NBU- t_0 Class 에 대한 검정법 연구

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ABSTRACT

A survival variable is a nonnegative random variable X with distribution function F and a survival function $\overline{F}=1-F$. This variable is said to be New Better than Used of specified age t_0 if $\overline{F}(x+t_0)\le \overline{F}(x)\cdot \overline{F}(t_0)$ for all $x\ge 0$ and a fixed t_0. We propose the test for H_0 : $\overline{F}(x+t_0)=\overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge 0$ against H_1 : $\overline{F}(x+t_0)\le \overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge 0$ when the specified age t_0 is unknown but can be estimated from the data when $t_0=\mu$, the mean of F, and also when $t_0=\xi_p$, the pth percentile of F. This test statistic, which is based on a linear function of the order statistics from the sample, is readily applied in the case of small sample. Also, this test statistic is more simple than the test statistic of Ahmad's test statistic (1998). Finally, the performance of this test is presented.

Key Words : order statistic, new better than used at t_0, survival function, percentiles, test statistic, performance of the test.

1. Introduction

A life is represented by a nonnegative random variable X with distribution function F and a survival function $\overline{F}=1-F$. Then, F is 'new bettet than used' (NBU) if, for all x, $t\ge 0$, $\overline{F}(x+t)\le\overline{F}(x)\cdot\overline{F}(t)$. Hollander, Park & Proschan (1986) introduced a larger class than NBU class. The distribution F is said to be new better than used of a specified age t_0 (NBU- t_0) if, for all $x\ge 0$, $\overline{F}(x+t_0)\le\overline{F}(x)\cdot\overline{F}(t_0)$. While $\overline{F}(x)=\exp(-\lambda x)$ is the only distribution such that $\overline{F}(x+t)=\overline{F}(x)\cdot\overline{F}(t)$ for all $x,t\ge 0$ (Barlow & Proschan, 1981), Hollander et al. (1986) showed that the family $\mathcal{A}$ of distributions such that $\overline{F}(x+t_0)$ = $\overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge 0$ and a fixed $t_0\ge 0$ includes precisely the following members:

(i) $\overline{F}_1(x)=\exp(-\lambda x)$ for all $x\ge 0$, $\lambda>0$;

(ii) all life distributions $\overline{F}_2$ such that $\overline{F}_2(t_0)=0$;

(iii) $\overline{F}_3(x)=\overline{G}(x)$ for $0\le x<t_0$ and $\overline{F}_3(x)=\overline{G}^j(t_0)\overline{G}(x-jt_0)$ for $jt_0\le x<(j+1)t_0$ $(j=1,2,\dots)$, where G is a life distribution.

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Hollander et al. (1986) proposed a test for $H_0: F \in \mathcal{A}$ against $H_1: F$ is $NBU-t_0$, where t_0 is a known value. This test was extended into a class of tests by Ebrahimi & Habbibullah (1990). Also Ahmad (1998) proposed a test for $H_0: F \in \mathcal{A}$ against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value.

In practice, however, one might be interested in the new better than used behaviour at an unknown but estimable point t_0. Such points would be, e.g. the pth percentile of F, denoted by ξ_p, or the mean life μ of F. The choice $t_0 = \xi_p$ may appear naturally in manufacturing where there is the concept of 'infant mortality', where items may improve over $(0, \xi_p)$ and then begin to decay, where p, the percentage of the item's life, may be known but ξ_p often not. Therefore we investigate the testing of $NBU-t_0$ alternatives when t_0 is not known but is estimable from the data.

When testing for $H_0: F \in \mathcal{A}$ against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value, Ahmad(1998) obtained the test statistic, $\hat{T}_k$, for the $NBU-t_0$ class based on U-statistic and discussed its properties. In this thesis, we propose a test statistic for $H_0: F \in \mathcal{A}$ against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value. Our test statistic, which is based on the order statistic from the sample, is readily applied in the case of small sample. Also, our test statistic is simpler than the test statistic of Ahmad.

In, section 2, we propose the test statistic for $H_0: F \in \mathcal{A}$ against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value and this test is implemented for a sample size ranging from $n=10$ to $n=60$

Finally in section 3, Monte Carlo simulations are conducted to evaluate the performance of the test for small sample size and to compare the powers of the our test and Ahmad's test and we give some conclusions and remarks for further researches.

2. Testing New Better than Used at t_0

Ahmad proved the following simple characterization of the $NBU-t_0$ class plays a major role in our developments.

Lemma 1 We have $\overline{F}$ is $NBU-t_0$ *if and only if, for all integers* $k \geq 1$,

$$\overline{F}(x+kt_0) \leq \overline{F}(x) \cdot \overline{F}^k(t_0).$$

Let ξ_p denote the pth percentile of F, that is, $\overline{F}(\xi_p) = 1-p$ for $0 \leq p \leq 1$. Consider the measure of departure from H_0, defined by

$$T_k^1(F) = \frac{1}{\mu}\int_0^\infty \{\overline{F}(x)\ \overline{F}^k(t_0) - \overline{F}(x+kt_0)\}\,dx$$

$$= \frac{1}{\mu}\ \overline{F}^k(t_0)\int_0^\infty \overline{F}(x)\,dx - \frac{1}{\mu}\int_0^\infty \overline{F}(x+kt_0)\,dx$$

When $t_0 = \xi_p$, we obtain

$$T_k^1(F) = (1-p)^k - \overline{G}(k\xi_p),$$

where $\overline{G}(x) = \int_x^\infty \overline{F}(y)\,dy/\mu$ and $G(x)$ is called as the renewal or equilibrium distribution corresponding to $F(x)$. Under H_0, $T_k^1(F) = 0$ and under H_1, $T_k^1(F) > 0$, since $F(x)$ is continuous.

Let $X_1, X_2, \ldots, X_n$ denote random sample from F. In the usual way we estimate ξ_p by $\hat{\xi}_p = X_{([np])}$, where $X_{(r)}$ denotes the rth order statistic in the sample and $[x]$ means the largest integer less than or equal to x. The empirical distribution $F_n(X_{(i)}) = i/n$, $i = 0,1,\ldots,n$ where $X_{(0)} = 0$. The empirical survival function is $\overline{F}_n(X_{(i)}) = (n-i)/n$, $i = 0,1,\ldots,n$. Let $D_j = (n-j+1)(X_{(j)} - X_{(j-1)})$, $j = 1,2,\ldots,n$. Now, $\overline{G}_n(X_{(i)}) = \sum_{j=i+1}^n D_j / \sum_{j=1}^n D_j$, $i = 1,\ldots,n-1$ and $\overline{G}_n(X_{(i)}) = 0$. Thus, we estimate $T_k^1(F)$ by

$$H_n^k = \widehat{T_k^1}(F_n) = (1-p)^k - \overline{G}(kX_{([np])}).$$

If $X_{(i)} \le kX_{([np])} \le X_{(i+1)}$, we can rewrite $\overline{G}(kX_{([np])})$ as followings;

$$\overline{G}(kX_{([np])}) = \left\{ \sum_{h=i+2}^n D_h + (n-i)(X_{(i+1)} - kX_{([np])}) \right\} / \sum_{j=1}^n D_j$$

Thus,

$$H_n^k = (1-p)^k - \left\{ \sum_{h=i+2}^n D_h + (n-i)(X_{(i+1)} - kX_{([np])}) \right\} / \sum_{j=1}^n D_j.$$

We use H_n^k to test H_0: $F \in \mathcal{E}$ against H_1: F is $NBU-\xi_p$ for $0 \le p \le 1$.

The critical values of the statistic H_n^k in the case of $k=1$ are given in Tables 2.1 through 2.4. These Tables contains lower and upper percentile points, based on Monte Carlo sampling with 10,000 replications.

Table 2.1 Critical values of the new better than used statistic H_n^1

in the case of p=0.05

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
20	-0.0500	-0.0499	-0.0497	-0.0436	-0.0417	-0.0368
25	-0.0500	-0.0499	-0.0498	-0.0460	-0.0447	-0.0417
30	-0.0500	-0.0499	-0.0499	-0.0472	-0.0464	-0.0442
35	-0.0500	-0.0500	-0.0499	-0.0480	-0.0474	-0.0459
40	-0.0493	-0.0480	-0.0465	0.0092	0.0264	0.0645
45	-0.0494	-0.0484	-0.0470	0.0017	0.0176	0.0538
50	-0.0500	-0.0485	-0.0474	-0.0030	0.0122	0.0452
55	-0.0496	-0.0487	-0.0477	-0.0067	0.0056	0.0348
60	-0.0473	-0.0439	-0.0407	-0.0154	0.0298	0.0621

Table 2.2 Critical values of the new better than used statistic H_n^1 in the case of p=0.10

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.0999	-0.0994	-0.0988	-0.0725	-0.0622	-0.0403
15	-0.1000	-0.0998	-0.0995	-0.0882	-0.0841	-0.0751
20	-0.0981	-0.0950	-0.0918	0.0227	0.0548	0.1291
25	-0.0985	-0.0961	-0.0936	-0.0023	0.0246	0.0889
30	-0.0942	-0.0865	-0.0803	-0.0338	0.0594	0.1139
35	-0.0952	-0.0884	-0.0832	-0.0150	0.0383	0.0919
40	-0.0877	-0.0774	-0.0699	0.0339	0.0553	0.1015
45	-0.0893	-0.0802	-0.0737	0.0196	0.0389	0.0820
50	-0.0818	-0.0697	-0.0621	0.0351	0.0539	0.0983
55	-0.0834	-0.0737	-0.0664	0.0216	0.0401	0.0748
60	-0.0766	-0.0646	-0.0567	0.0340	0.0516	0.0887

Table 2.3 Critical values of the new better than used statistic H_n^1 in the case of p=0.25

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.2440	-0.2348	-0.2266	0.0083	0.0704	0.1932
15	-0.2344	-0.2173	-0.2036	0.0205	0.0642	0.1642
20	-0.1972	-0.1678	-0.1458	0.0864	0.1260	0.1728
25	-0.1907	-0.1583	-0.1389	0.0704	0.1065	0.1755
30	-0.1803	-0.1526	-0.1344	0.0574	0.0916	0.1564
35	-0.1736	-0.1466	-0.1279	0.0473	0.0790	0.1394
40	-0.1527	-0.1210	-0.1016	0.0688	0.0956	0.1496
45	-0.1504	-0.1177	-0.1003	0.0609	0.0883	0.1349
50	-0.1438	-0.1158	-0.0993	0.0537	0.0789	0.1304
55	-0.1427	-0.1142	-0.0975	0.0461	0.0683	0.1157
60	-0.1230	-0.0976	-0.0812	0.0604	0.0830	0.1247

3. Powers of the New Test Statistic

Now, we carry out to estimate the empirical powers of the proposed test H_n^k by comparing with Ahmad's test $\widehat{T}_k$ at the significance levels $\alpha=0.05$ and $\alpha=0.10$ in the case of $k=1$ for Weibull and gamma alternatives given by

(a) Weibull distribution

$$F_1(x) = 1-\exp[-x^{(1+\theta)}], \quad x \geq 0,\ \theta \geq 0$$

(b) Gamma distribution

$$F_2(x) = \int_0^x (1/\Gamma(1+\theta)) e^{-t} t^{\theta} dt, \quad x \geq 0, \ \theta \geq 0$$

Table 2.4 Critical values of the new better than used statistic H_n^1 in the case of p=0.50

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.3682	-0.2961	-0.2524	0.1620	0.2156	0.3025
15	-0.3384	-0.2735	-0.2328	0.0986	0.1452	0.2293
20	-0.2789	-0.2083	-0.1688	0.1195	0.1608	0.2284
25	-0.2626	-0.2005	-0.1684	0.0883	0.1244	0.1871
30	-0.2277	-0.1661	-0.1354	0.1004	0.1324	0.1883
35	-0.2188	-0.1675	-0.1396	0.0776	0.1069	0.1638
40	-0.1931	-0.1440	-0.1156	0.0873	0.1145	0.1685
45	-0.1902	-0.1454	-0.1193	0.0711	0.0995	0.1502
50	-0.1745	-0.1274	-0.1020	0.0822	0.1055	0.1549
55	-0.1733	-0.1307	-0.1055	0.0679	0.0928	0.1385
60	-0.1562	-0.1164	-0.0925	0.0732	0.0971	0.1402

The random numbers for the two alternatives are generated from the IMSL subroutines. This study is done for n = *10(5)60* in the case of $k=1$ at *p=0,10* and *p=0.50.* In this simulation, 10,000 replications are performed for each value of design constants.

Tables 3.1 through 3.4 contain a comparative study of the sample powers for $\widehat{T}_k$ and H_n^k in the case of $k=1$ for $NBU-t_0$.

Table 3.1 Comparisons of small sample powers in Weibull at p=0.10

	$\theta=0.5$				$\theta=1.0$			
	$\alpha=0.10$		$\alpha=0.05$		$\alpha=0.10$		$\alpha=0.05$	
n	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$
10	.4271	.2407	.2744	.1224	.7025	.4152	.5547	.2642
15	.4822	.2486	.3379	.1566	.7813	.4556	.6603	.3381
20	.3508	.2897	.2431	.1920	.5229	.5836	.4074	.4568
25	.3813	.2939	.2680	.2072	.5587	.6021	.4463	.5066
30	.4894	.3516	.3657	.2433	.6999	.7022	.6017	.5960
35	.5107	.3703	.3839	.2561	.7324	.7285	.6314	.6252
40	.6085	.4209	.4839	.3000	.8303	.8020	.7569	.7082
45	.6255	.4270	.5064	.3057	.8468	.8209	.7786	.7240
50	.6930	.4695	.5793	.3514	.9098	.8643	.8584	.7947
55	.7127	.4709	.5939	.3377	.9163	.8763	.8643	.7916
60	.7686	.5126	.6573	.3840	.9490	.9154	.9134	.8549

Table 3.2 Comparisons of small sample powers in Weibull at p=0.50

	$\theta=0.5$				$\theta=1.0$			
	$\alpha=0.10$		$\alpha=0.05$		$\alpha=0.10$		$\alpha=0.05$	
n	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$
10	.2929	.2579	.1726	.1913	.4680	.4959	.3095	.4092
15	.4083	.3686	.2609	.2257	.6550	.6928	.4993	.5266
20	.4899	.3947	.3313	.3161	.7824	.7588	.6268	.6855
25	.5882	.4618	.4212	.3278	.8687	.8406	.7617	.7408
30	.6537	.5147	.4944	.3792	.9201	.8947	.8373	.8153
35	.7287	.5682	.5833	.4388	.9539	.9263	.9070	.8701
40	.7768	.5903	.6435	.4766	.9743	.9466	.9409	.9106
45	.8248	.6427	.6947	.5031	.9885	.9680	.9636	.9372
50	.8460	.6727	.7382	.5414	.9904	.9799	.9764	.9552
55	.8843	.7063	.7828	.5793	.9950	.9848	.9845	.9697
60	.9100	.7352	.8225	.6170	.9979	.9922	.9939	.9176

Table 3.3 Comparisons of small sample powers in gamma at p=0.10

	$\theta=0.5$				$\theta=1.0$			
	$\alpha=0.10$		$\alpha=0.05$		$\alpha=0.10$		$\alpha=0.05$	
n	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$
10	.2951	.1910	.1649	.0882	.4862	.2583	.3136	.1325
15	.3389	.1905	.2053	.1109	.5805	.2854	.4179	.1802
20	.2467	.2028	.1511	.1192	.3624	.3283	.2405	.2175
25	.2640	.2038	.1659	.1354	.3894	.3375	.2703	.2476
30	.3191	.2258	.2092	.1308	.4921	.4027	.3656	.2798
35	.3432	.2526	.2244	.1500	.5273	.4290	.3947	.3076
40	.4093	.2719	.2828	.1744	.6179	.4903	.4917	.3567
45	.4272	.2814	.3030	.1772	.6442	.5180	.5194	.3731
50	.4644	.2998	.3364	.1976	.7012	.5460	.5887	.4175
55	.4987	.3060	.3551	.1860	.7377	.5654	.6218	.4196
60	.5349	.3311	.3985	.2162	.7828	.6102	.6791	.4771

Tables 3.1 and 3.2 show that H_n^k performs better than $\widehat{T}_k$ in the cases of the most part in Weibull distributions. Also Tables 3.3 and 3.4 shows that H_n^k performs better than $\widehat{T}_k$ in the cases of the most part in gamma distributions. Therefore we recommend our proposed statistic H_n^k as test statistic for $NBU-t_0$ class in the case of $k=1$.

We will drive the limiting distribution and consistency of H_n^k which is based on the order statistics from the sample in further study.

Table 3.4 Comparisons of small sample powers in gamma at p=0.50

	$\theta = 0.5$				$\theta = 1.0$			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$	H_n^k	$\widehat{T}_k$
10	.1744	.1631	.0892	.1140	.2328	.2424	.1327	.1770
15	.2296	.2322	.1263	.1157	.3347	.3694	.2061	.2106
20	.2540	.2242	.1419	.1616	.3986	.3714	.2431	.2918
25	.2932	.2550	.1786	.1558	.4821	.4325	.3217	.3039
30	.3293	.2821	.2047	.1728	.5370	.4910	.3843	.3538
35	.3766	.3160	.2460	.2094	.6179	.5399	.4621	.4156
40	.4057	.3110	.2667	.2161	.6632	.5737	.5113	.4453
45	.4463	.3452	.2883	.2315	.7151	.6260	.5573	.4903
50	.4470	.3633	.3155	.2366	.7368	.6522	.6007	.5148
55	.4945	.3772	.3393	.2557	.7860	.6801	.6407	.5570
60	.5221	.4050	.3654	.2800	.8152	.7138	.6853	.5886

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