Proceedings of ITC-CSCC 2000, Pusan, Korea

A Neuro-Fuzzy System Reconstructing Nonlinear Functions from Chaotic Signals

Kei EGUCHI†, Fumio UENO†, Toru TABATA†, Hong Bin ZHU††, and Kaeko NAGAHAMA†

† Dept. of Information and Computer Sciences, Kumamoto National College of Technology,
Kumamoto-ken, 861-1102 Japan
Tel. : +81-96-242-6072
E-mail : eguti@eecs.kumamoto-u.ac.jp
†† Dept. of Electrical and Computer Engineering, Kumamoto University,
Kumamoto-shi, 860-8555 Japan

Abstract

In this paper, a neuro-fuzzy system for quantitative characterization of chaotic signals is proposed. The proposed system is differ from the previous methods in that the nonlinear functions of the nonlinear dynamical systems are calculated as the invariant factor. In the proposed neuro-fuzzy system, the nonlinear functions are determined by supervised learning. From the reconstructed nonlinear functions, the proposed system can generate extrapolated chaotic signals. This feature will help the study of nonlinear dynamical systems which require large number of chaotic data. To confirm the validity of the proposed system, nonlinear functions are reconstructed from 1-dimensional and 2-dimensional chaotic signals.

1 Introduction

Chaos is the most frequently encountered phenomenon in the study of nonlinear dynamical systems such as biological neural networks, power systems, and so on. To analyze these nonlinear dynamical systems which exhibit chaotic behavior, two major approaches are studied. One of these approaches is a system modeling. For example, many physical, biological and chemical processes are modeled by means of large ensembles of interacting chaotic cells [1]. Among others, Pivka has modeled the traveling waves by using resistor-coupled Chua's circuits [2]. Another approach is a quantitative characterization by extracting the invariant factors from the chaotic signals which are generated from nonlinear dynamical systems. In the quantitative characterization of the chaotic signals, the metric entropy, the dimensions and the spectrum of Lyapunov exponents, etc. have been used as the invariant factors [3],[4]. For example, Murayama et al. analyzed voluntary functions in the upper limbs using fractal dimensions [5]. Although the quantitative characterization and classification can be achieved effectively by using these invariant factors, we focused on the nonlinear functions as an invariant factor.

In this paper, a neuro-fuzzy system for quantitative characterization of chaotic signals is proposed. The proposed method is differ from the previous methods in that the nonlinear functions of the nonlinear dynamical systems are calculated as the invariant factor. Hence, by exploiting the reconstructed nonlinear functions, the proposed system can generate extrapolated chaotic signals. This feature will help the study of nonlinear dynamical systems which require large number of chaotic data such as chaos-control [6]. In the proposed system, the nonlinear functions are reconstructed by supervised learning. Furthermore, thanks to the efficient supervised learning, the proposed system can reconstruct the nonlinear functions from small number of input data. The validity of the proposed system is confirmed by numerical simulations.

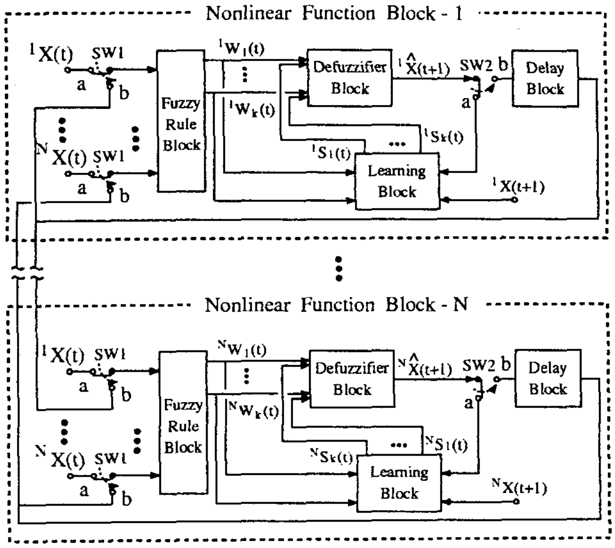

Fig.1 General architecture of the proposed neuro-fuzzy system.

2 Architecture

Figure 1 shows a general architecture of the proposed neuro-fuzzy system. The proposed system consists of N nonlinear function blocks. The nonlinear function

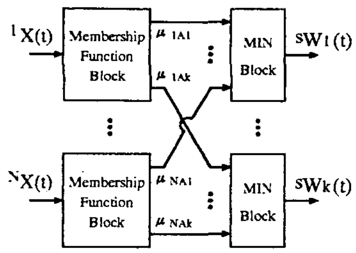

Fig.2 Architecture of the fuzzy rule block.

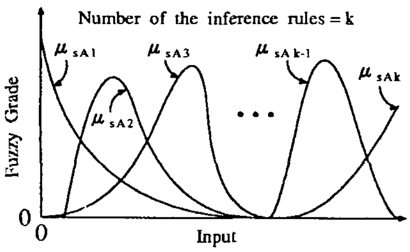

Fig.3 Membership functions.

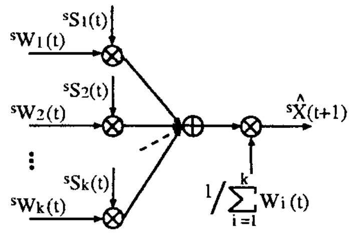

Fig.4 Architecture of the defuzzifier block.

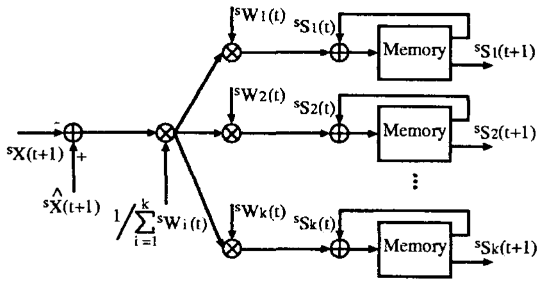

Fig.5 Architecture of the learning block.

block is built with a neuro-fuzzy based system consists of a fuzzy rule block, a defuzzifier block, a learning block, and a delay block. The nonlinear function blocks are in the learning process when the switches SW1's and SW2's are in the terminals **a**'s. After the learning process finished, the position of these switches is reversed. Then, the nonlinear function blocks function as the nonlinear functions, $F_s(\cdot)$ $(s = 1, \ldots, N)$.

2.1 Fuzzy Rule Block

The inference rules FR_i's used in the nonlinear function block are given by the form:

$$FR_i : \text{If } x \text{ is } A_i \text{ then } y \text{ is } B_i, \quad (i = 1, 2, \ldots, k) \quad (1)$$

where x is an input variable, y is an output variable, A_i is a fuzzy set defined by the membership functions, and B_i is a fuzzy singleton, $1/S_i(t)$.

Figure 2 shows the architecture of the fuzzy rule block. In the fuzzy rule block, the matching degrees $W_i(t)$'s are determined by the following equation:

$$W_i(t) = \min\ (\mu_{{}^1A_i}({}^1x(t)), \ldots, \mu_{{}^NA_i}({}^Nx(t))), \quad (i = 1, 2, \ldots, k) \quad (2)$$

where ${}^1x(t)$, ${}^2x(t)$, ..., and ${}^Nx(t)$ are the input values, ${}^sA_i$ $(s = 1, \ldots, N$ and $i = 1, \ldots, k)$ denotes the fuzzy label for the input ${}^sx^*$ in the ith fuzzy rule, $\mu_{{}^1A_i}$, $\mu_{{}^2A_i}$, ..., and $\mu_{{}^NA_i}$ are the membership functions as shown in Fig.3, and k is the number of the inference rules. In Fig.2, the membership function block calculates the membership functions. The MIN block realizes the minimum operation in Eq.(2).

2.2 Defuzzifier Block

Figure 4 shows the architecture of the defuzzifier block. In the defuzzifier block, the output fuzzy set, ${}^sW_1(t)/{}^sS_1(t) + \ldots + {}^sW_k(t)/{}^sS_k(t)$, is defuzzified by the center of area (COA) method, where ${}^sS_i(t)$ is the singleton's element, / is Zadeh's separator, and + is the union operation. The output of the defuzzifier block, ${}^s\hat{x}(t+1)$, corresponds to an estimated $t+1$th chaotic signal. The defuzzified output ${}^s\hat{x}(t+1)$ is given by

$${}^s\hat{x}(t+1) = \frac{\sum_{i=1}^{k} {}^sS_i(t)\ {}^sW_i(t)}{\sum_{i=1}^{k} {}^sW_i(t)}, \quad (s = 1, 2, \ldots, N) \quad (3)$$

where t denotes the count of the learning cycle.

2.3 Learning Block

Figure 5 shows the architecture of the learning block. The singleton's elements ${}^sS_i(t)$'s are determined in the learning block. In the learning process, the $t+1$th chaotic signals are given as the supervisor signals corresponding to the respective sets of sample inputs, ${}^1x(t)$'s, ${}^2x(t)$'s, ..., and ${}^Nx(t)$'s. The learning dynamics is expressed by the following equation:

$${}^sS_i(t+1) = {}^sS_i(t) + \frac{({}^s\hat{x}(t+1) - {}^sx(t+1))\ {}^sW_i(t)}{\sum_{i=1}^{k} {}^sW_i(t)}, \quad (4)$$

where ${}^sx(t+1)$ is the $t+1$th chaotic signal which is given as a supervisor signal. The singleton's elements ${}^sS_i(t+1)$ of Eq.(4) is stored in the memory. The learning process terminates when

$$|\ {}^sS_i(t+1) - {}^sS_i(t)| < \epsilon, \quad (5)$$

where ϵ is a parameter.

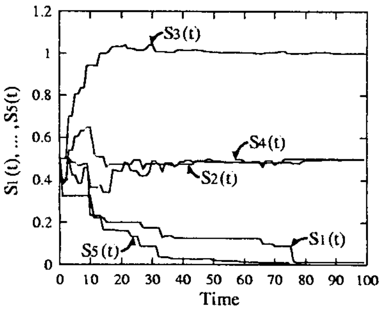

Fig.6 Convergence behaviors of the singletons.

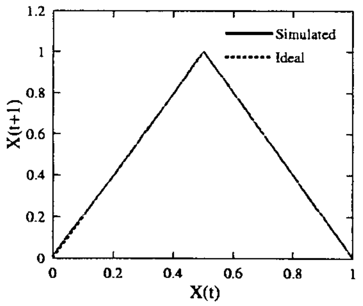

Fig.7 Nonlinear function reconstructed.

2.4 Delay Block

After the learning process finished, the nonlinear function blocks operate as a chaos generator. The proposed system generates the chaotic signals from the nonlinear functions which are determined by the supervised learning. The delay block produces a unit delay and feeds the delayed output.

3 Simulation

To confirm the validity of the proposed algorithm, numerical simulations were performed. In the numerical simulations, two types of the nonlinear functions are reconstructed from the chaotic signals. The chaotic signals are generated from the following equations:

$$\text{Tent map}: \ {}^1x(t+1) = 1-2|{}^1x(t)-0.5|, \quad (6)$$

$$\text{Hénon map}: \ {}^1x(t+1) = 1+{}^2x(t)-1.4\,{}^1x(t),$$
$$ {}^2x(t+1) = 0.3\,{}^1x(t). \quad (7)$$

These maps are the most famous nonlinear maps for which a rigorous proof of the chaotic behavior has been accomplished.

Firstly, the numerical simulations for the tent map were performed. In numerical simulations for the tent map, the following membership functions were used.

- Membership functions for the tent map:

$$\mu_{Ai}(x) = 1-4|x-(1/4)(i-1)|, \quad (i=1,\ldots,5) (8)$$

Figure 6 shows the convergence behavior of the singletons for the tent map. In Figs.6, the singleton's element values converged to their final values after 100 learning cycles. This result means that the proposed system requires only 100 input data to reconstruct the nonlinear function of the tent map. Figure 7 shows the examples of the reconstructed nonlinear functions obtained by using the final values of the singletons of Fig.6. Figure 8 shows the examples of the chaotic signals obtained by the reconstructed nonlinear functions in Fig.7. In Fig.8, the initial value, X_0, was set to 0.1.

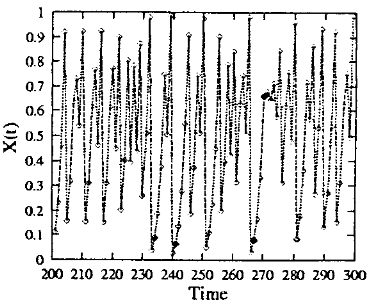

Fig.8 Chaotic signal generated from the reconstructed nonlinear function in Fig.7.

Secondly, numerical simulations for the Hénon map were performed. In numerical simulations for the Hénon map, the following membership functions were used.

- Membership functions for a Hénon map:

$$\mu_{{}^1Ai}({}^1x) = 1-3|{}^1x+(1/3)(i-11/2)|,$$
$$\mu_{{}^2Ai}({}^2x) = 1-9|{}^2x+(1/9)(i-11/2)|,$$
$$(i=1,\ldots,10). \quad (9)$$

Figure 9 shows the examples of the reconstructed nonlinear functions for the Hénon map.

Figure 10 shows the examples of the strange attractor obtained by the reconstructed nonlinear functions in Fig.9. In Fig.10, the initial values, X_0 and Y_0, were set to 0.1 and 0.1, respectively. As one can see from Figs.8 and 10, the proposed system can generate extrapolated chaotic signals from the reconstructed nonlinear functions. The precision of the reconstructed nonlinear function depends on the number of the singletons and the parameter ϵ. The error of the reconstructed nonlinear function can be reduced by increasing the number of the singletons.

4 Conclusion

In this paper, a neuro-fuzzy system reconstructing nonlinear functions from chaotic signals has been proposed. The simulation results showed that the proposed system can reconstruct the nonlinear functions from the chaotic signals and can generate the chaotic signals from the reconstructed nonlinear functions.

The circuit implementation of the proposed system is left to the future study.

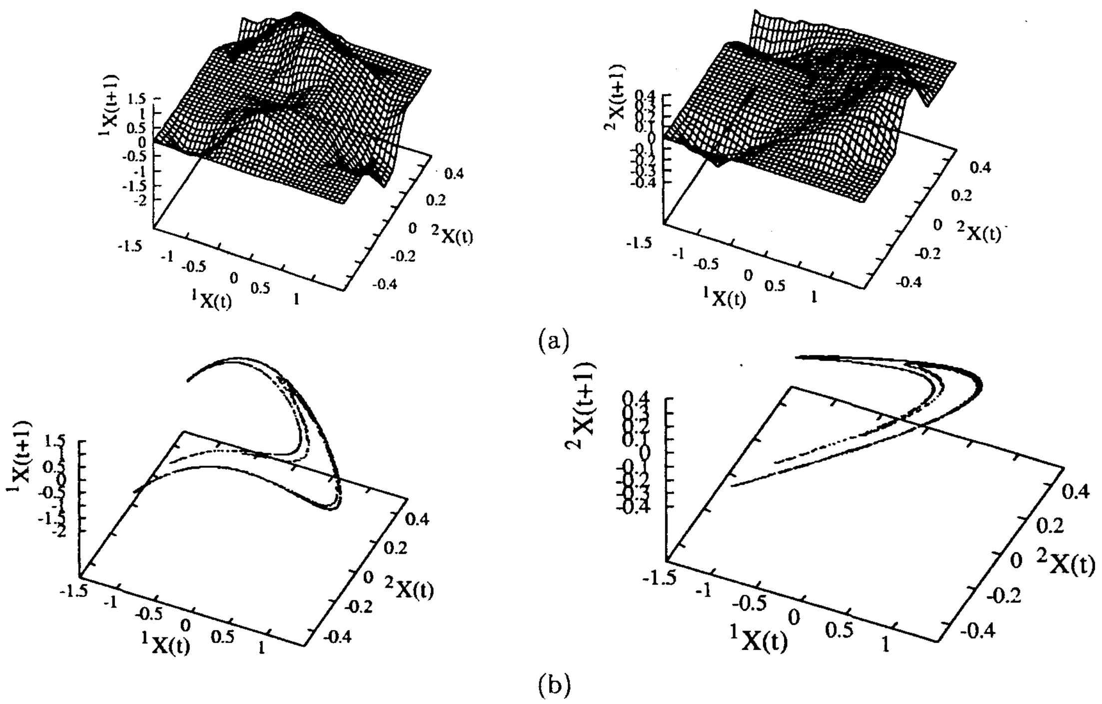

Fig.9 Nonlinear functions for a Hénon map. (a) Reconstructed nonlinear functions. (b) *Ideal* nonlinear functions obtained from Eq.(7).

References

[1] A.M.Dabrowski, W.R.Dabrowski, and M.J.Ogorzalek, "Dynamic phenomena in chain interconnections of Chua's circuits," IEEE Trans. Circuits & Syst.-I, vol.40, no.11, pp.868-871, Nov. 1993.

[2] L.Pivka, "Autowaves and spatio-temporal chaos in CNNs-part I: a tutorial," IEEE Trans. Circuits & Syst., vol.42, no.10, pp.638–649, Oct. 1995.

[3] T.Kohda and K.Murao, "Approach to time series analysis for one-dimensional chaos based on Frobenius-Perron operator," Trans. IEICE Japan, vol.E73, no.6, pp.793–800, Jun. 1990.

[4] A.I.Mees, "Rarsimonies dynamical reconstruction," Int. Journal of Bifurcation and Chaos, vol.3, pp.669–675, Jun. 1993.

[5] T.Igasaki, N.Murayama, R.Nakanishi, T.Mitsuishi, C.Okumura, F.Uwatoko, "Quantitative analysis of voluntary functions in the upper limbs using fractal dimensions", The XI International Congress of EMG and Clinical Neurophysiology, Prague, Czech Republic, 1999.

[6] K.Yuki, Y.Namba, and M.Miyazaki, "Controlling chaos for 3-dimensional unknown systems," Trans. IEE Japan, vol.119-C, no.6, pp.684–690, Jun. 1999.

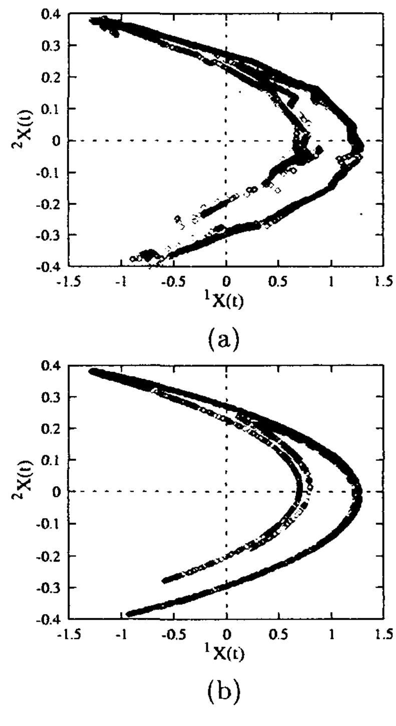

Fig.10 Strange attractor of the Hénon map. (a) Generated from the reconstructed nonlinear function. (b) Generated from the ideal nonlinear function.