A High Quality Mesh Generation for a Surface defined by Linear Lie Algebra

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Abstract: Recently, the research on computer graphics (CG) has been actively studied and developed. Namely, many surface/solid models have been proposed in the field of computer aided geometric design as well as the one of CG. Since it is difficult to visualize the complex shape exactly, an approximation by generating a set of meshes is usually used. Therefore it is important to guarantee the quality of the approximation in consideration of the computational cost.

In this paper, a mesh generation algorithm will be proposed for a surface defined by linear Lie algebra. The proposed algorithm which considers the quality in the meaning of validation of invariants obtained by the mesh.

1. Introduction

Recently, since technology of computer graphics (CG) can represent more complicated shape (surface/solid), CG has been used in various engineering fields. Therefore research on surface/solid modeling has been studied, which can represent more complex shape[1].

In the research field, a representation method using linear Lie algebra has been studied and developed, namely in the field of invariant 3-dimensional object recognition and representation[2][3].The method can represent the object (surface) as small number of parameters, so that it is also expected to be useful in the field of intelligent communication system.

However in general, it is difficult to visualize exactly the complicated shape defined by such method, because it takes huge computational cost in visualization. Therefore, an approximation by a set of meshes is often used. It is obviously seen that the quality of approximation depends on suitable size of meshes to the shape of the object.

In order to approximate the surface by linear Lie algebra, an adaptive method is proposed, which considers difference of neighbor normal vectors[4]. Although the method can represent a set of meshes with high quality, there are some cases under which it cannot guarantee the accuracy.

In this paper, we shall propose a high quality mesh generation method, for a surface defined by linear Lie algebra, which can be applied to more cases than previous method.

2. Meshing Method by Linear Lie Algebra

2.1 Linear Lie Algebra

Lie group is the C^∞ class differentiable manifold and normal/tangent vector field on the Lie group is called Lie algebra. Lie group and Lie algebra have the corresponding relation. Then, we can get the shape means global information (Lie group) from normal/tangent vector field means local information (Lie algebra).

In this paper, we treats the linear Lie algebra represented by the 1-order linear equation.

2.2 Application of Linear Lie Algebra to Surface Modeling

For a given point $\boldsymbol{p}$ on a surface by linear Lie algebra, its normal vector $\boldsymbol{v}$ is defined as follows:

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{p}, \qquad (1)$$

where $\boldsymbol{A}$ is a representation matrix such that

$$\boldsymbol{A} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \boldsymbol{P}_\theta \boldsymbol{Q}_\phi \boldsymbol{P}_\psi. \qquad (2)$$

Here, $\boldsymbol{P}_\theta$, $\boldsymbol{Q}_\phi$ and $\boldsymbol{P}_\psi$ are rotation matrices with respect to X, Y and Z axis, respectively, and a set of λ_1, λ_2, λ_3, θ, ϕ and ψ is called as invariant.

Since the surface is implicitly defined as a set of integral curves by Eq.(1), the neighbor point $\boldsymbol{p}'$ of $\boldsymbol{p}$ is obtained approximately as

$$\boldsymbol{p}' = \boldsymbol{p} + \Delta t \boldsymbol{w}, \qquad (3)$$

where $\boldsymbol{w}$ is a tangent vector at $\boldsymbol{p}$ and Δt is small enough (**Figure 1**).

When a set of points is calculated by Eq.(3) repeatedly, the shape of surface is obtained. However by the accumulated errors, the obtained shape is often different from the original one.

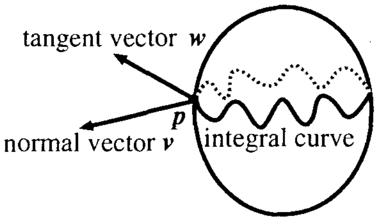

Figure 1: normal/tangent vector on the object surface

In order to guarantee the quality of approximation, in this paper, we propose the following algorithm.

3. Algorithm

Here we show an outline of the proposed algorithm as follows: First, we input some data, invariant and initial point p_0, default interval length Δt between the neighbor points and threshold. Then, calculate the normal vector v_0 on p_0 and generate neighbor point. Finally, examine significance and adjust Δt for that the error between two variants will be less than threshold.

Step1: Prior process

For the given surface by linear Lie algebra, suppose that a representation matrix $\boldsymbol{A}$, an initial point p_0 on the object surface, and an initial stepsize Δt are given. And let $i = 0$.

Step2: Calculation of initial point

Calculate a normal vector v_i at p_i using Eq.(1).

Step3: Making temporary neighbor points

Calculate three neighbor points $p_{i,j}(j = 1, 2, 3)$ of p_i by Eq.(3) (**Figure 2**).

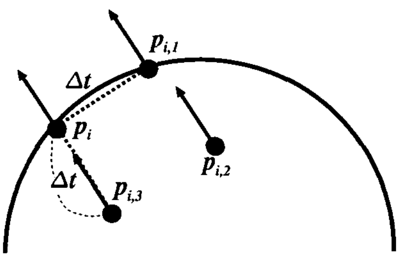

Figure 2: Generating neighbor points of p_i

Step4: Calculation of representation matrix

Calculate a representation matrix $\boldsymbol{A}_i$ by solving the following simultaneous equation

$$v_{i,j} = \boldsymbol{A}_i p_{i,j}, \quad j = 1, 2, 3. \tag{4}$$

Here we assume that

$$v_{i,j} = v_i, \quad j = 1, 2, 3, \tag{5}$$

i.e., p_i and $p_{i,j}$ are assumed to be located on the same plane.

Step5: Adjustment of Δt

If $||\boldsymbol{A} - \boldsymbol{A}_i||$ is greater than the given threshold, we consider Δt is too large. That is to say, p_i and $p_{i,j}$ aren't on the same surface. Then, return to **step 3** with $\Delta \leftarrow \alpha\Delta t$ $(0 < \alpha < 1)$(**Figure 3**).

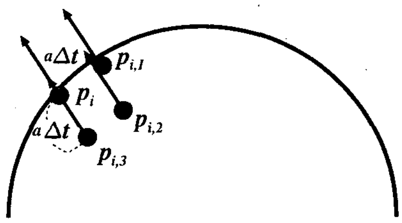

Figure 3: shortened Δt

Else a set of $p_{i,j}$ is adapted as the points defining the mesh with guaranteed quality.

Step6: Return to Step2

Let p_{i+1} be one of $p_{i,j}$. Return to **step2**.

From the above, the high quality meshes can be generated. This algorithm can generate the mesh automatically and adaptively, only by setting invariant data, initial point and some parameters.

Also, the previous method cannot reconstruct surface exactly in several case such as **Figure 4**, because this method guarantees significance by considering the difference of neighbor normal vectors. However, the proposed method considers the invariant. Then, our method can represent surface which includes such case.

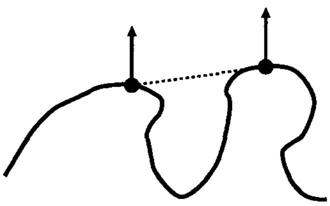

Figure 4: The case which contains the risk of error

4. Simulation

We here show some results by the proposed algorithm.

Figure 5 ~ **Figure 10** show images of simulation by the proposed method. Also **Table 1** shows the parameters for each figure.

Figure 5: Simulation 1

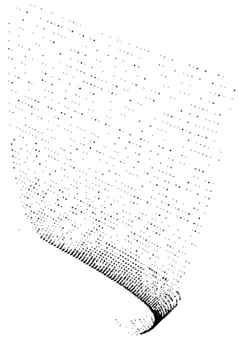

Figure 6: Simulation 2

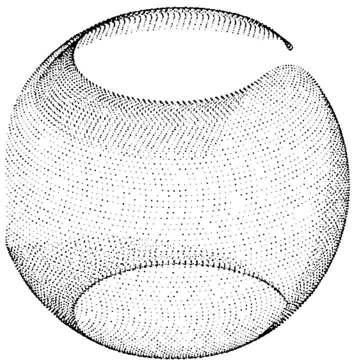

Figure 7: Simulation 3

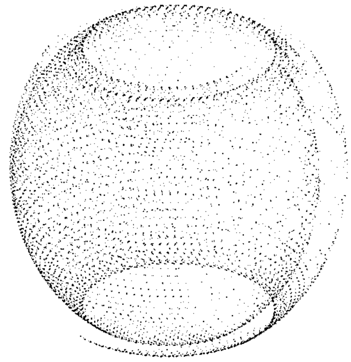

Figure 8: Simulation 4

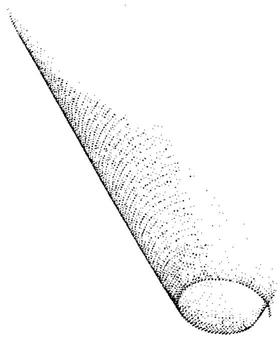

Figure 9: Simulation 5

Figure 5 and **Figure 6** show results of generation nonalgebraic shape. We can see that the surface consists of several size of patches. The area where the curvature is small consists of large patches, and the other area consists of small patches. Also, if we want to represent surface of **Figure 6** in high quality without our method, it needs 1.8×10^5 patches. The surface in **Figure 6** contains only 6.0×10^3 patches. Then, the proposed method can represent high quality surface rationally.

Figure 7 and **Figure 8** are results of simulations in the same environment except the threshold. The former has smaller threshold than the latter. From these images, we can see the difference of shape. **Figure 8** cannot represent the shape of sphere. In that way, we control the quality of surface only changing threshold.

Figure 10: Simulation 6

Table 1: invariant and threshold

Figure	invariant						threshold
5	-10	-10	20	-40	-100	-140	0.1
6	-2	3	-6	-30	100	75	0.1
7	1	1	1	0	0	0	0.1
8	1	1	1	0	0	0	0.8
9	5	6	-20	6	30	-30	0.1
10	5	6	-20	6	30	-30	0.1
	the initial size of Δt	**the maximal number of division Δt**					
5	10	3					
6	10	3					
7	10	3					
8	10	2					
9	2	2					
1o	8	4					

Figure 9 and **Figure 10** are results of simulations about the difference of initial size of Δt. We can see in these images that they represent the same shape of surface, in spite of the difference of initial size of Δt. Because, the proposed method judges significance by

invariant, so it isn't influenced step size.

All simulation take about 10 minutes without depending on parameters under the 443 MHz Ultra SPARC II*i* system.

5. Conclusion

In this paper, we have proposed a mesh generating algorithm with the guaranteed quality for the surface by linear Lie algebra. Since user can generate a set of high quality mesh from the invariant automatically and adaptively, they can reduce computational cost.

Especially, because we proposed the method which guarantees quality by considering invariant, it is seen that the quality becomes higher than the previous method.

In a future paper, the algorithm should be considered which solves the following problems:

1. Undefined stopping criterion for closed surface.
2. Recognition of small hollows which is less than Δt.
3. Application to higher order algebraic and non-algebraic shape.

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