Joint Pricing and Lot Sizing Policy under Order-Size-Dependent Delay in Payments

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Abstract

This paper deals with the problem of determining the retailer's optimal price and order size under the condition of order-size-dependent delay in payments. It is assumed that the length of delay is a function of the retailer's total amount of purchase. The constant price elasticity demand function is adopted which is a decreasing function of retail price. Investigation of the properties of an optimal solution allows us to develop an algorithm whose validity is illustrated through an example problem.

1. Introduction

The basic EOQ model is based on the implicit assumption that the retailer must pay for the items as soon as he receives them from a supplier. However, a common practice in industry is to provide a specific delay period for the payments after the items are delivered.

In this regard, a number of research papers appeared which deal with the EOQ problem under a fixed credit period. Chapman *et al.*(1985), Chung(1998) and Goyal(1985) analyzed the effects of trade credit on the optimal inventory policy. Also, Chu, Chung and Lan(1998) examined the EOQ model for deteriorating items under trade credit. The common assumption held by the above authors is that the demand is a known constant and, thus, they disregarded the effects of trade credit on the quantity demanded. The major reason for the supplier to offer a credit period to the retailers is to stimulate the demand for the product he produces. And so, the supplier usually expects that the increased sales volume can compensate the capital losses incurred during the credit period. The positive effects of credit period on the product demand can be integrated into the EOQ model through the consideration of retailing situations where the demand rate is a function of the selling price. The availability of the credit period from the supplier enables the retailer to choose the selling price from a wider range of option. Since the retailer's order size is affected by the demand rate of the product, the problems of determining the retail price and the order size are interdependent and must be solved simultaneously. Based upon the above observations, Kim, Hwang and Shinn(1995), and Hwang and Shinn(1997) examined the joint price and order size determination problem under the condition of permissible delay in payments assuming that the demand rate is a decreasing function of selling price.

All the research works mentioned above assumed that the supplier offers a certain fixed length of credit period. However, in Korea, some pharmaceutical companies and agricultural machines manufacturers associate the length of the credit period with the retailer's total amount of purchase, i.e., they offer a longer credit period for a large amount of purchase. In this regard, Hwang and Shinn(1996) evaluated the optimal lot sizing model under order-size-dependent delay in payments assuming that the demand rate is constant. The order-size-dependent delay in payments policy is based on the principle of economy of scale from the supplier's point of view and tends to make retailer's total amount of purchase large enough to qualify a certain credit period break.

This paper deals with the joint price and order size determination problem with permissible delay in payments where the length of delay is a function of the amount purchased by the retailer. In section 2, we formulate a relevant mathematical model. A solution algorithm is developed in section 3 based on the properties of an optimal solution. A numerical example is provided in section 4, which is followed by concluding remarks.

2. Development of the Model

The mathematical model of the joint price and order size determination problem is developed with the following assumptions and notations.

1) Replenishments are instantaneous with a known and constant lead time.
2) No shortages are allowed.
3) The inventory system deals with only one type of item.
4) The demand rate is represented by a constant price elasticity function of retail price.
5) The supplier allows a delay in payments for the items supplied where the length of delay is a function of the retailer's total amount of purchase.
6) The sales revenue generated during the credit period is deposited in an interest bearing account with rate I. At the end of the period, the credit is settled and the retailer starts paying the capital opportunity cost for the items in stock with rate $R(R \geq I)$.

P : unit retail price.
C : unit purchase cost.
S : ordering cost.
H : inventory carrying cost, excluding the capital opportunity cost.
R : capital opportunity cost(as a percentage).
I : earned interest rate(as a percentage).
D : annual demand rate, as a function of retail price(P), $D = KP^{-\beta}$,
where K is the scaling factor(> 0) and β is the index of price elasticity(> 0).

T : replenishment cycle time.

tc_j : credit period for the amount purchased TDC, $v_{j-1} \le TDC < v_j$,

where $tc_{j-1} < tc_j$, $j = 1,2,\Lambda ,m$ and $v_0 < v_1 < \Lambda < v_m$, $v_0 = 0$, $v_m = \infty$.

The retailer's objective is to maximize the annual net profit $\Pi(P,T)$ from the sales of the products. His annual net profit consists of the following five elements as stated by Hwang and Shinn(1996).

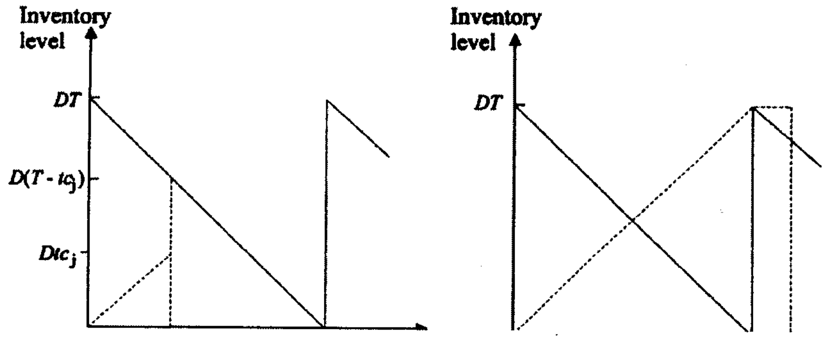
1) Annual sales revenue = DP.
2) Annual purchasing cost = DC.
3) Annual ordering cost = S / T.
4) Annual inventory carrying cost = $TDH / 2$.
5) Annual capital opportunity cost for $v_{j-1} \le TDC < v_j$
   (i) Case 1($tc_j \le T$): (see Figure 1. (a))
   the annual capital opportunity cost = $DC(R-I)tc_j^{\,2}/2T + DTRC/2 - DCRtc_j$.
   (ii) Case 2($tc_j > T$): (see Figure 1. (b))
   the annual capital opportunity cost = $DTIC/2 - DCItc_j$.

Then, depending on the relative size of tc_j to T, the annual net profit $\Pi(P,T)$ has two different expressions as follows:

1. Case 1($tc_j \le T$)

$$\Pi_{1,j}(P,T) = DP - DC - \frac{S}{T} - \frac{TDH}{2} - \left(\frac{C(R-I)Dtc_j^{\,2}}{2T} + \frac{CRDT}{2} - CRDtc_j \right) \qquad (1)$$

$$, \; TDC \in [v_{j-1}, v_j), \; j = 1,2,\Lambda ,m .$$



2. Case $2(tc_j > T)$

$$\Pi_{2,j}(P,T) = DP - DC - \frac{S}{T} - \frac{TDH}{2} - \left(\frac{CIDT}{2} - CIDtc_j\right) \qquad (2)$$

$$,\ TDC \in [v_{j-1}, v_j),\ j = 1,2,\Lambda\ ,m.$$

3. Analysis of the Model

The problem is to find an optimal retail price P^* and an optimal replenishment cycle time T^* which maximizes $\Pi(P,T)$. Once P^* and T^* are found, an optimal lot size Q^* can be obtained by the relation $Q = DT$. For a fixed P(let $P = P^0$), $\Pi_{i,j}(P^0,T)$ is a concave function of T for every i and j. Thus, there exists a unique value $T_{i,j}$, which maximizes $\Pi_{i,j}(P^0,T)$ and they are:

$$T_{1,j} = \sqrt{\left(2S + C(R-I)Dtc_j^2\right)/DH_1} \quad \text{where } H_1 = H + CR, \qquad (3)$$

$$T_{2,j} = \sqrt{2S/DH_2} \quad \text{where } H_2 = H + CI. \qquad (4)$$

So, as stated by Hwang and Shinn(1996), we have the following properties of $T_{i,j}$ and $\Pi_{i,j}(P^0,T)$.

Property 1. $T_{1,j} < T_{1,j+1}$ holds for j = 1, 2, ..., m-1.

Property 2. $T_{2,j} = T_{2,j+1}$ holds for j = 1, 2, ..., m-1.

Property 3. For any T, $\Pi_{i,j}(P^0,T) < \Pi_{i,j+1}(P^0,T)$, i = 1, 2 and j = 1, 2, ..., m-1.

Property 4. For any j, if $T_{1,j} \geq tc_j$, then $T_{2,j} \geq tc_j$, which implies that $\Pi_{2,j}(P^0,T)$ is increasing in T for $T < tc_j$. Also, if $T_{2,j} < tc_j$, then $T_{1,j} < tc_j$, which implies that $\Pi_{1,j}(P^0,T)$ is decreasing in T for $T \geq tc_j$.

Also, we have the following observations about the characteristics of the annual net profit function $\Pi_{i,j}(P^0,T)$ for T, $T \in I_j = \left\{ T \,\middle|\, v_{j-1}/DC \le T < v_j/DC \right\}$. Let k be the smallest index such that $T_{2,j} < tc_j$.

Observation 1. (for $j \ge k$)

(i) If $T_{2,j} < \frac{v_{j-1}}{DC}$, then $T = \frac{v_{j-1}}{DC}$ yields the maximum annual net profit for $T \in I_j$.

(ii) If $\frac{v_{j-1}}{DC} \le T_{2,j} < \frac{v_j}{DC}$, then $T = T_{2,j}$ yields the maximum annual net profit for $T \in I_j$

Observation 2. (for $j < k$)

(i) If $T_{1,j} < \frac{v_{j-1}}{DC}$, then $T = \frac{v_{j-1}}{DC}$ yields the maximum annual net profit for $T \in I_j$.

(ii) If $\frac{v_{j-1}}{DC} \le T_{1,j} < \frac{v_j}{DC}$, then $T = T_{1,j}$ yields the maximum annual net profit for $T \in I_j$.

(iii) If $tc_j < \frac{v_j}{DC} \le T_{1,j}$, then $T = \frac{v_j^-}{DC}$,where $v_j^- = v_j - \varepsilon$ and ε is a very small positive number, yields the maximum annual net profit for $T \in I_j$.

(iv) If $v_j/DC \le tc_j \le T_{1,j}$, then we do not need to consider T for $T \in I_j$ to find T^*.

Observation 3.(Search Stopping Rule)

(i) If $T = T_{1,j}$ yields the maximum annual net profit for $T \in I_j$, then $T^* \ge T_{1,j}$.

(ii) If $T = v_j^-/DC$ yields the maximum annual net profit for $T \in I_j$, then $T^* \ge v_j^-/DC$.

The observations state that for $P = P^0$ fixed, only the elements in set $B = \{T_{i,j}(P^0)$, v_{j-1}/DC , v_j^-/DC for $i = 1,2$ and $j = 1,2,\Lambda,m\}$ become candidates for an optimal replenishment cycle time $T^*(P^0)$ where $T_{i,j}(P^0)$ is obtained by substituting P with P^0 in equations (3) and (4) . Noting that some elements of B can be dropped from consideration in search of $T^*(P)$, we formulate the following conditions $T_{i,j}(P)$, v_{j-1}/DC and v_j^-/DC must satisfy to become a candidate of $T^*(P)$.

(C-1): The conditions for $T_{i,j}(P)$ to be a candidate of $T^*(P)$.

$$T_{1,j}(P) \ge tc_j \text{ and } v_{j-1}/DC \le T_{1,j}(P) < v_j/DC \quad \text{for Case 1} \qquad (5)$$

$T_{2,j}(P) < tc_j$ and $v_{j-1}/DC \le T_{2,j}(P) < v_j/DC$ for Case 2 (6)

(C-2): The conditions for v_{j-1}/DC to be a candidate of $T^*(P)$.

$v_{j-1}/DC \ge tc_j$ and $v_{j-1}/DC > T_{1,j}(P)$ for Case 1 (7)

$v_{j-1}/DC < tc_j$ and $v_{j-1}/DC > T_{2,j}(P)$ for Case 2 (8)

(C-3): The conditions for v_j^-/DC to be a candidate of $T^*(P)$.

$v_j/DC > tc_j$ and $v_j/DC \le T_{1,j}(P)$ for Case 1 (9)

Since the demand rate D is also a function of P, the inequality (5), (6), (7), (8) and (9) can

$$P \le P2_j \text{ where } P2_j = \left(\frac{KC(R-I)tc_j^2}{\sqrt{S^2 + C^{-1}(R-I)H_1 tc_j^2 v_{j-1}^2} - S} \right)^{\frac{1}{\beta}} \quad \text{if } R > I,$$

$$\left(2KSC^2 / H_1 v_{j-1}^2 \right)^{\frac{1}{\beta}} \quad \text{if } R = I,$$

from $\frac{v_{j-1}}{DC} \le T_{1,j}(P)$, (11)

$$P > P3_j \text{ where } P3_j = \left(\frac{KC(R-I)tc_j^2}{\sqrt{S^2 + C^{-1}(R-I)H_1 tc_j^2 v_j^2} - S} \right)^{\frac{1}{\beta}} \quad \text{if } R > I,$$

$$\left(2KSC^2 / H_1 v_j^2 \right)^{\frac{1}{\beta}} \quad \text{if } R = I,$$

from $T_{1,j}(P) < v_j/DC$, (12)

$P < P1_j$ where $P1_j = \left(KH_2 tc_j^2/2S\right)^{\frac{1}{\beta}}$ from $T_{2,j}(P) < tc_j$, (13)

$P > P4_j$ where $P4_j = \left(2KSC^2/H_2 v_j^2\right)^{\frac{1}{\beta}}$ from $T_{2,j}(P)DC < v_j$, (14)

$P \le P5_j$ where $P5_j = \left(2KSC^2/H_2 v_{j-1}^2\right)^{\frac{1}{\beta}}$ from $T_{2,j}(P)DC \ge v_{j-1}$,(15)

$P \ge P6_j$ where $P6_j = \left(KCtc_{j+1}/v_j\right)^{\frac{1}{\beta}}$ from $v_j \ge DCtc_{j+1}$, (16)

$P > P7_j$ where $P7_j = \left(KCtc_j / v_j\right)^{\frac{1}{\beta}}$ from $v_j > DCtc_j$. (17)

So, utilizing the price ranges in equations (10) to (17), we find the following price intervals which correspond to conditions (C-1), (C-2) and (C-3).

(PI-1): Price Interval on which $T_{i,j}(P)$ becomes a candidate for $T^*(P)$.

$$PI1_j = \{ P \mid P3_j < P \le P2_j \text{ and } P \ge P1_j \} \text{ for Case 1} \quad (18)$$

$$PI1_j = \{ P \mid P4_j < P \le P5_j \text{ and } P < P1_j \} \text{ for Case 2} \quad (19)$$

(PI-2): Price Interval on which v_{j-1}/DC becomes a candidate for $T^*(P)$.

$$PI2_j = \{ P \mid P > P2_j \text{ and } P \ge P6_{j-1} \} \quad \text{for Case 1} \quad (20)$$

$$PI2_i = \{ P \mid P > P5_i \text{ and } P < P6_{i-1} \} \quad \text{for Case 2} \quad (21)$$

The price intervals we present have a significant role in solving the model. Substituting T with the candidate values in $\Pi_{i,j}(P,T)$, we have the single variable functions and they are:

$$\Pi^0{}_{1,j}(P) = \Pi_{1,j}(P, T_{1,j}(P)) = D\{P - C(1 - Rtc_j)\} - \sqrt{H_1 D\{2S + DC(R - I)tc_j^{\,2}\}} \quad (23)$$

$$\Pi^0{}_{2,j}(P) = \Pi_{2,j}(P, T_{2,j}(P)) = D\{P - C(1 - Itc_j)\} - \sqrt{2H_2 DS} \quad (24)$$

$$\Pi_{1,j}(P, v_{j-1}/DC) = D\{P - C(1 - Rtc_j)\} - \frac{DCS}{v_{j-1}} - \frac{D^2C^2(R-I)tc_j^{\,2}}{2v_{j-1}} - \frac{H_1 v_{j-1}}{2C} \quad (25)$$

$$\Pi_{2,j}(P, v_{j-1}/DC) = D\{P - C(1 - Itc_j)\} - \frac{DCS}{v_{j-1}} - \frac{H_2 v_{j-1}}{2C}$$

(26)

$$\Pi_{1,j}(P, v_j^-/DC) = D\{P - C(1 - Rtc_j)\} - \frac{DCS}{v_j^-} - \frac{D^2C^2(R-I)tc_j^{\,2}}{2v_j^-} - \frac{H_1 v_j^-}{2C}. \quad (27)$$

So, an optimal solution (P^*, T^*) which maximizes $\Pi(P,T)$ is found by searching over $\Pi^0{}_{i,j}(P)$, $\Pi_{i,j}(P, v_{j-1}/DC)$ and $\Pi_{1,j}(P, v_j^-/DC)$, and

$$\max_{P,T} \Pi(P,T) = \max\left[\max_{\substack{P \in PI1_j \\ i,j}} \Pi^0{}_{i,j}(P),\ \max_{\substack{P \in PI2_j \\ i,j}} \Pi_{i,j}\left(P, \frac{v_{j-1}}{DC}\right),\ \max_{\substack{P \in PI3_j \\ j}} \Pi_{1,j}\left(P, \frac{v_j^-}{DC}\right) \right]. \quad (28)$$

For $\beta \le 1$, $\Pi^0{}_{i,j}(P)$, $\Pi_{i,j}(P, v_{j-1}/DC)$ and $\Pi_{1,j}(P, v_j^-/DC)$ are increasing function of P. Thus an optimal value $P_{i,j}$ occurs at the maximum point of the price interval corresponding to the candidate value. For $\beta > 1$, it can be shown that $\Pi^0{}_{i,j}(P)$, $\Pi_{i,j}(P, v_{j-1}/DC)$ and $\Pi_{1,j}(P, v_j^-/DC)$ are concave function of P if $P < C(1-Rtc_j)(\beta+1)/(\beta-1)$. So, $P_{i,j}$ is the one which has the minimum absolute value of $\Pi^0{}_{i,j}(P)'$, $\Pi_{i,j}(P, v_{j-1}/DC)'$ and $\Pi_{1,j}(P, v_j^-/DC)'$ on the price interval. Note that for problem with $\beta > 1$, the algorithm is valid only when $P < C(1-Rtc_m)\,(\beta+1)/(\beta-1)$. Based on the above results, we develop the following solution algorithm for determining an optimal solution (P^*, T^*).

Solution algorithm

Step 1: This step identifies all the candidate values T_0 of T satisfying $T_0 \geq tc_j$ (for Case 1).

1.1. Determine $P_{1,j}$ which maximizes $\Pi^0{}_{1,j}(P)$ among the following price intervals:

1.2. Determine $P_{1,j}$ which maximizes $\Pi_{1,j}(P, v_{j-1}/DC)$ among the following price intervals: $P \in PI2_j$ and $P \leq P_u$ with $T_0 = v_{j-1}/DC$, $j = 2,3,\Lambda,m$.

1.3. Determine $P_{1,j}$ which maximizes $\Pi_{1,j}(P, v_j^-/DC)$ among the following price intervals: $P \in PI3_j$ and $P \leq P_u$ with $T_0 = v_j^-/DC$, $j = 1,2,\Lambda,m-1$.

Step 2: This step identifies all the candidate values T_0 of T satisfying $T_0 < tc_j$ (for Case 2).

2.1. Determine $P_{2,j}$ which maximizes $\Pi^0{}_{2,j}(P)$ among the following price intervals: $P \in PI1_j$ and $P \leq P_u$ with $T_0 = T_{2,j}(P)$, $j = 1,2,\Lambda,m$.

2.2. Determine $P_{2,j}$ which maximizes $\Pi_{2,j}(P, v_{j-1}/DC)$ among the following price intervals: $P \in PI2_j$ and $P \leq P_u$ with $T_0 = v_{j-1}/DC$, $j = 2,3,\Lambda,m$.

Step 3: Select the optimal retail price(P^*) and replenishment cycle time(T^*) which gives the maximum annual net profit among those obtained in the previous steps.

4. Numerical Example

To illustrate the solution algorithms, the following problem is considered.

(1) $S = \$\,50$, $K = 2.5 * 10^5$, $C = \$\,3$, $H = \$\,0.1$, $R = 0.15 (= 15\%)$, $I = 0.1 (= 10\%)$.

(2) Supplier's credit schedule:

$tc_1 = 0.1$ for $0 \leq TDC < \$1500$

$tc_1 = 0.2$ for $\$1500 \leq TDC < \$3000$

$tc_1 = 0.3$ for $\$3000 \leq TDC$

Table 1. Results of Step 1.

j	$T = T_{1,j}(P)$: $P \in PI1_j$ and $P \le P_u$	$P_{1,j}$	$T_{1,j}(P_{1,j})$	$T = \frac{v_{j-1}}{DC}$: $P \in PI2_j$ and $P \le P_u$	$P_{1,j}$	$\frac{v_{j-1}}{DC}$	$T = \frac{v_j^-}{DC}$: $P \in PI3_j$ and $P \le P_u$	$P_{1,j}$	$\frac{v_j^-}{DC}$
1	ϕ	-	-	-	-	-	[4.79, 6.68]	5.12	0.118
2	[5.06, 6.68]	5.06	0.230	ϕ	-	-	[4.79, 5.05]	4.98	0.221*
3	ϕ	-	-	[5.63, 6.68]	5.63	0.301	-	-	-

*: Optimal solution for Case 1(Annual net profit = $8788).

Table 2. Results of Step 2.

j	$T = T_{2,j}(P)$: $P \in PI1_j$ and $P \le P_u$	$P_{1,j}$	$T_{2,j}(P)$	$T = v_{j-1}/DC$: $P \in PI2_j$ and $P \le P_u$	$P_{1,j}$	v_{j-1}/DC
1	ϕ	-	-	-	-	-
2	ϕ	-	-	ϕ	-	-
3	[illegible]	[illegible]	[illegible]*	[illegible]	[illegible]	[illegible]

The solution procedure with $\beta = 2.5$ and $P_u = C(1 - Rtc_3)(\beta + 1)/(\beta - 1) = 6.68$ generates the optimal solution. Table 1 and 2 show the results for each case and an optimal solution (P^*, T^*) becomes (4.93, 0.233) with its maximum annual net profit $8927.

5. Conclusion

This paper dealt with an optimal retailing policy when the retail demand of the product is represented by a constant price elasticity function of the retail price and the supplier permits delay in payments where the length of delay is a function of the retailer's total amount of purchase. The availability of the credit period according to the total retail volume can be justified by the principle of economy of scale. The credit policy tends to make the retailer's order size larger by inducing him to qualify for a longer credit period in his payments. After formulating the mathematical model, we proposed the solution procedure which leads to an optimal retailing policy. With an example problem, the validity of the algorithm is illustrated. The results showed that the annual net profit can be increased through a wise selection of both the retail price and the order size.

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