An Approach to Identify NARMA Models Based on Fuzzy Basis Functions

Worapoj Kreesuradej and Chokchai Wiwattanakantang
Faculty of Information Technology,
King Mongkut's Institute of Technology Ladkrabang,
Ladkrabang, Bangkok 10520 Thailand
Tel: +66-2-737-2551-4 ext. 522 Fax: +66-2-326-9074
Email: worapoj@it.kmitl.ac.th

Abstract: Most systems in the real world are non-linear and can be represented by the non-linear autoregressive moving average (NARMA) model. The extension of fuzzy system for modeling the system that is represented by NARMA model will be proposed in this paper. Here, fuzzy basis function (FBF) is used as fuzzy NARMA(p,q) model. Then, an approach to identify fuzzy NARMA models based on fuzzy basis functions is proposed. The efficacy of the proposed approach is shown from experimental results.
Keywords: fuzzy systems, modeling, fuzzy NARMA, time series

1. Introduction

In real-life most systems are non-linear. Since linear models cannot capture the behavior of limit data associated with non-linear systems, it is important to investigate the identification procedure for non-linear model [1], [5]. The NARMA model provides a basis for such a development.

Most applications of fuzzy modeling rely on the framework of autoregressive model or regression model [2], [3]. This is the case because the inputs value for the fuzzy models can be easily identified: they are simply the lagged values of the time series itself or the exogeneous inputs. Fuzzy models that are based on the framework of autoregressive model or regression model fail to model a system that is represented by NARMA model. As a result, high prediction error may occur when applied fuzzy AR model or fuzzy regression model for modeling NARMA process.

Here we propose an alternative approach for identifying NARMA models based on fuzzy basis function (FBF). Fuzzy systems are represented as series expansion of fuzzy basis functions. These fuzzy basis functions are capable of uniformly approximating any real continuous function on a compact set to arbitrary accuracy [3]. This means that NARMA can be approximated within an arbitrary accuracy by model based on FBF. In this paper, We performed two kinds of statistical tests—autocorrelation test and chi-squared test in order to measure the quality of fit.

2. Fuzzy Basis Function

In this paper, we consider a fuzzy system whose basic configuration is shown in Fig.1.

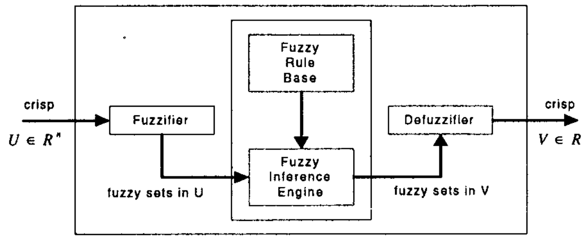

Fig. 1. Basic configuration of fuzzy systems.

If the fuzzy rule base consists of a collection of fuzzy IF-THEN rule:

$$R^l: \text{IF } x_1 \text{ is } F_1^l \text{ and} \ldots \text{and } x_n \text{ is } F_n^l \text{ THEN y is } G^l$$

where l=1, 2, ...L, F_i^l and G^l are labels of fuzzy sets in U and V respectively, then fuzzy logic systems with a center average defuzzifier, algebraic product inference, and singleton fuzzifier consist of all functions of the form

$$y(\bar{x}) = \frac{\sum_{l=1}^{L} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{L} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{1}$$

where $\mu_{F_i^l}$ and μ_{G^l} are membership function of F_i^l and G^l respectively and $\bar{y}^l$ is the point at which μ_{G^l} achieves its maximum value that is assumed to be one. Fuzzy basis functions (FBF) are defined as

$$p_j(\bar{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{L} \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i)\right)}, \quad j = 1, 2, 3, \ldots, L. \tag{2}$$

Therefore, the fuzzy system in the equation.(1) is equivalent to a fuzzy basis function (FBF) expansion:

$$y(\bar{x}) = \sum_{j=1}^{L} \theta_j p_j(\bar{x}) \tag{3}$$

Here, Least square (LS) algorithm is proposed to identify the parameters of the FBF expansion. To applied LS algorithm, the FBF expansion is rewrite as

$$y(\bar{x}) = \sum_{j=1}^{L} \theta_j p_j(\bar{x}) + \varepsilon$$

Given N input-output pairs, the matrix notation of the equation can be written as

$$Y = P\Theta + E, \tag{4}$$

Where

$$Y = \begin{bmatrix} y^1 \\ \cdot \\ \cdot \\ y^N \end{bmatrix}, P = \begin{bmatrix} p_1^1 & \cdot & \cdot & p_L^1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ p_1^N & \cdot & \cdot & p_L^N \end{bmatrix}, E = \begin{bmatrix} \varepsilon^1 \\ \cdot \\ \cdot \\ \varepsilon^N \end{bmatrix}.$$

Then, Θ must satisfy the following equation:

$$(P^TP)\,\Theta = P^TY$$

The derivation of the equation (5) can be found in Kreesuradej [7]. The inverse or pseudo-inverse of P^TP are usually utilized for finding the value of Θ.

One important property of FBF expansions is that FBF expansions are capable of approximating any real continuous function [3], [4]. This gives a justification for using FBF expansions to model a dynamic system that is usually described by continuous functions. In this work, the FBF expansions will be applied to modeling NARMA model.

3. Identification Procedure Of NARMA Model Based On Fuzzy Basis Functions

Here, FBF is proposed to model NARMA (p,q) model, which has the following form [6]:

$$y(t) = f(y(t-1),\ldots, y(t-n_y), e(t-1),\ldots e(t-n_e)) + e(t) \tag{5}$$

where $y(t)$ and $e(t)$ are the system output and prediction error, respectively; n_y and n_e are the maximum lags in the output and noise, respectively, $\{e(t)\}$ is assumed to be a white sequence, and $f(.)$ is some non-linear function. The identification procedure can be summarized as follows

(i) Choose n_y and n_e. Initially the set of

$$y(t) = [y(t-1) \cdots y(t-n_y)]^T$$

FBF model is selected using the LS algorithm and the initial model is used to generate the initial prediction error sequence $\{\varepsilon^{(0)}(t)\}$.

(ii) An iterative loop is then entered to update the model. At the k th iteration

$$y(t) = [y(t-1) \cdots y(t-n_y), \varepsilon^{(k-1)}(t-1) \cdots \varepsilon^{(k-1)}(t-n_e)]^T$$

FBF model is selected by LS algorithm and this gives rise to the prediction error sequence $\{\varepsilon^{(k)}(t)\}$. Typically two to four iterations are sufficient. [5]

The model validity tests are performed to assess the model. If the model is considered adequate the procedure is terminated. Otherwise go to step(i).

4. Experiment Results

As an experiment, the first 700 points of data generated from the Eq.(6) are used to identify the fuzzy NARMA based on the proposed approach. Then, the next 100 points of data are used to test the fuzzy NARMA. For comparison purpose, the same data set is also used to identify the fuzzy NAR model. Then, the chi-squared test and autocorrelations of residuals are used to validate both models.

The results are given in Table 1 for both fuzzy NAR model and fuzzy NARMA model. The outputs of simulation and the model response are shown in Figs 2. and 3. The correlation tests and chi-squared tests are shown in Figs 4. and 5. respectively.

From the results, the fuzzy NARMA model provides lower mean square error (MSE) and better standard deviation (STD) of errors than the fuzzy NAR model. According to the chi-squared tests and the correlation tests in Figs 2, 3, 4 and 5, the fuzzy NARMA model is better than the fuzzy NAR model.

$$y(t) = 1.4y(t-1)\exp\left(\frac{-y^2(t-1)}{6}\right) + e(t) + 0.9e(t-1)\exp\left(\frac{e^2(t-1)}{3}\right) \tag{6}$$

Table 1.Predicive modeling results

MODEL	MSE		STD	
(Noise STD=0.5057)	TRAINING	TESTING	TRAINING	TESTING
Fuzzy NAR Model	0.3168	0.3969	0.5652	0.6297
Fuzzy NARMA Model	0.2496	0.3188	0.4999	0.5619

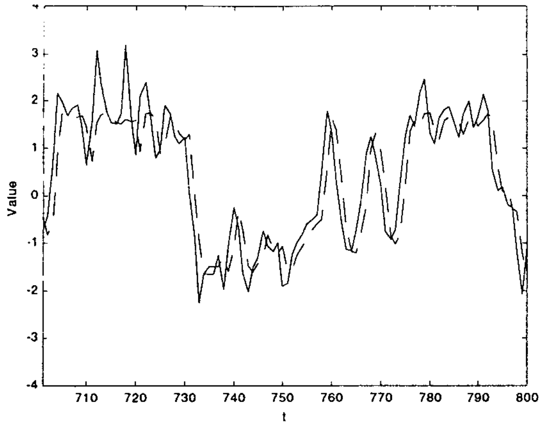

Fig 2. Outputs of simulation (solid line) and Fuzzy NAR Model (dashed line)

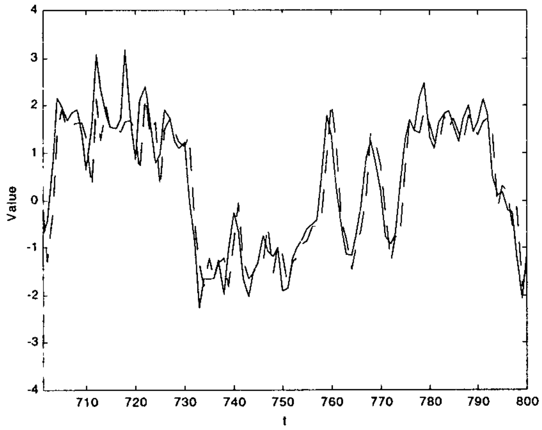

Fig 3. Outputs of simulation (solid line) and the identification model : Fuzzy NARMA Model (dashed line)

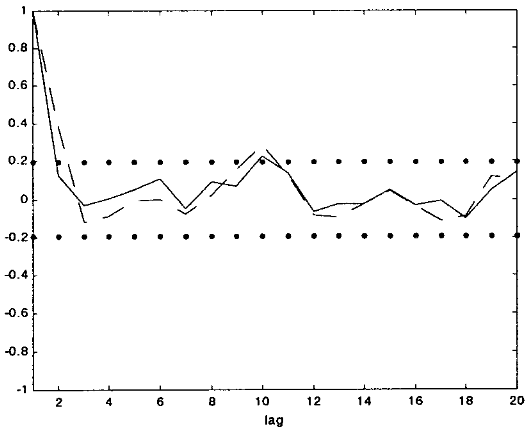

Fig 4. Autocorrelations of residuals; dotted line is 95 % confidence band, dashed line is the correlation of the Fuzzy NAR Model and solid line is the correlation of the Fuzzy NARMA Model

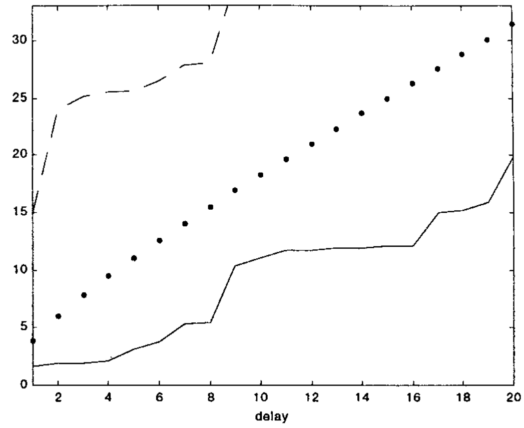

Fig 5. Chi-squared tests; $\omega(t) = e(t-1)$; Dotted line is 95 % confidence limit, dashed line is the value of ζ of the Fuzzy NAR Model and solid line is the value of ζ of the Fuzzy NARMA Model

5. Conclusions

In this paper, an approach to identify NARMA models based on fuzzy basis functions is proposed. From the simulation results, the fuzzy NARMA model successfully captures the behavior of the NARMA In the future, the further comprehensive study and testing the proposed model with complex time series model will be reported.

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