# Performance of OFDM with Macrodiversity Selection on a Shadowed Multipath Channel

Wooncheol Hwang, Hongku Kang, Seung-Geun Kim, and Kiseon Kim Dept. of Information and Communications, K-JIST Tel: +82-62-970-2264, Fax: +82-62-970-2204

E-mail: cloud@geguri.kjist.ac.kr

#### **Abstract**

In this paper, we investigate performance of OFDM employing macrodiversity selection on a shadowed multipath channel to reduce the effects of long-term shadowing. Three different situations are considered: independent shadowing with equidistant ports, independent shadowing with non-equidistant ports, and correlated shadowing. Numerical results show that OFDM performance is improved as the number of macrodiversity ports increases. In the situation where the user is much closer to certain port(s), performance becomes worse compared to that of the equidistant situation. In addition, as the correlation increases among the ports, the advantage of macrodiversity is reduced.

#### I. Introduction

In multipath environments, orthogonal frequency division multiplexing (OFDM) is a quite attractive transmission scheme for high-speed communications because it can remove or at least reduce the effects of intersymbol interference (ISI) caused by the dispersive channel [1, 2]. Besides the multipath effects, however, there is another factor called shadowing that severely deteriorates the system performance. Shadowing means slow variation of the mean signal power over the spatial environment due to topographical variation along the transmission path. It is a large-scale effect well described by lognormal distribution, while the multipath fading is a small-scale one.

One well-known countermeasure to combat the channel shadowing is the macroscopic diversity (or macrodiversity) [3-6]. It is a kind of "large-scale" space diversity employing multiple widely separated

receiving sites. In this scheme, several signals are received at different sites (radio-ports) with different long-term shadowing experienced on those signals. By combining, or selecting from, these received signals, the signal variability is reduced, thus the deleterious effects of shadowing can be mitigated.

Macrodiversity selection is one common scheme of macrodiversity which selects the radio-port with the smallest shadowing effect (or the largest local mean signal power) to set up the communication link. In this scheme, a number of transmitters are dedicated to serve a user, but only one actually transmits at any given time. This can be achieved by letting the radio-port that receives the strongest mean signal from the user be the one to transmit back to it.

In this paper, we analyze the performance of OFDM employing macrodiversity selection on a shadowed multipath channel. In Section II, system and channel models are described. The symbol error rate (SER) of OFDM is analyzed for three different macrodiversity situations in Section III, and numerical results are shown in Section IV. Finally, conclusions are given in Section V.

### II. System Description

We consider an OFDM system employing the macrodiversity selection to reduce the adverse effect of shadowing. It is assumed that there are P geographically distributed radio-ports which make up a macrodiversity group. At any given time, the "best" port that has the largest long-term local mean signal power is selected by the macrodiversity switch which is connected to all ports by cables. Once the best port is selected, the signal received by way of that port is processed for demodulation.

The transmitted complex baseband OFDM signal can be represented as

$$s(t) = \sum_{m=-\infty}^{\infty} \sum_{i=0}^{N-1} \frac{A}{\sqrt{T}} a_{m,i} e^{j2\pi f_i t} p(t - mT)$$
 (1)

where N is the number of subchannels, A is a constant related to the signal power, T is an OFDM symbol duration,  $a_{m,i}$  is a data symbol of the i-th subchannel in the m-th signaling interval [mT, (m+1)T),  $f_i = i/T$  is the i-th subcarrier frequency, and p(t) is a rectangular pulse with length T and amplitude 1. We assume that the data symbol is QPSK modulated, i.e.,  $a_{m,i}$  takes its value from a set  $\{\pm 1 \pm i\}$ .

There are P channels between the transmitter and the receiver although only one is activated at any given time. To represent each channel between the user and the l-th macrodiversity port, a shadowed two-path model is used having the following impulse response.

$$h_l(t) = \sqrt{x_l} \left[ \delta(t) + b \delta(t - \tau) \right], \quad 1 \le l \le P$$
 (2)

where  $x_l$  is a lognormal random variable representing the shadowing, and b and  $\tau$  are the attenuation coefficient and the delay of a delayed path, respectively. Letting  $g_l = 10 \log x_l$ ,  $g_l$  is a Gaussian random variable with mean  $M_l$  (dB) and standard deviation  $\sigma_l$  (dB). In addition, additive white Gaussian noise (AWGN) with one-sided power spectral density  $N_0$  is added on the channel.

By the macrodiversity selection, the port with the largest local mean power  $g = \max\{g_1, g_2, \cdots, g_P\}$  is repeatedly selected to provide service to the user. Selection of the best port is equivalent to the selection of the best channel. Thus, assuming that the macrodiversity selection perfectly works, the impulse response of the equivalent channel can be represented as

$$h(t) = \sqrt{x} \left[ \delta(t) + b \delta(t - \tau) \right] \tag{3}$$

where  $x = \max\{x_1, x_2, \dots, x_P\} = 10^{g/10}$ .

In these reasons, we regard that the OFDM signal in Eq.(1) arrives at the receiver through the "single" channel of Eq.(3). In the receiver, the received signal passes through a bank of correlators (fast Fourier transform operation) to yield decision variables. Then, the detected symbols are converted to a serial fashion by a parallel-to-serial converter.

# III. Performance Evaluation

#### 3.1 SER Analysis

In the receiver, the decision variable of the k-th subchannel in the n-th signaling interval is represented as [2]

$$z_{n,k} = 10^{g/10} A a_{n,k} + D_{n,k} + G_k \tag{4}$$

where  $D_{n,k}$  is an ISI component, and  $G_k$  is an interchannel interference (ICI) component combined with an AWGN component.  $D_{n,k}$  is given by [2]

$$D_{n,k} = 10^{g/20} Ab e^{-j\phi} \left( a_{n-1,k} - \frac{\tau}{T} + a_{n,k} - \frac{T-\tau}{T} \right)$$
 (5)

where  $\phi$  is a uniform random variable in  $[0, 2\pi]$ . For sufficiently large N,  $G_k$  can be approximated as a zero-mean Gaussian random variable with following variance [2].

$$\sigma_{G_{i}}^{2} = \frac{N_{0}}{2} + \frac{10^{g/10} (Ab)^{2}}{\pi^{2}} \sum_{i=0, i \neq k}^{N-1} \frac{1 - \cos\left(\frac{2\pi(i-k)\tau}{T}\right)}{(i-k)^{2}}$$
(6)

For fixed g, the probability of error at one branch (in-phase or quadrature) is calculated as [2]

$$P_{br,k}(g) = \frac{1}{16\pi} \int_0^{2\pi} \sum_{l=1}^4 \sum_{m=1}^2 Q\left(\frac{10^{g/20}A}{\sigma_{G_k}} \left[1 - b\left(\frac{\tau}{T} \times (\alpha_l^i + 1) - 1\right)\cos\phi - b\left(\frac{\tau}{T}(\alpha_l^i - \beta_m^i) + \beta_m^i\right)\sin\phi\right]\right) d\phi$$
(7)

where  $Q(\cdot)$  is the tail distribution of a zero-mean Gaussian random variable with unity variance,  $\alpha_l$  and  $\beta_m$  are complex constants given by  $\alpha_1 = 1 + j$ ,  $\alpha_2 = 1 - j$ ,  $\alpha_3 = -1 + j$ ,  $\alpha_4 = -1 - j$ ,  $\beta_1 = j$ ,  $\beta_2 = -j$ . The superscripts r and i in Eq.(7) stand for the real part and the imaginary part, respectively. By averaging  $P_{br,k}(g)$  over g, the branch error probability of the k-th subchannel,  $P_{br,k}$ , is obtained.

$$P_{br,k} = \int_{-\infty}^{\infty} P_{br,k}(g) f_G(g) dg$$
 (8)

where  $f_G(g)$  is a probability density function (pdf) of g which is treated in detail in the following subsection. Then, the SER of the k-th subchannel,  $P_{s,k}$ , is calculated from  $P_{br,k}$  as follows.

$$P_{s,k} = 1 - (1 - P_{br,k})^2 \tag{9}$$

Finally, the overall SER is obtained by averaging  $P_{s,k}$  over k.

$$P_{s} = \frac{1}{N} \sum_{k=0}^{N-1} P_{s,k} \tag{10}$$

# 3.2 Distribution of the Largest Local Mean

Three different macrodiversity situations are considered in this paper. In all situations, it is assumed that all ports experience equally severe shadowing, i.e.,  $\sigma_l = \sigma$  for  $l = 1, \dots, P$ .

## Situation I

- P ports experience statistically independent shadowing.
- The user is at a point which is equidistant from all serving ports making up a macrodiversity group, i.e.,  $M_l = M$  for  $l = 1, 2, \dots, P$ .

In this situation, the pdf of the largest local mean g is given by [4, 5]

$$f_G(g) = \frac{P}{\sqrt{2\pi\sigma}} \left[ 1 - Q \left( \frac{g - M}{\sigma} \right) \right]^{P-1} \exp \left[ -\frac{(g - M)^2}{2\sigma^2} \right]$$
(11)

#### Situation II

- · There are 3 ports with independent shadowing.
- · The user is much closer to certain port(s):
- 1) one dominant port ( $M_1 = M_2 + \Delta M$ ,  $M_2 = M_3$ )
- 2) two dominant ports ( $M_2 = M_3 = M_1 + \Delta M$ )

In this situation, the pdf of g is given by [5]

$$f_G(g) = \frac{1}{\sqrt{2\pi\sigma}} \left[ 1 - Q \left( \frac{g - M_2}{\sigma} \right) \right] \times \left\{ \left[ 1 - Q \left( \frac{g - M_2}{\sigma} \right) \right] \exp \left[ - \frac{(g - M_1)^2}{2\sigma^2} \right] \right.$$

$$\left. + 2 \left[ 1 - Q \left( \frac{g - M_1}{\sigma} \right) \right] \exp \left[ - \frac{(g - M_2)^2}{2\sigma^2} \right] \right\}$$

The assumption of independent shadowing among ports may sometimes be violated because of insufficient spacing of the ports. Thus, we consider a correlated shadowing situation for the simplest case. Situation III

• There are 2 ports with correlated shadowing. The correlation coefficient is given by

$$r = \frac{E[(g_1 - M_1)(g_2 - M_2)]}{\sigma^2}$$
 (13)

• The user is at a point which is equidistant from the two ports, i.e.,  $M_1 = M_2 = M$ .

In this situation, the pdf of g is given by [6]

$$f_G(g) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \left[ 1 - Q \left( \frac{1-r}{\sqrt{1-r^2}} \cdot \frac{g-M}{\sigma} \right) \right] \times \exp \left[ -\frac{(g-M)^2}{2\sigma^2} \right]$$
(14)

The density functions in Eqs.(11), (12), and (14) are used for the calculation of  $P_{br,k}$  in Eq.(8), and eventually for the overall SER given in Eq.(10).

Table 1 System and channel parameter values used for the numerical examples

	Parameter	Value
System	Data Rate	155 Mbps
	Number of subcarriers	128
	Modulation Type	QPSK
	Symbol Duration T	1.65 μs
Channel	Delay τ	50 ns
	Attenuation Coefficient b	0.2
	Standard Deviation of Shadowing σ	3.2 dB

#### IV. Numerical Results

Table 1 presents the parameter values used for numerical examples. 128-carrier OFDM supporting 155 Mbps was assumed. The channel parameters were set to b = 0.2 and  $\tau = 50$  ns, and the standard deviation of shadowing was set to  $\sigma = 3.2$ dB. Fig. 1 shows the numerical result for Situation I. We can see that OFDM performance is improved as the number of macrodiversity ports increases. For example, 3.0 dB, 4.7 dB and 5.7 dB  $E_s/N_0$  gains are obtained at SER=10<sup>-5</sup> by using 2-, 3-, and 4-port macrodiversity selection, respectively. Fig. 2 and Fig. 3 correspond to Situation II. It is shown that, in a situation where the user is much closer to certain port(s), performance becomes worse compared to that of the equidistant-port situation. Performance of one-dominant-port case with  $\Delta M > 10$  dB is almost the same as that of no diversity case (Fig. 2). Also, performance of two-dominant-port case with  $\Delta M$ 10 dB is almost the same as that of equidistant 2-port diversity case (Fig. 3). Fig. 4 shows the effect of port correlation (Situation III). It is shown that, as the correlation increases, macrodiversity gain becomes small. Two-port macrodiversity with fully correlated shadowing (r=1) yields the same performance as no macrodiversity.

# V. Conclusion

In this paper, we have investigated performance of OFDM using the macrodiversity selection to reduce the effects of shadowing. 3 different macrodiversity situations were addressed, and numerical examples were illustrated for 128-carrier OFDM suffering from shadowing of 3.2 dB standard deviation.

The numerical results showed performance

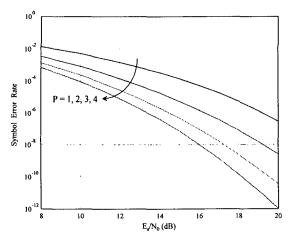
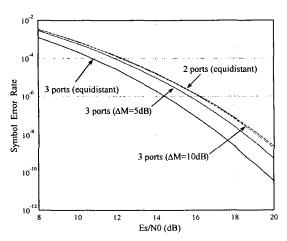


Fig. 1 Performance of OFDM with macrodiversity (Situation I: equidistant ports with independent shadowing)

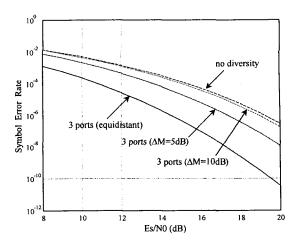


**Fig. 3** Performance of OFDM with 3-port macrodiversity (Situation II-2: non-equidistant ports with independent shadowing, two dominant ports,  $M_2$  =  $M_3$  =  $M_1$  +  $\Delta M$ )

improvement on the shadowed environment by use of macrodiversity selection. 3.0 dB, 4.7 dB and 5.7 dB  $E_s/N_0$  gains were obtained at SER= $10^{-5}$  by using 2-, 3-, and 4-port macrodiversity selection for the equidistant-port situation. In a situation where the user was much closer to certain port(s), performance became worse compared to that of the equidistant-port situation. In addition, as the correlation increased among the ports, the advantage of macrodiversity was reduced.

#### References

 L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. COM-33,



**Fig. 2** Performance of OFDM with 3-port macrodiversity (Situation II-1: non-equidistant ports with independent shadowing, one dominant port,  $M_2=M_3=M_1-\varDelta M$ )

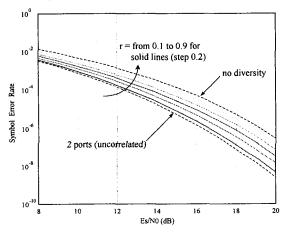


Fig. 4 Performance of OFDM with 2-port macrodiversity (Situation III: equidistant ports with correlated shadowing)

- no. 7, pp. 665-675, July 1985.
- [2] W. Hwang and K. Kim, "Performance analysis of OFDM on the shadowed multipath channels," *IEEE Trans. Consumer Electron.*, vol. 44, no. 4, pp. 1323–1328, Nov. 1998.
- [3] W. C. Jakes, Ed., Microwave Mobile Communications. IEEE Press, 1974.
- [4] W.-P. Yung, "Probability of bit error for MPSK modulation with diversity reception in Rayleigh fading and log-normal shadowing channel," *IEEE Trans. Commun.*, vol. 38, no. 7, pp. 933-937, July 1990.
- [5] A. M. D. Turkmani, "Probability of error for M-branch macroscopic selection diversity," *IEE Proceedings-I*, vol. 139, no. 1, pp. 71–78, Feb. 1992.
- [6] L.-C. Wang, G. L. Stüber, and C.-T. Lea, "Effects of Rician fading and branch correlation on a local-meanbased macrodiversity cellular system," *IEEE Trans.* Veh. Technol., vol. 48, no. 2, pp. 429-436, Mar. 1999.