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An interpolation 1-D kernel with quadratic polynomials

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Abstract: Sampling rate conversion widely used in subband coding, A/D and D/A transitions etc. is an important techniques Nyquist filters and the filter banks have been used for the sampling converter. However, they need many memories and, whenever the sampling rate is changed, it is necessary to design filters. So the objective of this paper is to present a new kernel that is quick to evaluate and has a good stopband performance.

1. Introduction

Sampling rate conversion widely used in subband coding [6], A/D and D/A transitions [5] etc. is an important techniques. Sampling rate conversion is often divided into subprocesses: reconstruction and sampling. The former creates a continuous function from the discrete signal data and the later samples this to create a new resampled signal data. Interpolation kernel is the process of estimating the intermediate values of a continuous event from discrete samples. In image processing, reconstruction of a piecewise continuous function from discrete signal data is often taken to be a linear combination of the input signal data and reconstruction kernel. Piecewise local polynomials are used extensively for reconstruction in signal data resampling applications because they are simple, quick to evaluate and easy to implement. However, an enough amount of the stopband attenuation cannot be achieved with the previous proposed kernels [1]-[4]. Therefore, it is unsuitable to applications of the communication signal processing like data transmission systems.

Sampling rate conversion using the filter banks is proposed in the field of digital signal processing [7]. However, in sampling rate conversion with a rational factor, the computational complexity may become very large.

On the other hand, an ideal reconstruction kernel to obtain a continuous signal from the discrete signal data is a sinc function, which is obtained by the Fourier transform of a rectangular wave. However, since this kernel has an infinite length polynomials, it cannot be actually realized. Then, the design method of the nyquist filter which is approximated the frequency domain simultaneously with the time domain is needed. The Nyquist filters have an implementation of large stopband attenuation. However, they need a lot of amounts of the calculation for reconstruction in signal data resampling when a high order filter is used. Moreover, whenever the sampling rate is changed, it is necessary to design the Nyquist filter again.

In this paper, we propose a kernel with linear phase characteristics using picewise local polynomials. The proposed kernel is approximated to each piece by any quadratic functions. The kernel has a good stopband performance because we are designing the kernel in the frequency domain. Linear programming technique is used for the design of the kernel. The kernel obtained is simple, less memory and easy to implement. Finally, the usefulness of the proposed kernel is verified through the examples.

2. Interpolation

Reconstruction of a piecewise continuous function from discrete data is taken to be a linear combination of input data and a reconstruction kernel. For unit spaced samples, this is

$$f(x) = \sum_{i=-\infty}^{\infty} f_i y(x-i) \tag{1}$$

where f_i are the sample values and $y(x)$ is the reconstruction kernel.

The proposed kernel with linear phase characteristics is approximated to each piece by using any quadratic functions as follows:

$$y(x)=\begin{cases} a_{1,1}x^2+b_{1,1}x+c_{1,1} & (0\leqq |x|\leq \frac{1}{N}) \\ \vdots & \\ a_{1,n}x^2+b_{1,n}x+c_{1,n} & (\frac{n-1}{N}\leqq |x|\leq 1) \\ \vdots & \\ a_{s,n}x^2+b_{s,n}x+c_{s,n} & (s-1+\frac{n-1}{N}\leqq |x|\leq s-1+\frac{n}{N}) \\ \vdots & \\ a_{S,N}x^2+b_{S,N}x+c_{S,N} & (S-1+\frac{N-1}{N}\leq |x|\leq S) \end{cases} \tag{2}$$

where N and S are the number of necessary polynomials for one sampling section and the numbers of sampling sections, respectively. Moreover, $a_{s,n}$, $b_{s,n}$ and $c_{s,n}$ are the coefficient of quadratic function. To produce a useful kernel from this general form, we need to apply restrictions to eq. (2). Following [5] and [6], the restrictions are

1) $y(x) = y_{1,0}$ for $x = 0$

2) $y(x) = y_{s,n}$ for $x = \dfrac{n}{N}$

3) $y(x) = 0$ for $x = s$

4) C0-Continuous.strictions.

We notice from conditions 1) and 3) that the kernel is zero intersymbol interference. Moreover, we notice from conditions 2) that each quadratic function starts and ends at same points. Substituting the above four restrictions in eq. (2), we obtain eq.(3).

The rough sketch of this kernel, eq. (3), is shown in Fig. 1.

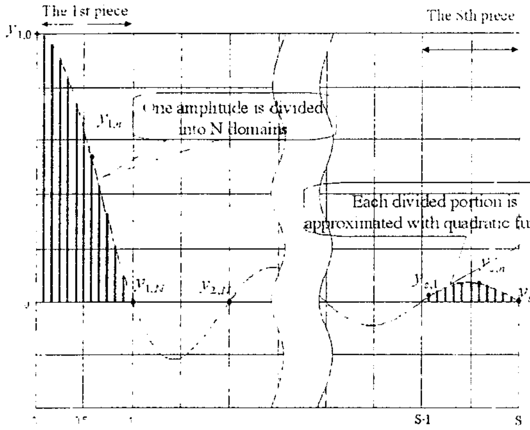

Fig. 1 The outline figure of a kernel

Here, although it can also approximate in a time domain as it is, Sinc function is infinite length and it is impossible to approximate them all as a matter of fact, as shown in the upper outline figure, it starts by limited length, and if approximated as it is, a close error will arise. Therefore, in order to determine a coefficient by the method proposed this time, $a_{s,n}$ and $y_{s,n}$ of (3) are optimized in a frequency domain. The frequency characteristic of eq. (3) is

$$Y(\omega) = \sum_{k=-SL}^{SL} y(kT)\cos(k\omega T) \tag{4}$$

where $T = \dfrac{1}{L}$ and L is a number of evaluation points to one sampling section. Let the frequency characteristics of an ideal lowpass filter be

$$D(\omega) = \begin{cases} 1 & (passband) \\ 0 & (stopband) \end{cases} \tag{5}$$

and let $W(\omega)$ and δ be the desired weighting value for approximating error at frequency ω and the maximum allowable approximation error. Accordingly, it has to satisfy the following set of linear inequalities

$$-\delta \le W(\omega_i)[D(\omega_i) - Y(\omega_i)] \le \delta \,. \tag{6}$$

In addition, M is grid points($i = 0, 1, \cdots, M$). To optimize coefficients $a_{s,m}$ and $y_{s,m}$ of eq. (3), we may use eq. (6) to formulate the following linear program:

Maximize $(-\delta)$

Subject to

$$-W(\omega_i)\sum_{k=-SL}^{SL} y(kT)\cos(k\omega_i T) - \delta \le -W(\omega_i)D(\omega_i) \tag{7a}$$

$i = 0, 1, \cdots, M/2$

$$W(\omega_i)\sum_{k=-SL}^{SL} y(kT)\cos(k\omega_i T) - \delta \le W(\omega_i)D(\omega_i) \tag{7b}$$

$i = 0, 1, \cdots, M/2$.

The approximation problem of eq. (7) can be solved by using the standard linear programming techniques.

3. The example of a design

In this section, to show the proposed kernel effectiveness, we consider about the following designs.

3.1. Example 1

[Specifications]

N: 2, S: 5, L: 10, roll-off rate:0.25,

passband edge:0.75, stopband edge:1.25,

weight:1.

$$y(x) = \begin{cases} a_{1,1}\left\{x^2 - \dfrac{1}{N}(x - \dfrac{1}{N}) - \dfrac{1}{N^2}\right\} + y_{1,1}Nx - y_{1,0}(Nx - 1) & (0 \le x \le \dfrac{1}{N}) \\ \vdots & \\ a_{1,n}\left[x^2 - N\left\{(\dfrac{n}{N})^2 - (\dfrac{n-1}{N})^2\right\}(x - \dfrac{n}{N}) - (\dfrac{n}{N})^2\right] + y_{1,n}(Nx + 1 - n) - y_{1,n-1}(Nx - n) & (\dfrac{n-1}{N} \le x \le \dfrac{n}{N}) \\ \vdots & \\ a_{s,n}\left[x^2 - N\left\{(s - 1 + \dfrac{n}{N})^2 - (s - 1 + \dfrac{n-1}{N})^2\right\}\left\{x - (s - 1 + \dfrac{n}{N})\right\} - (s - 1 + \dfrac{n}{N})^2\right] & \\ + y_{s,n}\left\{Nx + 1 - N(s - 1 + \dfrac{n}{N})\right\} - y_{s,n-1}\left\{Nx - N(s - 1 + \dfrac{n}{N})\right\} & (s - 1 + \dfrac{n-1}{N} \le x \le s - 1 + \dfrac{n}{N}) \\ \vdots & \\ a_{s,N}\left[x^2 - N\left\{s^2 - (s - 1 + \dfrac{N-1}{N})^2\right\}\{x - S\} - S^2\right] & \\ + y_{S,N}\{Nx + 1 - NS\} - y_{S,N-1}N(x - S) & (S - 1 + \dfrac{N-1}{N} \le x \le S) \end{cases} \tag{3}$$

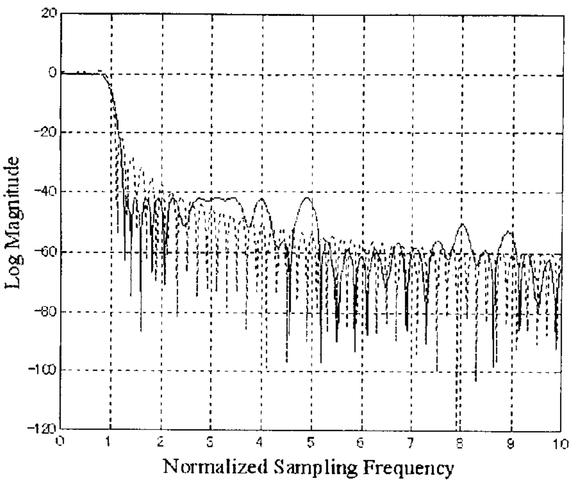

Fig. 2 The amplitude characteristics of the proposed kernel

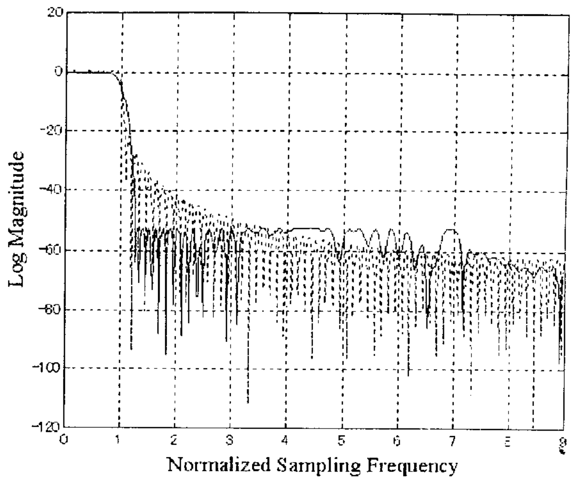

Fig. 4 The amplitude characteristics of the proposed kernel

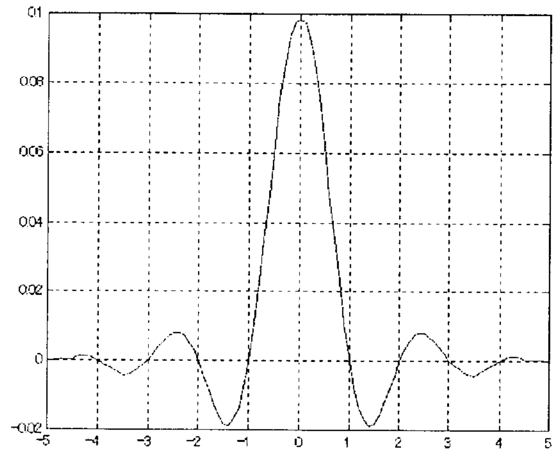

Fig. 3 The proposed kernel

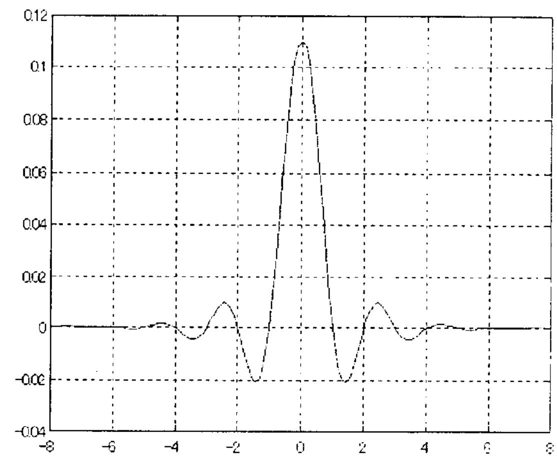

Fig. 5 The proposed kernel

Fig. 2 and 3 show the frequency characteristics and the kernel, respectively. In Fig. 2, the sampling rate is normalized as 1. In Fig. 1, solid line shows the amplitude characteristics of the proposed kernel and dash line is the amplitude characteristics Sinc function with L=8. It is clear from Fig. 2 that the stopband attenuation of Sinc function is worse than one of the proposed kernel. Moreover, it is clear that the proposed the kernel is zero intersymbol interference because it is $y(x)=0$ when x is the integer values other than 0 from Fig.3. On the other hand, it is necessary to design the filter of 50 orders to obtain the amplitude characteristic equal with the proposed kernel by using the Niquist filter. Therefore, the proposed kernel has fewer memory compared with Niquist filter. However, even if the sampling section, S, is increased, the stopband attenuation of the proposed kernel (N=2) does not exceed about -40dB.

3.2. Example 2

Here, the design example of increasing N is shown.

[Specifications]

$N:3$, $S:8$, $L:9$, roll-off rate:0.25,

passband edge: 0.75, stopband edge:1.25,

weight:1

Fig. 4 and 5 is the frequency characteristics and the kernel, respectively. The frequency characteristic of Sinc function is also shown in Fig. 4. It is clear from Figs. 2 and 4 that the stopband attenuation is large by increasing N. It is clear from Fig.3 as well as example 1 that the proposed the kernel is zero intersymbol interference because it is $y(x)=0$ when x is the integer values other than 0.

3.3. Example 3

Here, the relation between N and the maximum stopband attenuation is shown.

[Specifications]

$S:12$, $L:30$, roll-off rate:0.25,

passband edge: 0.75, stopband edge:1.25,

weight:1

Fig. 6 shows the relation between N and the maximum stopband attenuation. It is clear from Fig. 6 that the maximum stopband attenuation increases when N increases. However, you note that the amount of the memory increases when N increases. Therefore, it is necessary to limit the value of N by the purpose.

3.4. Evaluation as a sampling converter

Here, The effect of the proposed kernel when the sampling rate changes is shown

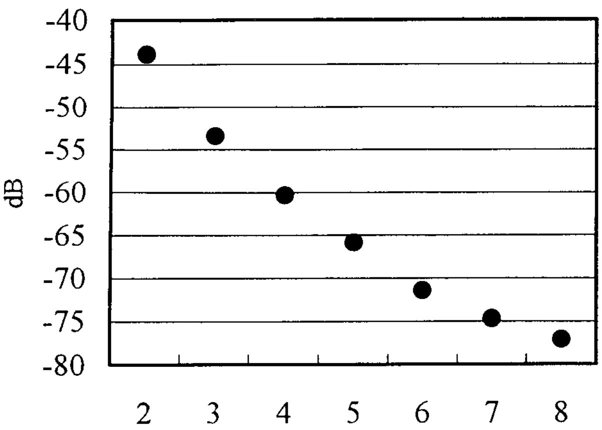

Fig. 6 the relation between N and the maximum stopband attenuation

We think about specifications the following with the sampling rate different.

N : 2, S : 8 , roll-off rate:0.25,

passband edge: 0.75, stopband edge:1.25, weight:1

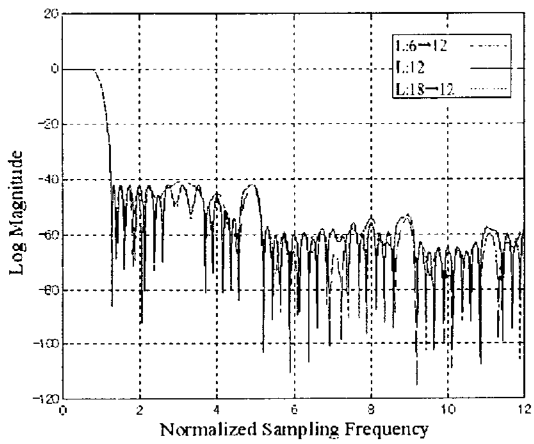

Fig. 7 The comparison figure of the frequency characteristic when changing a sampling rate

In Fig. 7, the solid line shows the frequency response of the kernel when the sampling rate is L=12. In addition, the dash line and dot line show the frequency response of the kernel when the sampling rate changes from L=6 into L=12 and when the sampling rate changes from L=18 into L=12, respectively. However, you note that the coefficient of the quadratic polynomial at L=8 designed first is used even if the sampling rate changes from L=8 into L=12. When the sampling rate changes from L=8 into L=12, the coefficient of the quadratic polynomial at L=8 is used. It is clear Fig. 7 that there is no change in the frequency response even if the sampling rate is different from the kernel designed. This means that the proposed kernel needs not be redesigned even if the sampling rate changes. That is, the proposed kernel is robustness to the change in the sampling rate.

4. Conclusion

In this paper, we proposed a kernel with linear phase characteristics using picewise local polynomials. The proposed kernel is approximated to each piece by any quadratic functions. The kernel has a good stopband performance because we are designing the kernel in the frequency domain. Linear programming technique is used for the design of the kernel. The kernel obtained is simple, less memory and easy to implement. Moreover, even if the sampling rate changes, the proposed kernel needs not to redesign the kernel. The usefulness of the proposed kernel is verified through the examples.

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