A Decoupling Method of Separable-Denominator Two-Dimensional Systems

Atsushi Kawakami
Computer and Control Systems Core
Division of Electrical Engineering, Faculty of Engineering
Kanazawa Institute of Technology
7-1, Ougigaoka, Nonoichi-machi, Ishikawa-gun, Ishikawa-ken, 921-8501 JAPAN
Phone: 076-294-6705 Ext-2288, Fax: 076-294-6707
E-mail: kit85003@kitnet.kanazawa-it.ac.jp

Abstract: The decoupling of the systems is to let the inputs and outputs correspond one to one, by performing the state feedback or the output feedback on the given systems. In this paper, we propose a method for decoupling the separable-denominator two-dimensional systems. And, we study about the realization dimension of the dynamical feedback and the dynamical feedforward performed for decoupling.

1. INTRODUCTION

The decoupling of the systems is to let the transfer function matrices of the multi-input multi-output systems be the invertible diagonal matrices, by performing the state feedback or the output feedback. That is to say, it is to let the inputs and outputs correspond one to one. This decoupling is the practical theory which was proposed in order to simplify the control of the tarboprop, boiler and many researches have been done since 1960's [1]-[3].

The consideration on the decoupling problem of the multivariable control systems with time delays was reported, too [4].

In this paper, we propose a method for decoupling the separable-denominator two-dimensional systems. We regard the one-dimensional transfer function matrices, which are calculated backward from the given separable-denominator two-dimensional systems, as the variable systems. Then, these variable systems are the realization of the separable-denominator two-dimensional transfer function matrices which are the outer description of the given two-dimensional systems, with respect to one variable. In the stage of these variable systems, we calculate the dynamical feedback, the dynamical feedforward performed for decoupling. Then, we study about the realization dimension of the dynamical feedback and the dynamical feedforward.

Methods proposed in this paper, can be applied to the decoupling of two-dimensional digital systems, mixed lumped and distributed circuits, delay-differential systems and circuits containing variable elements, etc.

Since many industrial multivariable systems (chemical processes, etc) actually involve multiple delays, the decoupling theory of two-dimensional systems should be valuable in designing controllers for such systems of any kind, i.e. optimal, suboptimal, modal, etc.

2. A DECOUPLING METHOD OF THE SEPARABLE-DENOMINATOR TWO-DIMENSIONAL SYSTEMS

2.1 The case of the first state-space realization form

We decouple the two-dimensional systems in the next equation.

$$\left\{\begin{aligned} \begin{bmatrix} \mathbb{X}_s \\ \hline \mathbb{X}_z \end{bmatrix} &= \left[\begin{array}{c|c} \mathbb{A}_{11} & \mathbb{O} \\ \hline \mathbb{A}_{21} & \mathbb{A}_{22} \end{array}\right] \begin{bmatrix} \mathbb{W}_s \\ \hline \mathbb{W}_z \end{bmatrix} + \begin{bmatrix} \mathbb{B}_1 \\ \hline \mathbb{B}_2 \end{bmatrix} \mathbb{U} \\ \mathbb{Y} &= \left[\begin{array}{c|c} \mathbb{C}_1 & \mathbb{C}_2 \end{array}\right] \begin{bmatrix} \mathbb{W}_s \\ \hline \mathbb{W}_z \end{bmatrix} + \mathbb{D}\mathbb{U} \end{aligned}\right. \tag{1}$$

Equation (1) has the outer description described by the transfer function matrix as in the next equation.

$$\mathbb{G}(s,z)/\{f_1(s)f_2(z)\} \qquad (2)$$

We rewrite (1) as in the next equation.

$$\begin{cases} \mathbb{X}_z = \mathbb{A}_{22}\mathbb{W}_z + [\mathbb{A}_{21} \mid \mathbb{B}_2]\begin{bmatrix}\mathbb{W}_s\\ \hline \mathbb{U}\end{bmatrix} \\ \begin{bmatrix}\mathbb{X}_s\\ \hline \mathbb{Y}\end{bmatrix} = \begin{bmatrix}\mathbb{O}\\ \hline \mathbb{C}_2\end{bmatrix}\mathbb{W}_z + \left[\begin{array}{c|c}\mathbb{A}_{11} & \mathbb{B}_1\\ \hline \mathbb{C}_1 & \mathbb{D}\end{array}\right]\begin{bmatrix}\mathbb{W}_s\\ \hline \mathbb{U}\end{bmatrix}\end{cases} \qquad (3)$$

From (3), we calculate the transfer function matrix as in the next equation.

$$\begin{bmatrix}\mathbb{X}_s\\ \hline \mathbb{Y}\end{bmatrix} = \left\{\begin{bmatrix}\mathbb{O}\\ \hline \mathbb{C}_2\end{bmatrix}(z\mathbb{I}_{np}-\mathbb{A}_{22})^{-1}[\mathbb{A}_{21} \mid \mathbb{B}_2] + \left[\begin{array}{c|c}\mathbb{A}_{11} & \mathbb{B}_1\\ \hline \mathbb{C}_1 & \mathbb{D}\end{array}\right]\right\}\begin{bmatrix}\mathbb{W}_s\\ \hline \mathbb{U}\end{bmatrix} \qquad (4)$$

$$\equiv \left[\frac{g_{ij}(z)}{f_{ij}(z)}\right]\begin{bmatrix}\mathbb{W}_s\\ \hline \mathbb{U}\end{bmatrix} \qquad (5)$$

We describe (5) as in the next equation.

$$\begin{bmatrix}\mathbb{X}_s\\ \hline \mathbb{Y}\end{bmatrix} = \left[\begin{array}{c|c}\mathbb{A} & \mathbb{B}\\ \hline \mathbb{C}(z) & \mathbb{D}(z)\end{array}\right]\begin{bmatrix}\mathbb{W}_s\\ \hline \mathbb{U}\end{bmatrix} \qquad (6)$$

We regard (6) as the variable system in the next equation.

$$\begin{cases}\mathbb{X}_s = \mathbb{A}\mathbb{W}_s + \mathbb{B}\mathbb{U}\\ \mathbb{Y} = \mathbb{C}(z)\mathbb{W}_s + \mathbb{D}(z)\mathbb{U}\end{cases} \qquad (7)$$

From (7), we calculate the two-dimensional transfer function matrix

$$\mathbb{C}(z)\{s\mathbb{I}_{mp} - \mathbb{A}\}^{-1}\mathbb{B} + \mathbb{D}(z). \qquad (8)$$

Equation (8) is equal to (2).

Consequently, it can be understood that equation (7) is a realization of (2) with respect to s.

We call (7) the first realization and call (1) the second realization.

Equation (7) is the realization by the controllable companion form.

We consider the sufficient condition so that equation (1) can be decoupled.

It is necessary that equation (7) can be decoupled so that equation (1) can be decoupled.

In the case of (7), the decoupling indices in the next equation

$$d_i = \begin{cases}\min\{j : \mathbb{C}_i(z)\mathbb{A}^j\mathbb{B} \neq \mathbb{O},\\ \qquad j = 0, 1, \cdots, mp-1\}\\ mp-1 : \mathbb{C}_i(z)\mathbb{A}^j\mathbb{B} = \mathbb{O} \quad \forall j\end{cases} \qquad (9)$$

are all 0, unless that among $f_1(s)$, $f_2(z)$, $\mathbb{G}(s,z)$ in (2), very special relation exist.

We restrict ourselves the system, in which

$$d_1 = d_2 = \cdots\cdots = d_p = 0 \qquad (10)$$

is satisfied.

In this case, the following [Condition 1] is the sufficient condition so that equation (1) can be decoupled.

[Condition 1] : $\mathbb{C}(z)\mathbb{B}$ is nonsingular and its inverse matrix is proper.

From now on, we decouple the system which satisfies [Condition 1].

We perform the dynamical feedback

$$\mathbb{U} = \mathbb{F}(z)\mathbb{W}_s + \mathbb{G}(z)\mathbb{V} \qquad (11)$$

to (1). In (11),

$$\mathbb{G}(z) = \{\mathbb{C}(z)\mathbb{B}\}^{-1} \qquad (12)$$

$$\mathbb{F}(z) = -\mathbb{G}(z)\mathbb{C}(z)\mathbb{A}. \qquad (13)$$

We perform the dynamical feedforward

$$\mathbb{y}=\mathbb{z}+\mathbb{P}(z)\mathbb{w}_s+\mathbb{H}(z)\mathbb{v} \tag{14}$$

to (1). In (14),

$$\mathbb{P}(z)=\mathbb{D}(z)\mathbb{F}(z) \tag{15}$$

$$\mathbb{H}(z)=\mathbb{D}(z)\mathbb{G}(z). \tag{16}$$

Concretely, we rewrite (11)

$$\mathbb{u}=\left[\mathbb{F}(z) \,\vdots\, \mathbb{G}(z)\right]\begin{bmatrix}\mathbb{w}_s\\ \hline \mathbb{v}\end{bmatrix} \tag{17}$$

and realize (17).

In $\mathbb{C}(z)$ in (12), denominator is at most np degree polynomial. Numerator is at most np degree polynomial. Therefore, $\mathbb{C}(z)$ is a proper rational function matrix.

In the next, $\mathbb{F}(z)$ in (13) is proper. Its denominator is at most np degree polynomial.

Consequently, equation (17) is a proper transfer function matrix and its McMillan degree is at most np^2.

We realize (17) with the minimal dimension.

$$\begin{cases}\hat{\mathbb{x}}_z=\hat{\mathbb{A}}\,\hat{\mathbb{w}}_z+\left(\hat{\mathbb{B}}_1 \,\vdots\, \hat{\mathbb{B}}_2\right)\left(\mathbb{w}_s^T \,\vdots\, \mathbb{v}^T\right)^T\\ \mathbb{u}=\hat{\mathbb{C}}\,\hat{\mathbb{w}}_z+\left(\hat{\mathbb{D}}_1 \,\vdots\, \hat{\mathbb{D}}_2\right)\left(\mathbb{w}_s^T \,\vdots\, \mathbb{v}^T\right)^T\end{cases} \tag{18}$$

In the next, we rewrite (14).

$$\mathbb{y}-\mathbb{z}=\left[\mathbb{P}(z) \,\vdots\, \mathbb{H}(z)\right]\begin{bmatrix}\mathbb{w}_s\\ \hline \mathbb{v}\end{bmatrix} \tag{19}$$

$\mathbb{P}(z)$ in (15), $\mathbb{H}(z)$ in (16) are proper, therefore, equation (19) is proper.

Then, the McMillan degree of (19) is at most n(1+p)p degree.

We realize (19) with the minimal dimension.

$$\begin{cases}\tilde{\mathbb{x}}_z=\tilde{\mathbb{A}}\,\tilde{\mathbb{w}}_z+\left(\tilde{\mathbb{B}}_1 \,\vdots\, \tilde{\mathbb{B}}_2\right)\begin{bmatrix}\mathbb{w}_s\\ \hline \mathbb{v}\end{bmatrix}\\ \mathbb{y}-\mathbb{z}=\tilde{\mathbb{C}}\,\tilde{\mathbb{w}}_z+\left(\tilde{\mathbb{D}}_1 \,\vdots\, \tilde{\mathbb{D}}_2\right)\begin{bmatrix}\mathbb{w}_s\\ \hline \mathbb{v}\end{bmatrix}\end{cases} \tag{20}$$

By performing (18) as the feedback, (20) as the feedforward, to (1), the system in the next equation is obtained.

$$\begin{cases}\begin{bmatrix}\mathbb{x}_s\\ \hline \bar{\mathbb{x}}_z\end{bmatrix}=\begin{bmatrix}\bar{\mathbb{A}}_{11} & \bar{\mathbb{A}}_{12}\\ \hline \bar{\mathbb{A}}_{21} & \bar{\mathbb{A}}_{22}\end{bmatrix}\begin{bmatrix}\mathbb{w}_s\\ \hline \bar{\mathbb{w}}_z\end{bmatrix}+\begin{bmatrix}\bar{\mathbb{B}}_1\\ \hline \bar{\mathbb{B}}_2\end{bmatrix}\mathbb{v}\\ \mathbb{z}=\left(\bar{\mathbb{C}}_1 \,\vdots\, \bar{\mathbb{C}}_2\right)\left(\mathbb{w}_s^T \,\vdots\, \bar{\mathbb{w}}_z^T\right)^T+\bar{\mathbb{D}}\mathbb{v}\end{cases} \tag{21}$$

From (21), we calculate the transfer function matrix like

$$\left(\bar{\mathbb{C}}_1 \,\vdots\, \bar{\mathbb{C}}_2\right)\begin{bmatrix}s\mathbb{I}_{mp}-\bar{\mathbb{A}}_{11} & -\bar{\mathbb{A}}_{12}\\ \hline -\bar{\mathbb{A}}_{21} & z\mathbb{I}_{\hat{n}}-\bar{\mathbb{A}}_{22}\end{bmatrix}^{-1}\cdot\left(\bar{\mathbb{B}}_1^T \,\vdots\, \bar{\mathbb{B}}_2^T\right)^T+\bar{\mathbb{D}} \tag{22}$$

$$(\hat{n}=2np(p+1)).$$

The next equation is obtained.

$$\begin{bmatrix}z_1\\ \vdots\\ z_p\end{bmatrix}=\begin{bmatrix}1/s & & \mathbb{O}\\ & \ddots & \\ \mathbb{O} & & 1/s\end{bmatrix}\begin{bmatrix}v_1\\ \vdots\\ v_p\end{bmatrix} \tag{23}$$

2.2 The case of the second state-space realization form

We decouple the two-dimensional systems in the next equation.

$$\begin{cases} \begin{bmatrix} \mathbb{X}_s \\ \hdashline \mathbb{X}_z \end{bmatrix} = \left[\begin{array}{c:c} \mathbb{A}'_{11} & \mathbb{A}'_{12} \\ \hdashline \mathbb{O} & \mathbb{A}'_{22} \end{array}\right] \begin{bmatrix} \mathbb{W}_s \\ \hdashline \mathbb{W}_z \end{bmatrix} + \begin{bmatrix} \mathbb{B}'_1 \\ \hdashline \mathbb{B}'_2 \end{bmatrix} \mathbb{U} \\ \mathbb{Y} = \left[\begin{array}{c:c} \mathbb{C}'_1 & \mathbb{C}'_2 \end{array}\right] \begin{bmatrix} \mathbb{W}_s \\ \hdashline \mathbb{W}_z \end{bmatrix} + \mathbb{D}'\mathbb{U} \end{cases} \qquad (24)$$

In this case, equation (7) is a realization by the observable companion form.

In this case, [Condition 1] is a sufficient condition so that equation (1) can be decoupled, too.

We decouple the system which satisfies [Condition 1], too.

$\mathbb{G}(z)$ in (11) is the same with (12).

$\mathbb{F}(z)$ in (11) is

$$\mathbb{F}(z) = -\mathbb{G}(z)\mathbb{C}\mathbb{A}. \qquad (25)$$

$\mathbb{P}(z)$ in (14) is

$$\mathbb{P}(z) = \mathbb{D}(z)\mathbb{F}(z). \qquad (26)$$

$\mathbb{H}(z)$ in (14) is the same with (16).

Consequently, equation (17) is proper and its McMillan degree is at most np^2.

Equation (19) is proper, too and its McMillan degree is at most np^2.

From now on, we decouple alike the case of the first state-space realization form.

3. CONCLUSION

In this paper, we proposed a method for decoupling separable-denominator two-dimensional systems and calculated the realization dimension of the dynamical feedback, dynamical feedforward.

In future, we will consider the case that equation (7) has decoupling indices which are not 0, and obtain the necessary and sufficient conditions so that separable-denominator two-dimensional systems can be decoupled.

4. REFERENCES

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