# Performance Evaluation on Throughput of a Petri Net Modeled Food Business

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**Abstract:** A workflow expresses the flow of persons and things related to a business. To improve efficiency of a business, it is important to grasp and evaluate the actual situation of the current business. Till now, researches on workflows have been done almost on office business [1] and these results can not be simply applied to food business. Besides, it is also important how to evaluate a food business workflow with a specific standard. In this paper, we propose a modeling method of food business by using hierarchical Petri net. Then we propose a concept, called throughput, as a standard to evaluate the workflows. Finally we show a method how to compute throughput and meanwhile apply a Petri net tool, Design/CPN, to do simulation of computing throughput. Our simulation result shows the modeling method and computation method of throughput are reasonable and useful.

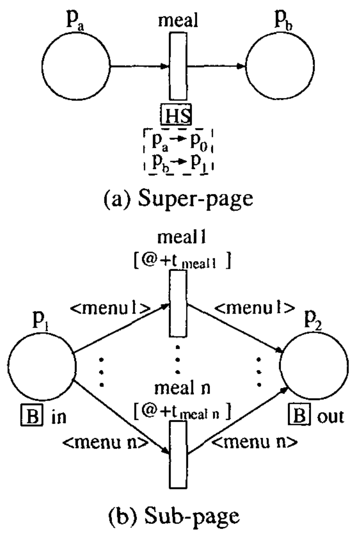



Figure 1: Hierarchical Petri Net

## 1. Introduction

A workflow expresses the flow of persons and things related to a business. To improve efficiency of a business, it is important to grasp and evaluate the actual situation of the current business. Till now, researches on workflows have been done almost on office business and these results can not be simply applied to food business. This is because, in a workflow of food business there are many factors to be considered, such employee, customer and multiple kinds of menus and cookings, and further theses factors are complicatedly interconnected. Therefore we need a new method to model the workflow and further to model it from global and local point of views in order to grasp its current situation. On the other hand, to improve the current business we must first evaluate whether the current business is efficient or not. Thus a standard to evaluate the current business is also important.

In this paper, we propose a modeling method of food business by using both colored timed Petri net and hierarchical Petri net. Then we propose a concept, called throughput, that is considered as a standard to measure the efficiency as well as the processing ability of workflows. Finally we show a method how to compute throughput. And meanwhile we apply a Petri net tool, Design/CPN, to do simulation of computing throughput in order to verify if our computation method is useful or not.

## 2. Definition and Modeling

We are to apply timed colored Petri net [2][3] to model the workflow of food businesses, since colored Petri net can express different flow of executions of systems, such the multiple flows in a food business.

**Definition 1:** A timed colored Petri net TCPN is a timed Petri net $TCPN=(\Sigma, P, T, A, E, I, R)$, where, $\Sigma$ is a set of color of tokens, $P$ is a set of places representing queue of token, $T$ is a set of transitions representing process and change of state, $A$ is a set of arcs representing connection relations between places and transitions, $E$ is a function shows color of tokens that arc passes, $I$ is a function shows initial marking, $R$ is a function shows time that needs to fire of transition. □

In a TCPN, each flow can be represented as a subnet having individual color, and all flows are folded into one net. Thus modeling workflows by colored Petri nets can make it easy to grasp the multiple flows inside of a workflow. We have done investigation for an actual restaurant that has the elements as shown in the following.

- Process: The task and action such as "order" and "cook".
- Object: Persons and things that are processed (e.g. customer).
- Worker: Persons who carry out the processes (e.g. employee).
- Flow of process: The order of process.
- Processing time: The time needed in executing the processes.
- Kind of food: Kind of food that customers ordered.

These elements can be represented by the clauses of TCPN. That is, transition, token, time function and token color represent respectively the Process, Object, Worker, Processing time, and Kind of food. On the other hand, generally such a modeled workflow is too detailed and large to grasp the main flow of the business. To solve this problem, we use hierarchical net structure [4][5]. The hierarchical Petri net has multiple layers, each its layer consists of one or more timed colored Petri nets. We call the top layer as super-page and call the others as sub-pages. A sub-page corresponds to a transition in the super-page. Figure 1 shows an example a super-page and the sub-page. In this figure, the processing called "meal" is represented by one transition in a super-page, which consists of plural transitions with different time functions for different kinds of foods in sub-page. Using hierarchical Petri net, the whole workflow can be explicitly expressed and analyzed easily for the super-page as well as the sub-pages if needed.

Figure 2 shows the hierarchical Petri net model of the workflow of the restaurant that we have investigated. In this figure, after the transition "order", tokens are firstly attached with colors and, each colored token then moves on its own decided way (sub-pages) throughout the net in turn, such as "cook", "carry" and "meal". Finally, tokens' colors are bleached at "payment".

## 3. Evaluation and Results

In this section, we first define a concept, "throughput", and then propose a computation method for it.

### 3.1 Definition of standard to evaluate

We have done an investigation for the same restaurant as modeled above by focusing its daytime business. Especially for the customer taking lunch, they wish to reach the table and finish the lunch as earlier as possible. On the other hand the customers are crowded and meanwhile the employees and the tables are limited at lunch time. Therefore compared with the quality of services it is ever important how to satisfy customers' such requirement. The way to solve this problem is to firstly evaluate the efficiency of current workflow to find out its bottleneck, and then to improve the workflow. Here We define the "throughput" of the workflow, and consider it as an important standard.

**Definition 2:** The throughput expresses processing ability of workflow of a restauant, and is defined as number of customers going out from the restaurant in unit time, when the customers are accumulating sufficiently. □

We are to obtain the maximum value of throughput from the super-page. The followings roughly show the steps of the computation.

(i) Dissolve the problem of competition of the resources to be assigned to plural transitions inside a sub-page. If the resources are used by plural sub-pages, distribute the resources to individual sub-page previously.

(ii) Compute processing ability of each transition. The processing ability is to be expressed by the number of tokens (object) that can be used in unit time by the transition.

(iii) Compute time function of each transition inside the super-page. This is done by measuring the processing ability of all the transitions of a sub-page.

(iv) Find the transition with maximum value of the time function in the super-page. Thus throughput is the reciprocal of this value.

### 3.2 Computation of throughput

First for step (i). Here resource competition means that, in a Petri net modeled workflow the resource tokens are placed at a same place and they provide firings of plural transitions $t_1, t_2, \cdots, t_n$. At first we assume resource competition occurs only inside a single sub-page. Suppose $\lambda_i$ be the firing probability of transition $t_i$ under the condition that the sum of probabilities of total transitions inside a sub-page is 1 ($\sum_{k=1}^{n} \lambda_k = 1$). Then we divide the resources according to the rate of each transition $t_i$ that is computed by the following equation:

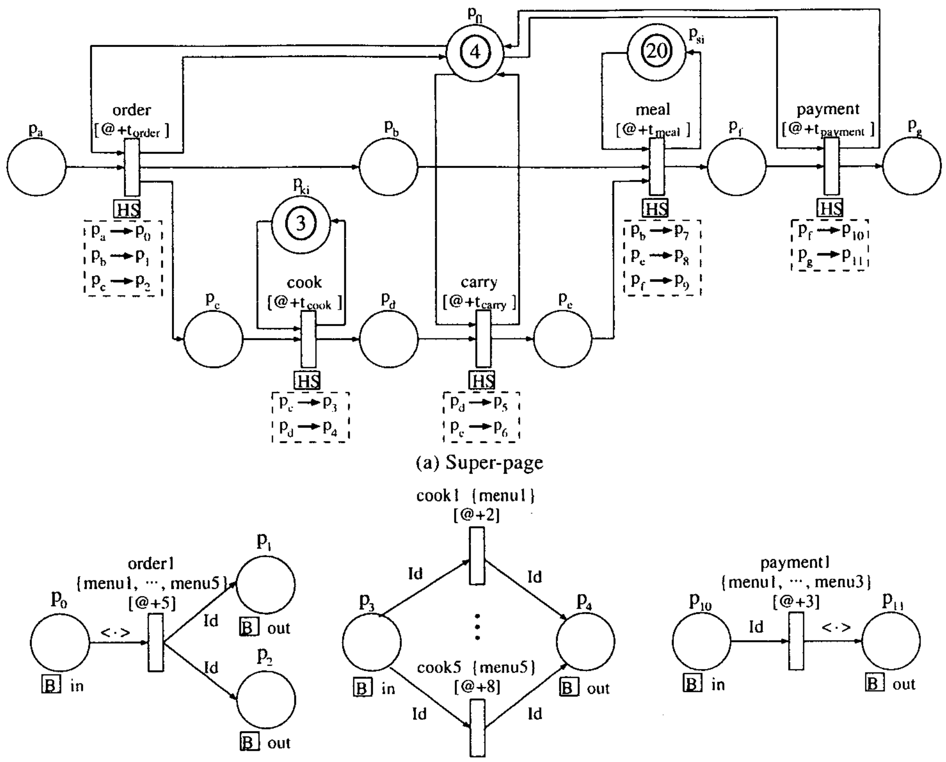
$$a_i = \frac{\lambda_i \tau_i}{\sum_{k=1}^{n} \lambda_k \tau_k} \quad (i = 1, 2, \cdots, n)$$

where $\tau_i$ is the time function of transition $t_i$.

Now we consider the case that resource competition occurs among plural sub-pages. Suppose there are two sub-pages, $X$ and $Y$, that use the same resources, then we determine the rates in order to distribute the resources to the two sub-pages as follows:

$$A_X = \frac{\sum_{k=1}^{n_X} \lambda_k \tau_k}{\sum_{i=1}^{n_X} \lambda_i \tau_i + \sum_{j=1}^{n_Y} \lambda_j \tau_j}$$

$$A_Y = \frac{\sum_{k=1}^{n_Y} \lambda_k \tau_k}{\sum_{i=1}^{n_X} \lambda_i \tau_i + \sum_{j=1}^{n_Y} \lambda_j \tau_j}$$




(i) Sub-page "order" (ii) Sub-page "cook" (iii) Sub-page "payment"

(b) Sub-page

Figure 2: Hierarchical Petri Net Model of Food Business

where $A_X$ and $A_Y$ are respectively for sub-pages $X$ and $Y$. These rates are so used in the distribution that if there are total $\gamma$ resources then $X$ and $Y$ occupy $\gamma \times A_X$ and $\gamma \times A_Y$ resources respectively. After distributing the resources to individual sub-pages, the resource competition problem become simple and we need only to solve it as occurring inside a single sub-page.

In step (ii), we compute the processing ability for each transition. The processing ability is denoted by $p_i$ and represented by the number of object (token) that can be processed per unit time by the transition $t_i$. If $t_i$ is assigned a single resource then it processes $1/\tau_i$ tokens per unit time. Since $\gamma a_i$ tokens can be assigned, the number of tokens $p_i$ that are processed by transition $t_i$ in unit time is as following:

$$p_i = \frac{a_i r}{\tau_i}$$

Here in step (iii), we compute time function of each transition inside a super-page. Previously by steps (i) and (ii), the processing ability of each transition inside individual sub-page is obtained, so we can compute total processing ability of a sub-page by the following equation:

$$P = \sum_{i=1}^{n} p_i$$

Therefore, the time needed in processing one token is

$$T = \frac{1}{P}.$$

This $T$ is treated as the time function of transition inside a super-page, which represents a sub-page.

In the last step, we compute the throughput. Stepping up here, a super-page becomes such one that is free of resource competition. So that tokens that entered to a super-page is only blocked at the transition that have the largest time function if there are not accumulated at any other transitions. Therefore, according to Definition 2, the throughput $H$ is as following.

$$H = \frac{1}{max(T_1, \cdots, T_n)}$$

## 4. Simulation and Discussion

To check the justifiability of our computation method of throughput, we have done experiments by simulating firing of Petri net modeled workflow to obtain throughputs and compare them with ones computed by our method. The modeled workflow used in the experiments is as shown in Fig. 2, and the parameters of the model are decided as will be described in the following. In the experiments, we change the value of token arrival interval time from large to small in order to find the maximum

Table 1: Processing Time

| kind of food / process | menu 1 | menu 2 | menu 3 | menu 4 | menu 5 |
|---|---|---|---|---|---|
| **order** | 5 | | | | |
| **cook** | 2 | 3 | 5 | 7 | 8 |
| **carry** | 2 | 3 | 3 | 4 | 5 |
| **meal** | 20 | | | | |
| **payment** | 3 | | | | |

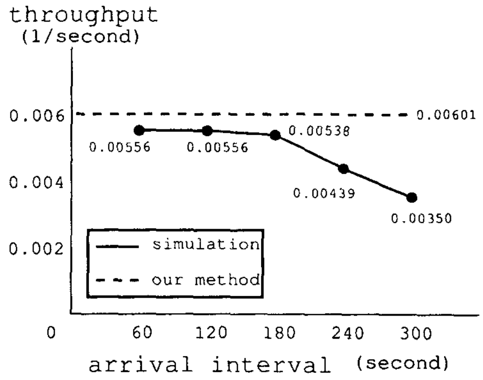



Figure 3: Comparison of the throughput

processing ability of the workflow, which is measured by tokens' output interval times. The followings are the explanation of the parameters.

- Simulation tool: Design/CPN version 4.0.1
- Number of resource: 4 employees in floor, 3 employees in kitchen and 20 seats are assumed, where employees in floor have to work for the processes of "order", "carry" and "payment", employees in kitchen work for process "cook" and seat are for process "meal".
- Order rate of cooking: $\lambda_1 = 0.3, \lambda_2 = \lambda_3 = \lambda_4 = 0.2, \lambda_5 = 0.1$
- Processing time: See Table 1.
- Arrival interval: $d$ =60, 120, 180, 240, 300 (second) are used.

Fig. 3 shows the experimental results and computed throughput by our method. As can be found from Fig. 3, the curve of simulation goes up with decreasing of token arrival interval and becomes almost no change when the arrival interval becomes smaller than 180 (second). This means the workflow can carry out its processing for one customer about every 180 seconds without asking customers to wait, i.e. it is the limit of the processing ability of the workflow. The throughput obtained by simulation is obviously 0.00556 and the difference between the simulation and our computation method is 0.00045. The ratio of the difference to the value of simulation is about 8%. Therefore we can say our method is reasonable and useful.

## 5. Conclusion

We have proposed a method to model workflow of food businesses by using both timed colored Petri net and hierarchical Petri net that can express both the top layer, super-page, and the detailed lower layers, sub-pages. To evaluate the efficiency and the capability, we have proposed a concept, called throughput, that can be used to measure the capacity how much a food business can provide service for customers. In order to easily obtain throughput, we have given a method that is carried out as: (i) determining the parameters of the super-page through analyzing the sub-pages; and (ii) finding the transition with minimum processing ability (can be also treated as bottleneck) in the super-page. Applying this method, we have evaluated workflow for an actual restaurant, meanwhile simulated the workflow in the sub-pages by Design/CPN. Comparing calculated values and the simulated ones, we find that the values by our method is quite near to the simulated ones, that is our method is useful and can be used in the evaluation of workflows. As our future work, we need to study how to change the workflows in order to improve the efficiency by applying the evaluation standard, throughput.

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