A STUDY FOR THE SYMMETRIC AND THE ASYMMETRIC RESONANCE PROPERTIES OF THE NUCLEAR FISSION PROBABILITY IN FISSIONABLE THORIUM (^{233}Th) , URANIUM (^{235}U), AND PLUTONIUM (^{239}Pu)

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In the present short paper, the authors would like to report a useful study for the symmetric and the asymmetric properties of the fission probability in nuclear fission reaction. Since such properties of the fission probability was not known, but it was not understood or analyzed well so far. However, in fact, the experimental result of the fission process and the resonance fission, the variation of values of the fission cross section $\sigma^{o,f}$ is quite often asymmetric and deviates from that of the symmetric fission cross section. The fission cross section $\sigma^{o,f}$ is given as

$$\sigma^{o,f} = \frac{\pi}{K^2} \frac{g\Gamma^n_N \Gamma^n_N}{(E-E_N)^2 + (\Gamma^{nf}_N/2)^2} \quad - (1)$$

where E is the incident neutron energy (E = $\hbar^2$ $K^2/(2M)$ and Plancks constant $\hbar$ (h = h/(2π)), E_N is the resonance energy, g is the statistical weight factor, Γ^n_N is the neutron resonance width, Γ^f_N is the fission resonance width, and Γ^γ_N is the radiation resonance width.

To understand the symmetric and the asymmetric properties of the fission probabilities (P^f_N) will be helpful for us to study further safety controls for both nuclear weapons and reactors as well as, in particular, to design and test the nuclear explosives. In fission experiment, the most direct obtainable information on the fission reaction or process is the fission probability (P^f_N), which is given by the ratio of the fission resonance width (Γ^f_N) to the neutron resonance width (Γ^n_N) plus the fission resonance width (Γ^f_N), that is,

$$\Gamma^f_N = \frac{\Gamma^n_N}{\Gamma^n_N + \Gamma^f_N} \quad - (2a)$$

$$= \frac{1}{1+\alpha} \quad - (2b)$$

where, in the vicinity of a resonance, the factor α is given as Γ^n_N / Γ^f_N Here, it may be noted that the fission probability is not a function of only the neutron resonance width and the fission resonance width, but also it includes all possible decay or reaction modes.

For example, considering another radiation mode in the fission process, the fission probability

Of Eq. (2a) is rewritten as

$$P_N^f = \frac{\Gamma_N^{f\gamma}}{\Gamma_N^n + \Gamma_N^f} \qquad - (3)$$

where $\Gamma_N^{f\gamma} = \Gamma_N^n + \Gamma_N^\gamma$ with $\Gamma_N^f > \Gamma_N^\gamma$ and Γ_N^γ is the radiation resonance width. Now, we consider the symmetric and the asymmetric properties of the fission probability of Eq. (2a). It comes from the symmetric and the asymmetric properties of the resonance parameters, i.e., Γ_N^f, Γ_N^n ,and Γ_N^γ. These resonance parameters break up into the folowing two segments: (i) the symmetric resonance widths and (ii) the asymmetric resonance widths. Therefore, because the neutron resonance width and the fission resonance width (including the radiation resonance width) divide by these two properties of (i) and (ii), that is, the symmetric resonance widths ($\Gamma_N^{n,s}$, $\Gamma_N^{f,s}$,and $\Gamma_N^{\gamma,s}$) and the asymmetric resonance widths ($\Gamma_N^{n,a}$, $\Gamma_N^{f,a}$(D),and $\Gamma_N^{\gamma,a}$(D)), there exist both the symmetric and the asymmetric resonance cases in the fission probabilities of Eqs. (2a) and (3). First of all, we consider the symmetric and the asymmetric neutron and fission resonance widths including the radiation resonance widths. These resonance widths are:

(a) for the symmetric resonance case

$$\frac{\Gamma_N^{n,s}}{2} = \frac{\hbar^2 K r_s}{M r_s^2} \qquad - (4a)$$

$$\frac{\Gamma_N^{f,s}}{2} = \frac{\hbar^2 q_f K r_s}{M r_s^2} \qquad - (4b)$$

$$\frac{\Gamma_N^{s}}{2} = \frac{\hbar^2 K r_s}{M r_s^2} \qquad - (4c)$$

where, again, h is Plancks costant (h=h/(2π), k is the incident neutron wave numbers (here, the incident neutron energy E is given as E = $h^2 K^2/(2M)$), M is the reduced neutron mass, γ_s is nuclear surface radius, and q_f and q_γ are the fission and radiation phase shift factors, respectively.

(b) for the asymmetric resonance case

$$\frac{\Gamma_N^{n,as}}{2} = \frac{lK/KlD}{\pi} \qquad - (5a)$$

$$\frac{\Gamma_N^{f,as}(D)}{2} = \frac{q_f D}{\pi} \qquad - (5b)$$

$$\frac{\Gamma_N^{\gamma,as}}{2} = \frac{q_\gamma D}{\pi} \qquad - (5c)$$

where, again, q_f and q_γ are the fission and radiation phase shift factors, respectively, D is the level spacing or the separation between the energy levels, K is given as

K = 0.2187 (40 (in MeV) + E (in MeV))) - (5d)

Next, we consider the respective symmetric and asymmetric fission probabilities (i.e. $P_N^{f,s}$,and $P_N^{f,a}$) in Eq. (2a). First, on substituting the symmetric resonance widths of Eqs. (4a) and (4b) into the fission probability of Eq. (2a), the result for the symmetric fission probability is obtained as

$$P_N^{f,s} = \frac{\Gamma_N^{f,s}}{\Gamma_N^{f,s} + \Gamma_N^{f,s}} \qquad - (6a)$$

$$= \frac{\dfrac{2h^2 q_s K r_s}{Mr_s^2}}{\dfrac{2h^2 K r_s}{Mr_s^2} + \dfrac{2h^2 q_f K r_s}{Mr_s^2}} \qquad - (6b)$$

Secondly, if the radiation process is included, the symmetric fission probability also will be written as

$$P_N^{f\gamma,s} = \frac{\Gamma_N^{f\gamma,s}}{\Gamma_N^{n,s} + \Gamma_N^{f\gamma,s}} \qquad - (7a)$$

$$= \frac{\dfrac{2h^2(q_f + q_\gamma K r_s}{Mr_s^2}}{\dfrac{2h^2 K r_s}{Mr_s^2} + \dfrac{2h^2(q_f + q_\gamma)K r_s}{Mr_s^2}} \qquad - (7b)$$

where $\Gamma_N^{f\gamma,s} = \Gamma_N^{f,s} + \Gamma_N^{\gamma,s}$. Now, from Eq. (6a), it can be seen that the symmetric fission probability of Eq. (7b) is mainly the function of the respective fission and radiation phase shift q_f and q_γ , and the incident neutron energy E (= h^2 K^2/(2M)). Here, at low energy E, the factor K of Eq. (5d) can be considered to be a constant factor and Mr_s^2 is the same as the moment of inertia. Of course, Eq. (7b) shows the fission probability included the radiation process depends on the fission and radiation phase shift factors, i.e., q_f and q .

Secondly, we consider the asymmetric fission probability ($P_N^{f,a}$). Now, on substituting Eqs. (5a)and(5b) into Eq. (2a), the result for the asymmetric probability ($P_N^{f,a}$) is obtained as

$$P_N^{f,a} = \frac{\Gamma_N^{F,a}}{\Gamma_N^{n,a} + \Gamma_N^{f,a}} \qquad \text{- (8a)}$$

$$= \frac{2\, q_f D/\pi}{2lK/KlD/\pi + 2\, q_f D/\pi} \qquad \text{- (8b)}$$

In addition, if the radiation process is considerd, the asymmetric fission probability can be written as, from use of Eq. (5c),

$$P_N^{f\gamma,a} = \frac{\Gamma_N^{f\gamma,a}}{\Gamma_N^{n,a} + \Gamma_N^{f\gamma,a}} \qquad \text{- (9a)}$$

$$= \frac{2(q_f + q_\gamma)D/\pi}{2lK/KlD/\pi + 2(q_f + q_\gamma)D/\pi} \qquad \text{- (9b)}$$

Here, it can be seen that the asymmetric fission probability of Eq. (9b) depends on the phase shifts q_f and q_γ, the neutron energy E, and, particularly, the level spacing D. It is of very interest. Here, it is noted that K is considered to be constant at low E.

As a result, the fission probabilities of Eqs. (7a) and (7b), and Eqs. (8a) and (8b) show clearly the symmetric and the asymmetric fission probabilities, respectively. It indicates that the value of a fission probability can be observed either the symmetric case or the asymmetric one in experiment. In conclusion, we may hope that our present theoretical result for the symmetric and the asymmetric fission probabilities may prove helpful in further detailed exprimental study of the fission probabilities.

REFERENCES

1. P. J. and A. S. Jensen, " Elements of Nuclei: Many-Body Physics with the Strong Interaction" (Addison-Wesley, Redwood City, Melon Park, Reading, New York, Amsterdam, Don Mills, 1987).
2. E. Segre, Nuclei and Particles (W. A. Benjamin, New York, Amsterdam, 1965).
3. K. S. Krane, Introductory Nuclear Physics (John Wiley, New York, Chichester, Brisbane, Toronto, Singapore, 1988).
4. G. R. Choppin and J. Rydberg, Nuclear Chemistry: Theory and Application (Pergamon, Oxford, New York, Toronto, Sydney, Paris, Frankfurt, 1980).
5. A. P. Arya, Fundamentals of Nuclear Physics (Allyn and Bacon, 1966).
6. E. B. Paul, Nuclear and Particle Physics (North-Holland, Amsterdam, London, 1969).
7. W. E. Meyerhof, Elements of Nuclear Physics (McGraw-Hill, New York, ST. Louis, London, San Francisco, Toronto, Sydney, 1967).
8. Ho Jin Choi and Koan-Sik Joo, Advanced Nuclear Fission Theory: A Study of Nuclear Resonance Fission Process Using Regge Pole Method (this book will be published soon).
9. Ho Jin Choi, A Study of The Fission Probability Using Regge Pole method (unpublished).