.5	0.966130	0.893240	0.989103	0.955331	0.860516	0.985726
0.6	0.971385	0.904266	0.991095	0.961802	0.872836	0.988268
0.7	0.976697	0.915568	0.993094	0.968361	0.885514	0.990822
0.8	0.982068	0.927156	0.995101	0.975010	0.898565	0.993390
0.9	0.987498	0.939041	0.997117	0.981751	0.912007	0.995971
1.0	0.992988	0.951234	0.999141	0.988586	0.925857	0.998566

Table 2. MLE and 95% Confidence Limits for A_H when $\alpha = 0.6$ and 1.0

p	α =0.6			α =1		
	MLE of A_H	LCL	UCL	MLE of A_H	LCL	UCL
0.0	0.908359	0.768971	0.967233	0.888015	0.726983	0.959374
0.1	0.915339	0.780231	0.970210	0.896028	0.738497	0.962981
0.2	0.922427	0.791826	0.973206	0.904187	0.750382	0.966615
0.3	0.929626	0.803770	0.976220	0.912496	0.762655	0.970276
0.4	0.936938	0.816080	0.979253	0.920959	0.775337	0.973966
0.5	0.944365	0.828773	0.982305	0.929581	0.788447	0.977683
0.6	0.951912	0.841867	0.985376	0.938365	0.802009	0.981429
0.7	0.959580	0.855382	0.988467	0.947318	0.816045	0.985204
0.8	0.967373	0.869337	0.991576	0.956442	0.830581	0.989008
0.9	0.975293	0.883756	0.994706	0.965744	0.845644	0.992841
1.0	0.983344	0.898660	0.997855	0.975229	0.861264	0.996704

4. CONCLUSION

In this paper, we consider the hot standby redundant system with the function of switchover processing. Using the same reliability modeling technique as Lim [10] and Lim and Koh [11], we also include the system failure ,caused by the failure of control module, in our availability modeling. Assuming a constant failure rate for each of the units comprising the system, we derive the steady state availability of the system considered. We also construct a $100(1-\gamma)\%$ confidence limit on steady state availability for the system. Using the failure and repair data, the maximum likelihood estimate of availability and $100(1-\gamma)\%$ confidence limits for the availability are obtained for various values of p and α and for $\gamma = 0.05$.

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