평행평판내의 수정멱법칙 유체에 대한 열전달 및 압력강하의 예측

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Predictions of Heat Transfer and Pressure Drop between Parallel Plates with Modified Power Law Fluids

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1. Introduction

The predictions of pressure drop and heat transfer to fluids flowing in ducts are important in many engineering applications. For fully developed laminar flow of Newtonian and non-Newtonian power law fluids flow in a square duct, the solutions are well known for both the classical boundary conditions of constant wall temperature (CWT) and constant wall heat flux (CHF) and the pressure drop.

For Newtonian fluids, pressure drop and heat transfer coefficients were calculated by Shah and London[1], Rothfus *et.al*[2], etc.. For power law fluids, Chandrupatla[3], Wheeler and Wissler[4], Kozicki[5], and Kozicki *et.al*[6] obtained those analytically and experimentally.

Non-Newtonian fluids usually have been assumed as power law fluids in the analysis. But, many non-Newtonian fluids have viscous properties which are different in the various shear rate ranges.

Although a power law model has been used extensively for calculating velocity profile and heat transfer coefficient in engineering, it has significant disadvantages that it only applies to the power law region in the flow curve and the apparent viscosity at the centeroid of the duct becomes infinite.

A constitutive equation is one that relates the shear stress or apparent viscosity in a fluid to the shear rate through the rheological properties of the fluid. A convenient way to depict the constitutive equation is to plot a curve of apparent viscosity against shear rate. Fig. 1 shows such a graph which is indicative of the behavior of many purely viscous pseudoplastic fluids. In the lower shear rate range, the fluid is Newtonian and in the higher shear rate range the fluids acts as a power law fluid. Between these region is a transition range.

The present research attempts to correct this situation by presenting a solution should have the characteristics that at low velocities (low shear rates) the Newtonian solution is an asymptote while at large shear rates the power law solution is an asymptote. In addition, the solution should predict the appropriate pressure drop and heat transfer behavior in the transition zone. Finally a parameter is needed to predict the shear rate range in terms of the operating characteristics of the system.

The purpose of present study is to extend our knowledge analytically by presenting solutions for fluids having the rheological characteristics illustrated Fig. 1 and develop the relationships between the friction factor and Reynolds number and the heat transfer coefficients for a Modified Power Law fluid. For a circular tube (Brewster and Irvine[7]), and concentric annulus (Capobianchi and Irvine[8]) such solutions are available.

When using a particular constitutive equation, it is necessary to determine that the equation correctly describes the relation between the apparent viscosity and the shear rate for the particular fluid being considered. Thus it is required to measure the rheological properties in the constitutive equation and compare the equation of predictions with the experimental values of the apparent viscosity vs. the shear rate. This was done for the CMC (Sodium Carboxymethyl Cellulose) solutions by Park[9].

2. Analysis

A number of constitutive equation can describe the apparent viscous-shear rate relation for fluids such as shown in Figure 1. A convenient and useful equation is the "modified power law model" which to the authors' knowledge was first used by Dunleavy and Middleman[10].

$$\eta_a = \frac{\eta_0}{1 + \frac{\eta_0}{K}\dot{\gamma}^{1-n}} \tag{1}$$

Inspection of Equation(1) reveals that at low rates ($\eta_0/K\,\dot{\gamma}^{1-n} \ll 1$) the apparent viscosity becomes equal to η_0 and the fluid is operating in the Newtonian region of Figure 1. At higher shear rates ($\eta_0/K\,\dot{\gamma}^{1-n} \gg 1$) the fluid becomes a power law fluid where $\eta_a = K\,\dot{\gamma}^{1-n}$. At intermediate shear rates, there is a transition zone. An additional advantages of the modified power law over constitutive equations such as Ellis, Sutterby, Cross, etc., is that the familiar Newtonian and power law Reynolds are retained in the analysis.

Velocity Profile

For infinite parallel plates as shown in Figure 2, the fully developed shear stress field is described by the momentum equation.

$$\frac{d}{dy}\left(\eta_a \frac{du}{dy}\right) = -\frac{dp}{dx} \tag{2}$$

The following dimensionless quantities now be defined

$$y^+ = \frac{y}{a}, \qquad f_a = \frac{2a\frac{dp}{dx}}{\rho \bar{u}^2}$$

$$u^+ = \frac{u}{\bar{u}}, \qquad Re_g = \frac{\rho \bar{u}^{2-n} a^n}{K}$$

$$Re_a = \frac{\rho \bar{u} a}{\eta_0}, \qquad Re_M = \frac{\rho \bar{u} a}{\eta^*}$$

$$\eta^* = \frac{\eta_0}{1+\beta}, \qquad \eta_a^+ = \frac{1+\beta}{1+\beta\left(\frac{du^+}{dy^+}\right)^{1-n}}$$

$$\beta = \frac{\eta_0}{K}\left(\frac{\bar{u}}{a}\right)^{1-n}, \qquad u^{++} = \frac{u^+}{f_a\, Re_g/2} \tag{3}$$

For Newtonian fluid, the continuity equation is

$$\bar{u} = \frac{1}{a}\int_0^a u\,dy \tag{4}$$

Dimensionless form of Equation(4) becomes

$$\int_0^1 u^+ dy^+ = 1 \tag{5}$$

Since $\eta_a = \eta_0$, Equation(2) becomes

$$\frac{d^2u^+}{dy^{+2}} = -\frac{f_a\, Re_a}{2}, \qquad \frac{d^2u^{++}}{dy^{+2}} = -1 \tag{6}$$

Equation(5) becomes

$$\int_0^1 u^{++} dy^+ = \frac{2}{f_a\, Re_g} \tag{7}$$

For a modified power law fluid, Equation(2) may be written in dimensionless form as

$$\frac{d}{dy^+}\left(\eta_a{}^+ \frac{du^{++}}{dy^+}\right) = -1 \tag{8}$$

with boundary conditions

$$u^{++}(1) = 0, \quad u^{++\prime}(0) = 0$$

The continuity equation remains the same as Equation(7).

$$\eta_a{}^+ = \frac{1+\beta}{1+\beta\left(\frac{f_a\, Re_g}{2}\right)^{1-n}\left(\frac{du^{++}}{dy^+}\right)^{1-n}} \tag{9}$$

The parameter β is the shear rate parameter that determines whether the fluid system is operating in the Newtonian, transition or power law regions. As β becomes small, Re_M approaches the Newtonian Reynolds number Re and as β becomes large, Re_M approaches the power law Reynolds number Re_g.

Energy Equation(CHF)

The energy equation for constant heat flux can be written as

$$k\frac{\partial^2 T}{\partial y^2} = \rho c_p u \frac{\partial T}{\partial x} \tag{10}$$

with boundary conditions

$$T(a) = T_w, \quad T'(0) = 0$$

The temperature field is fully developed when

$$\frac{\partial}{\partial x}\left(\frac{T - T_w}{T_b - T_w}\right) = 0 \tag{11}$$

where h = const.

$$(T_b - T_w)\left(\frac{\partial T}{\partial x} - \frac{\partial T_w}{\partial x}\right) - (T - T_w)\left(\frac{\partial T_b}{\partial x} - \frac{\partial T_w}{\partial x}\right) = 0$$

From $q_w = h(T_w - T_b) = const.$

$$\frac{dT_w}{dx} = \frac{dT_b}{dx}$$

From Equation (11)

$$\frac{dT_w}{dx} = \frac{dT_b}{dx} = \frac{dT}{dx}$$

Defining a dimensionless temperature, $T^+ = \frac{T - T_w}{T_b - T_w}$, Equation(10) becomes

$$\frac{d^2 T^+}{dy^{+2}} = -u^+ Nu_a \tag{12}$$

with boundary conditions

$$T^+(1) = 0, \quad T^{+\prime}(0) = 0$$

Defining new dimensionless temperature, $T^{++} = \frac{T^+}{Nu_a}$, Equation(12) becomes

$$\frac{d^2 T^{++}}{dy^{+2}} = -u^+ \tag{13}$$

with boundary conditions

$$T^{++}(1) = 0, \quad T^{++\prime}(0) = 0$$

From the definition of bulk temperature

$$T_b = \frac{1}{ua} \int_0^a uTdy,$$

$$\int_0^1 u^+ T^{++} dy^+ = \frac{1}{Nu_a} \tag{14}$$

From Equation(13) and (14), the Nusselt number for constant heat flux can be obtained.

Energy Equation (CWT)

$$\frac{\partial}{\partial x}\left(\frac{T - T_w}{T_b - T_w}\right) = 0$$

From T_w = const.,

$$\frac{\partial T}{\partial x} = \frac{T - T_w}{T_b - T_w} \frac{\partial T_b}{\partial x}$$

Equation (10) becomes

$$\frac{d^2 T^+}{dy^{+2}} = -u^+ T^+ Nu_a \tag{15}$$

with boundary conditions

$$T^+(1) = 0, \quad T^{+\prime}(0) = 0$$

Integrating Equation(15), then

$$\int_0^1 \frac{\partial^2 T^+}{\partial y^{+2}} dy^+ = -\int_0^1 u^+ T^+ Nu_a dy^+ = -Nu_a$$

$$\left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=1} - \left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=0} = T^{+\prime}(1) = -Nu_a \qquad (16)$$

From Equation(15) and (16), the Nusselt number for constant wall temperature can be obtained.

3. Results and Discussion

The graphical result of the calculations discussed above are shown in Figure 4,5 and 6. As expected, the numerical solutions at small values of β approach the Newtonian analytical solutions.

$$f_a Re_a = 6$$

$$Nu_{CHF} = 2.06$$

$$Nu_{CWT} = 1.88$$

The numerical solution at large values of β approach the power law fluid analytical solutions[11].

$$f_a Re_a = 2\left(\frac{1+2n}{n}\right)^n$$

$$Nu_{CHF} = 2.06\left(\frac{1+2n}{3n}\right)^{1/3}$$

$$Nu_{CWT} = 1.88\left(\frac{1+2n}{3n}\right)^{1/3}$$

It is interesting to note that the transition region (approximately $10^{-2.5} \le \beta \le 10^{2.5}$) is useful to estimate whether the fluid is a fully developed Newtonian fluid ($\beta \le 10^{-2.5}$) or a fully developed power law fluid ($\beta \ge 10^{2.5}$).

4. Conclusions

Numerical solutions for laminar fully developed flow are presented for the friction factor time Reynolds number and the Nusselt number (CHF and CWT) for MPL (Modified Power Law) fluid flowing within infinite parallel plates.

By using the constitutive equation, solutions are obtained which are applicable over a wide shear rate range of pseudoplastic fluids from the Newtonian behavior at a lower shear rate range to the power law behavior at a higher shear rate range. A shear rate parameter can be used for the prediction of the shear rate range for a specified set of operating conditions.

Nomenclature

a one half of slot width [m]

c_p specific heat [J/kgK]

f_a Darcy friction factor ($2\frac{dp}{dx} a/\rho \bar{u}^2$) [-]

h heat transfer coefficient ($\frac{q_w}{T_w - T_b}$) [W/m^{2}K]

k thermal conductivity [W/mK]

K power law consistency [Ns^n/m^2]

n power law flow index [-]

N numerical nodal point [-]

Nu_a Nusselt number (ha/k) [-]

p pressure [N/m^2]

q_w wall heat flux [W/m^2]

Re_a Newtonian Reynolds number ($\rho \bar{u} a/\eta_0$) [-]

Re_g power law Reynolds number ($\rho \bar{u}^{2-n} a^n/K$) [-]

Re_M modified Reynolds number ($\rho \bar{u} a/\eta^*$) [-]

T temperature [K]

T^+ dimensionless temperature [-]

T^{++} dimensionless temperature [-]

u velocity in flow direction [m/s]

$\bar{u}$ duct average velocity [m/s]

u^+ dimensionless velocity in flow direction ($u/\bar{u}$) [-]

u^{++} dimensionless velocity in flow direction ($\frac{u^+}{(f_a Re_g)/2}$) [-]

x coordinate in flow direction [m]

y coordinate in flow transverse direction [m]

y^+ dimensionless coordinate in flow transverse direction [-]

Greek symbols

α thermal diffusivity [m^2/s]

β shear rate parameter ($\beta = \frac{\eta_0}{K}(\frac{\bar{u}}{a})^{1-n}$) [-]

$\dot{\gamma}$ shear rate [1/s]

η_a apparent viscosity ($\tau/\dot{\gamma}$) [Ns/m^2]

η_0 zero shear rate viscosity [Ns/m^2] [-]

η^* reference viscosity $\left(\frac{\eta_0}{1+\beta}\right)$ [Ns/m^2]

η^+ dimensionless viscosity (η_a/η^*) [-]

ρ fluid density [kg/m^3]

τ shear stress [N/m^2]

Subscripts

a slot width

b bulk temperature

g generalized Reynolds number

M modified Reynolds number

w wall condition

Superscripts

+ dimensionless quantities

++ dimensionless quantities

′ derivative

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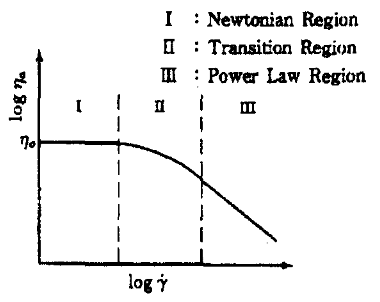

Figure 1. Typical Flow Curve of Pseudoplastic Fluid

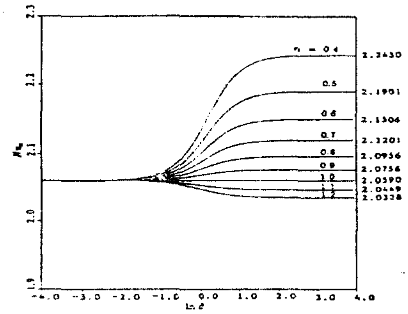

Figure 4. Variation of the fully developed Nusselt numbers with β and n (CHF)

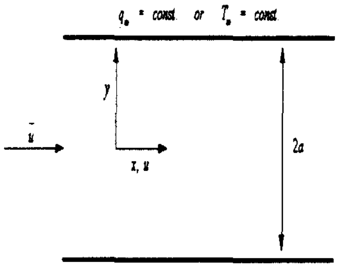

Figure 2. Coordinate system for parallel plates

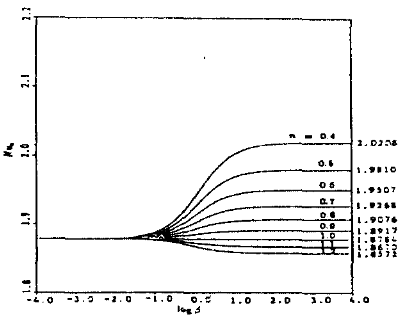

Figure 5. Variation of the fully developed Nusselt numbers with β and n (CWT)

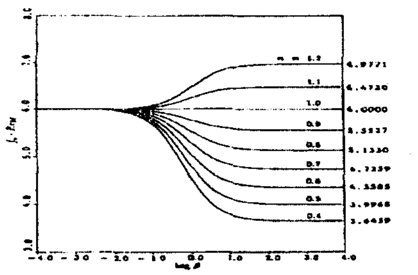

Figure 3. Variation of the fully developed $f_a\ Re_M$ with β and n