# RECONSTRUCTION OF LIMITED-ANGLE CT IMAGES BY AN ADAPTIVE RESILIENT BACK-PROPAGATION ALGORITHM

*Kazunori Matsuo*, Zensho Nakao*, Yen-Wei Chen*, & Fath El Alem F. Ali*†

*Department of Electrical & Electronics Engineering
Faculty of Engineering, University of the Ryukyus
Okinawa 903-0213, Japan
Phone: (+81) 98-895-8698 Fax: (+81) 98-895-8708
Email: {matsuo,nakao}@augusta.eee.u-ryukyu.ac.jp
†Department of Management and Infomation Systems
Meio University, Okinawa 905-8585, Japan

## ABSTRACT

A new and modified neural network model is proposed for CT image reconstruction from four projection directions only. The model uses the Resilient Back-Propagation (Rprop) algorithm, which is derived from the original Back-Propagation, for adaptation of its weights. In addition to the error in projection directions of the image being reconstructed, the proposed network makes use of errors in pixels between an image which passed the median filter and the reconstructed one. Improved reconstruction was obtained, and the proposed method was found to be very effective in CT image reconstruction when the given number of projection directions is very limited.

## 1. INTRODUCTION

In computed tomography (CT) applications, some projection data of an unknown multi-dimensional distribution (target image) are assumed to be given. Several techniques are known for reconstructing the original image by manipulating the projection data(Figure 1). When the number of projection directions is limited to three or four, Genetic Algorithms (GAs)[3], Algebraic Reconstruction Technique (ART), and Simulated Annealing have been used for reconstruction to date.

Here, a neural network model is applied to gray CT image reconstruction from only four projection data, and is based on the previously proposed Back-Propagation (BP) Algorithm based method[2]. The new and modified neural network model uses the Resilient Back-propagation (Rprop)[5] to modify the weights instead of BP. Rprop, which is derived from BP, can find global minima faster than BP and avoid local minima. In addition to Rprop, the system adopts the median filter as another teaching signal[4]. By using the filter, it can handle clusters of pixels of an image.

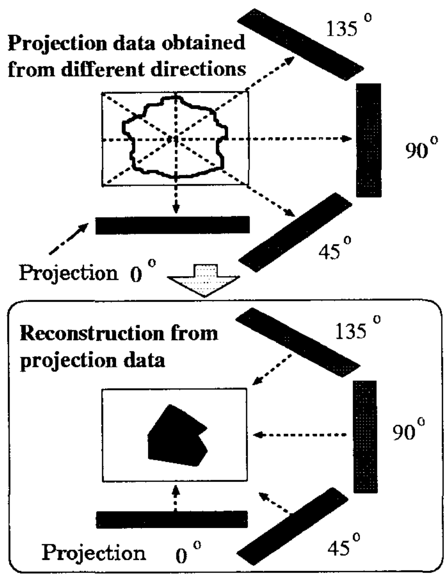


Figure 1: Reconstruction of an Image from Projections

## 2. IMAGE RECONSTRUCTION SYSTEM

### 2.1. Structure of the Network

The system for the reconstruction of the images which we use is structured with three layers of neurons, as is depicted in Figure 2. The number of the neurons of the first layer is equal to the projection data which we get from an original image. The next layer is the hidden layer and has a same number of the neurons as the pixels of an image.

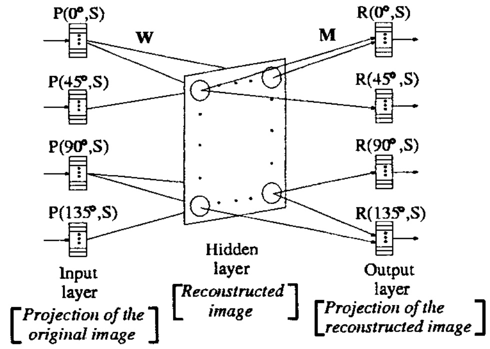


Figure 2: The Structure of the Network

The number of neurons of the output layer is the same as that of the input layer. The output from the output layer is the projection data from the reconstructed image.

The connection weights between the first and the second layer are denoted by $w_{ji}$, which is adapted through the Rprop learning process. The weights between the second and the third layer are denoted by $w_{kj}$, which are kept constant through the learning process. The weights $w_{kj}$ are determined by the method explained in section 2.2, and the projection data can be computed from the second layer, and entered to the third layer.

This system is based on the simple idea that if the error between the projections of the original image and of the reconstructed image is small, the reconstructed image will be similar to the original image. The input to the system is the projection data of the original unknown image as an input to the first layer.

## 2.2. Weight Initialization

The weights $w_{kj}$, which connect the second and the third layer, are used for computing the projection data of the reconstructed image as input to the output layer. The method for initialization of the weights is explained with Figure 3. Suppose that 25(5×5) squares give one image, each square representing one pixel. The image on the left is before rotation and the one on the right is after rotation. For 0 degree projection the coordinates (4.0, 2.0) of the pixel marked do not change and remain on the same point at 0 degree, and this pixel will project the entire area (1.0) to the 4th box under the image and this value is taken as a weight for the 0 degree projection.

After the 45 degree rotation, the pixel (4.0, 2.0) moves to the coordinates (4.4, 3.0). This pixel is projected into two separate boxes with area ratio 0.6 and 0.4. That is to say, the pixel on the coordinate (4,0,2.0) is related to the pixel

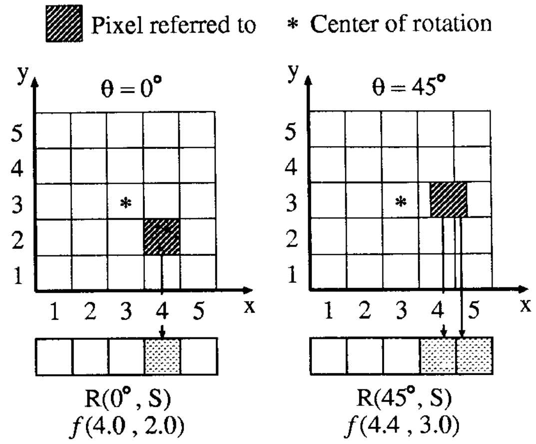


Figure 3: The Initialization of the Weights

on $R(45°, 4), R(45°, 5)$ (these representing the 4th and the 5th projection data for 45 degree projection direction, respectively) with weights 0.6 and 0.4. Likewise, weights of all the pixels are determined and computed for all directions.

## 2.3. The Learning Method of the Network

The network is to renew the weights with Rprop. Suppose that $y_i$ is the output of the $i$-th unit of the input layer and $w_{ji}$ is the connection weight between nodes $i$ and $j$, then the output $y_j$ of the $j$-th unit in the hidden layer is given by

$$y_j = \frac{1}{1 + exp(-x_j)}, \qquad y_j = \sum_i y_i w_{ji}$$

The output $y_k$ of the $k$-th unit in the output layer becomes

$$y_k = \sum_j y_j m_{kj}$$

and, the corresponding error function $E_k$ is given by

$$E_k = \frac{1}{2}(y_k - d_k)^2$$

and, the total error becomes

$$E = \sum_k E_k$$

Using the total error $E$, the weights $w_{ji}$ are renewed with the Rprop algorithm:

$$\frac{\partial E}{\partial w_{ji}} = \sum_k [(y_k - d_k) w_{kj}] y_j (1 - y_j) y_i$$

$\forall\ i, j : \Delta_{ji}(t) = \Delta_0$
$\forall\ i, j : \frac{\partial E}{\partial w_{ji}}(t-1) = 0$
**Repeat**

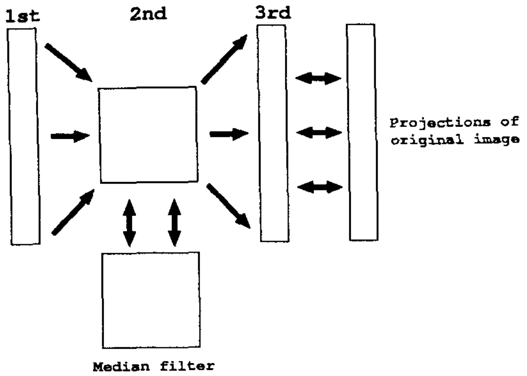


Figure 4: The system with the median filter

**Compute Gradient** $\frac{\partial E}{\partial w_{ji}}(t)$
**For all weights and biases**{
  **if** $(\frac{\partial E}{\partial w_{ji}}(t-1) * \frac{\partial E}{\partial w_{ji}}(t) > 0)$ **then** {
    $\Delta_{ji}(t) = \textbf{minimum}(\Delta_{ji}(t-1) * \eta^+, \Delta_{max})$
    $\Delta w_{ji}(t) = -\ \textbf{sign}\ (\frac{\partial E}{\partial w_{ji}}(t)) * \Delta_{ji}(t)$
    $w_{ji}(t+1) = w_{ji}(t) + \Delta w_{ji}(t)$
    $\frac{\partial E}{\partial w_{ji}}(t-1) = \frac{\partial E}{\partial w_{ji}}(t)$
  }
  **else if** $(\frac{\partial E}{\partial w_{ji}}(t-1) * \frac{\partial E}{\partial w_{ji}}(t) < 0)$ **then** {
    $\Delta_{ji}(t) = \textbf{maximum}(\Delta_{ji}(t-1) * \eta^-, \Delta_{min})$
    $\frac{\partial E}{\partial w_{ji}}(t-1) = 0$
  }
  **else if** $(\frac{\partial E}{\partial w_{ji}}(t-1) * \frac{\partial E}{\partial w_{ji}}(t) = 0)$ **then** {
    $\Delta w_{ji}(t) = -\ \textbf{sign}\ (\frac{\partial E}{\partial w_{ji}}(t)) * \Delta_{ji}(t)$
    $w_{ji}(t+1) = w_{ji}(t) + \Delta w_{ji}(t)$
    $\frac{\partial E}{\partial w_{ji}}(t-1) = \frac{\partial E}{\partial w_{ji}}(t)$
  }
}
**Until (converged)**

## 2.4. Enhancement of the System

### *2.4.1. The Resilient Back-Propagation System with Median Filter*

The new network which is proposed here is explained with Figure 4. In this case, the network adopts the Median filter.

In the older version of the system, only the projection data of the unknown image was used as a teaching signal. This system has two teaching signals, not only for the output layer but also for the hidden layer on which the reconstructed image appears. The reconstructed image passes the median filter and the resultant image is used as a teaching signal. The learning method with the median filter is explained below. Suppose that $y_j$ is the the $j$-th node in the hidden layer (a pixel of the reconstructed image) and $g_j$ is a $j$-th pixel value of the reconstructed image which passed the median filter, and that the total error is given by:

Table 1: Parameters

| | |
|---|---|
| $\alpha$ | $5.0e^{-7}$ |
| $\Delta_0$ | $5.0e^{-3}$ |
| $[\Delta_{min}, \Delta_{max}]$ | $[0.5, 5.0e^{-8}]$ |
| $[\eta^-, \eta^+]$ | $[0.5, 1.2]$ |
| $\beta$ | $5.0e^{-6}$ |

$$E_{(m)j} = \frac{1}{2}(y_j - g_j)^2$$

The sum of $E_{(m)j}$ is the error of the image:

$$E_{(m)} = \sum_j E_{(m)j}$$

and $w_{ji}$ is renewed as follows:

$$\begin{aligned} w'_{ji} &= w_{ji} \pm \Delta_{ji} - \beta \frac{\partial E_{(m)}}{\partial w_{ji}} \\ &= w_{ji} \pm \Delta_{ji} - \beta \frac{1}{N}(y_j - g_j) y_i \end{aligned}$$

where the parameter $\beta$ is a scaling constant.

### *2.4.2. Examination of the parameter*

We optimize the parameter empirically. Now, The initial value of weight $W_{ji}$ is determined by using the values of weight $M$. The magnification $M2W\_Bias$ is an important parameter:

$$W_{ji(init)} = M_{kj} * M2W\_Bias$$

First of all, the system makes about 100 trials. It repeats again with changing the parameter value. Thus an optimal value of the parameter is searched

The efficacy of the parameter is demonstrated by experiments.

## 3. EXPERIMENTS AND THE RESULTS

We show reconstruction results for two images. The number of the projections is four at angles $0°$, $45°$, $90°$, and $135°$.

Figures 5 and 6 show reconstruction results, and the simulation was done with the parameter values in Table 1. The proposed system reconstructs the images satisfactorily despite the smaller number of projection directions. The system with optimized parameter brings about good results, even without the median filter.

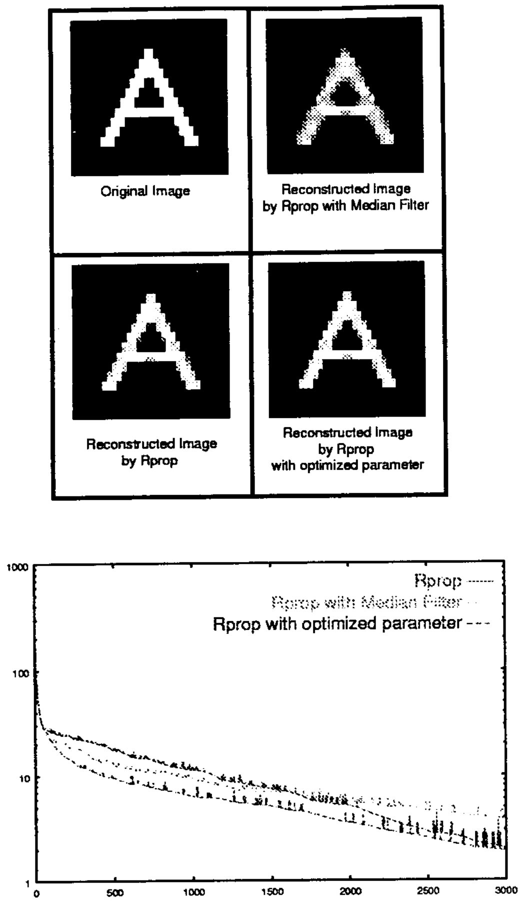


Figure 5: Results(A)

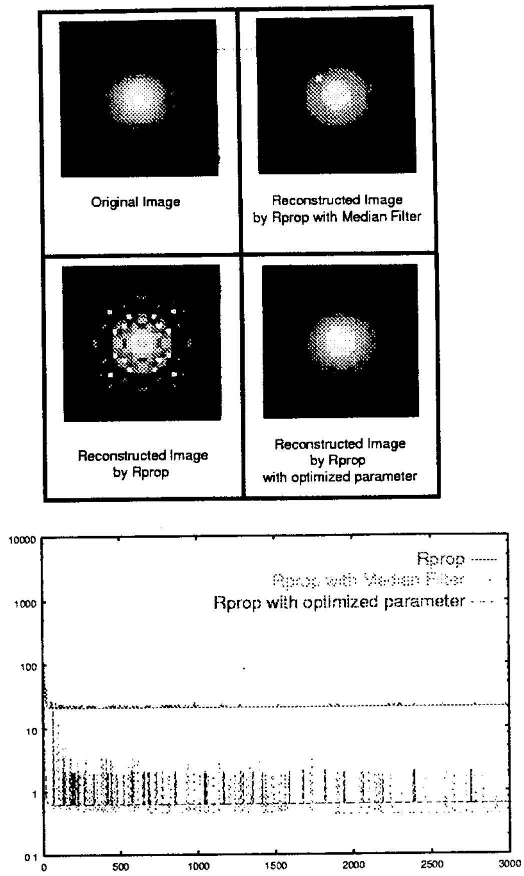


Figure 6: Results(Gauss)

## 4. CONCLUSIONS AND FUTURE WORK

Effectiveness of the Resilient Backpropagation system with/without median filter was demonstrated. Performance of the Resilient Backpropagation could be improved by optimizing the magnification parameter. Adopting a new algorithm for modifying the weight $w_{ji}$ and other type of filters are considered at present.

## REFERENCES


[1] Beale, R., & Jackson, T.: *Neural Computing: An Introduction*, IOP Publishing Ltd, 1990.

[2] Nakao, Z., Chen, Y.W., Tobaru, S., Ali, F.E.A.F., & Tengan, T.: CT image reconstruction by back propagation, *IASTED International Conference on Artificial Intelligence, Expert Systems and Neural Networks* (Honolulu, Hawaii, USA), pp.285-286, 1996.

[3] Nakao, Z., Chen, Y.W., & Ali, F.E.A.F.: Evolutionary reconstruction of plane binary images from projections, *J. Jpn. Soc. Fuzzy Theo. and Sys.*, vol.8, No.4, pp.687-694, 1996.

[4] Ohkawa, I., Tobaru, S., Nakao, Z., & Y.W., Chen: Reconstruction of CT Images by the Back-Propagation Algorithm, *The 2nd International Conference on Knowledge-based Intelligent Electronic Systems* (Adelaide, Australia), Vol.3,pp.150-154, 1998.

[5] Riedmiller, M., & Brain, H.: A Direct Adaptive Method for Faster Backpropagation Learning: The RPROP Algorithm, *IEEE International Conference on Neural Networks* (San Francisco, CA, U.S.A.), 1993.

[6] Matsuo, K., Nakao, Z., Y.W., Chen., & Ohkawa, I: Reconstruction of CT Images by the Resilient Back-Propagation Algorithm, *Proc of IWSCI'99* (Muroran, Japan),pp.198-203, 1999.