카오스 단축압출기의 혼합성능 정량화

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Measures of Mixing in Chaos Single-Screw Extruder

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Introduction

Chaos Screw(CS) is one of the successful examples of chaotic system in transport phenomena and it has practical importance in the polymer extrusion process. We already proposed a rigorous dynamical system modelling and the related dynamics concerning perturbed homoclinic orbits, resonance bands, KAM tori. We also showed how the conformation of these structures is determined by the practical design variables, e.g. the helix angle, barrier configurations, the adverse pressure gradient [1,2].

In this presentation, we propose useful concepts in quantifying the mixing performance and the mixedness of mixture of spatially-periodic continuous dynamical systems such as the PPM, the Kenics Mixer, and Chaos Screw. Our analysis starts with analysis of a solution map with its transversality. Then, we deal with some general mathematical properties of the solution map; flux preservation, orientation preservation, commutative property and symmetry. Using them, we will give an answer to the following fundamental question about the CS system as a mixer.

- How much volume of fluids is transported from one region to another during one period?

(Evidently if the system is not chaotic, this kind of transports cannot occur.) The analyses are basically different from those of well-known time-periodic systems, because the CS dynamical systems do satisfy flux-preserving property, not area-preserving property. In addition, we investigate the residence time distribution (RTD), its dependency on chaotic structures (homoclinic tangle, resonance bands, and KAM tori) and thereby dependencies on practical design variables.

Kinematic aspects of the solution map

When the axial velocity $w(\boldsymbol{x})$ does not change its sign in the cross-sectional domain of interest, say $w(\boldsymbol{x}) > 0$, one can construct a solution map which is a continuous map $\varphi : \Sigma \mapsto \Sigma$ such that

$$X(z) = \varphi(z, z_0)X_0, \quad \text{where} \quad z > z_0 \quad \text{and} \quad X = (x, y) \in \Sigma \subset \Re^2. \tag{1}$$

Σ is a cross section identified by the z coordinate of the CS system and X_0 is the cross-sectional position (x_0, y_0) of the initial point located on z_0. $\varphi(z, z_0)$ maps the

initial point (X_0, z_0) to (X, z) along the material line. The solution map (1) is a basic tool in describing the CS system and there are four purely kinematic properties in this map which are very important to understand the material transports in the system.
Flux preservation: A subdomain $D \subset \Sigma_{z_0}$ is mapped by the solution map to give another subdomain $\varphi(z, z_0)(D)$ on the Σ_z plane. There is no reason that D and $\varphi(z, z_0)(D)$ have the same area, that is, the map is *not* area-preserving. If we denote the flux through D by $\mathcal{F}(D)$ (and the flux through $\varphi(z, z_0)(D)$ by $\mathcal{F}(\varphi(z, z_0)(D))$), then from the divergence theorem, we have

$$\mathcal{F}(D) = \int_D w(\boldsymbol{x})\mathrm{d}A = \int_{\varphi(z,z_0)(D)} w(\boldsymbol{x})\mathrm{d}A = \mathcal{F}(\varphi(z, z_0)(D)), \tag{2}$$

for all z greater than z_0 and for all the choice of D.
Orientation preservation: From the continuity of $(\mathrm{d}x/\mathrm{d}z)$, $(\mathrm{d}y/\mathrm{d}z)$ and their partial derivatives with respect to x and y, relative ordering of material points under the solution map cannot be changed in continuous dynamical system. In the mathematical terms, the determinant of the solution map is always positive.
Symmetries From the symmetry of the velocity fields in CS, the solution map has the following symmetry:

$$\tilde{\varphi}(-z, -z_0) = \boldsymbol{S}_y \varphi(z, z_0) \boldsymbol{S}_y. \tag{3}$$

In this case, $\boldsymbol{S}_y$ is two-dimensional reflection symmetry about y axis, i.e., $\boldsymbol{S}_y : (x, y) \mapsto (-x, y)$ and $\tilde{\varphi}$ is the backward solution map which satisfies

$$\varphi(s, t)\tilde{\varphi}(t, s) = \tilde{\varphi}(t, s)\varphi(s, t) = id., \quad s > t. \tag{4}$$

Commutative property If we introduce the spatial period of the velocity field, say λ, it is clear that

$$\varphi(\lambda, 0)\varphi(z, 0) = \varphi(z, 0)\varphi(\lambda, 0) = \varphi(\lambda + z, 0), \tag{5a}$$

$$\{\varphi(\lambda, 0)\}^n = \varphi(n\lambda, 0), \quad n = \pm 1, \pm 2, \dots . \tag{5b}$$

In general, $\varphi(n\lambda + \phi_0, \phi_0)$ forms a commutative group and called mapping at a period. In this kind of mapping, X_0 is a fixed point of the mapping $\varphi(n\lambda + \phi_0, \phi_0)$, if and only if the orbit $\boldsymbol{x}(t)$ with initial condition $\boldsymbol{x}_0 = (X_0, \phi_0)$ is periodic in the dynamical system, and the fixed point has the same stability type as that of the corresponding orbit[3]. Let us define the commutative solution map $\varphi(\lambda + \phi_0, \phi_0)$ as the Poincaré map P_{ϕ_0} of the system. Since it is derived from the solution map, the Poincaré map is also expected to show flux- and orientation-preservation and to have symmetry structure. Arguments concerning the flux- and orientation-preservation are quite clear and we have the symmetry of the Poincaré map

$$P^n_{\phi_0} = \boldsymbol{S}_y P^{-n}_{1-\phi_0} \boldsymbol{S}_y \tag{6}$$

Transports across distinct regions

Fig.1 indicates lobe dynamics of homoclinic tangle at $\phi = .5$, when helix angle $\alpha = 15^o$ and the fraction of no-barrier zone (perturbation) $\beta = 0.05$ with the absence of adverse pressure gradient ($\kappa = 0$). There are three distinct and bounded regions denoted by $A_{O,L,R}$ the subscripts O, L, R indicating outer, left and right regions, respectively. $\boldsymbol{M}$ denotes mapping by one period. If we denote $\mathcal{F}(D)$ for flux through the region D, we have the following material transport relations in this case.

Transports between A_O and A_R: Under one iteration of $\boldsymbol{M}$, the only flux that can be transported from A_O into A_R is $\mathcal{F}(E_R)$. Similarly, under one iteration of $\boldsymbol{M}$, the only flux that can be transported from A_R into A_O is $\mathcal{F}(D_R)$.

Transports between A_O and A_L: Under one iteration of $\boldsymbol{M}$, the only flux that can be transported from A_O into A_L is $\mathcal{F}(E_L)$. Similarly, under one iteration of $\boldsymbol{M}$, the only flux that can be transported from A_L into A_O is $\mathcal{F}(D_L)$.

Transports between A_L and A_R: Under one iteration of $\boldsymbol{M}$, the total flux that can be transported from A_L into A_R is

$$\mathcal{F}_{L,R} = \sum_{k=0}^{\infty} \left[\mathcal{F}(\boldsymbol{M}^{k+2} D_L \cap \boldsymbol{M}^{-1} E_R) \right].$$

Similarly, under one iteration of $\boldsymbol{M}$, the total flux that can be transported from A_R into A_L is

$$\mathcal{F}_{R,L} = \sum_{l=0}^{\infty} \left[\mathcal{F}(\boldsymbol{M} D_R \cap \boldsymbol{M}^{-1-l} E_L) \right].$$

From symmetry, we have the following mass conservation law in this case.

$$\mathcal{F}(E_R) + \sum_{k=0}^{\infty} \left[\mathcal{F}(\boldsymbol{M}^{k+2} D_L \cap \boldsymbol{M}^{-1} E_R) \right] - \mathcal{F}(D_R) - \sum_{l=0}^{\infty} \left[\mathcal{F}(\boldsymbol{M} D_R \cap \boldsymbol{M}^{-1-l} E_L) \right] = 0.$$

Residence-Time Distribution

Distribution of residence times of a passive scalar can be produced by the coupling between a chaotic cross-sectional flow and the axial flows. In Fig.2(a) each particle on the $x = 1$ axis uniquely labels the Lagrangian initial coordinate. Integrating the dynamical system for a fixed amount of z and final exit time of each particle is plotted versus its index. At early times, smooth residence-time distribution curve can be found being due to the difference in velocity in the axia direction. A later times, however, increaing structure is visible. Adjacent particles have dramatically different final exit time distributions. Compare it with the Poincaré section Fig.2(b). It illustrates the hierarchical distribution determined to the dynamical structures. The configuration of dynamical structure, e.g. resonance bands, KAM tori, are directly determined by the frequency-ratio distribution which is a function of the real design parameters (the helix angle, adverse pressure gradient and amount of perturbation) [1].

References

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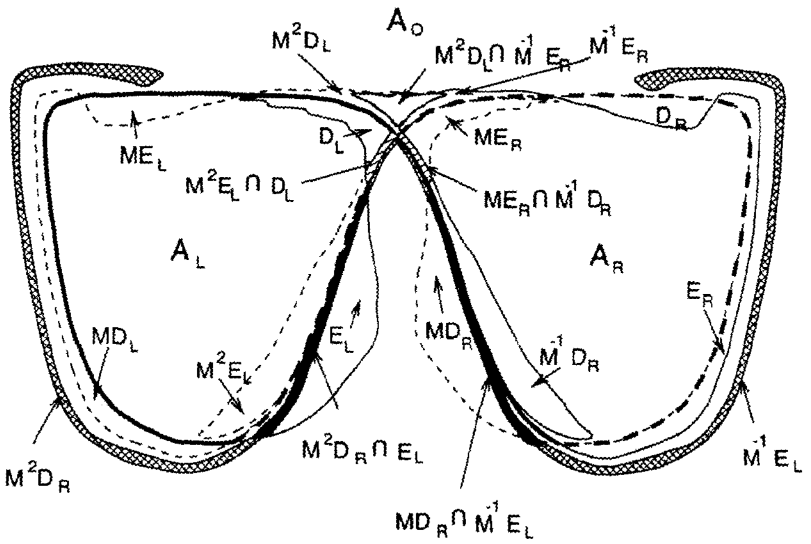
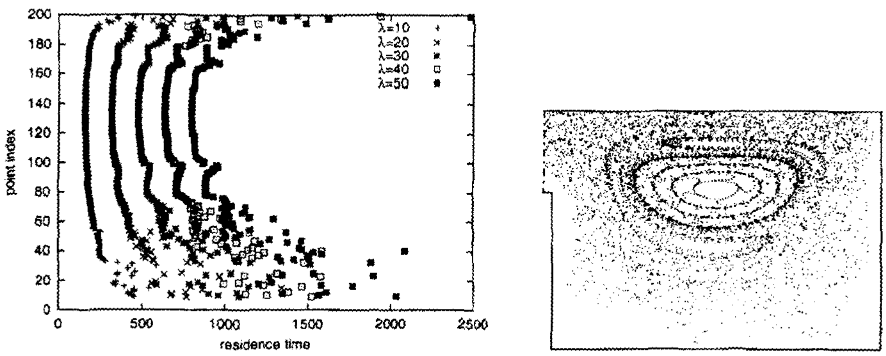


그림 1: Lobe dynamics of homoclinic tangle at $\phi = .5$ when $\alpha = \pi/12$, $\beta = 0.05$, $\kappa = 0$.



(a) (b)

그림 2: (a) Residence time distribution when $\alpha = 15°, \kappa = 0, \beta = 0.02$ and the corresponding Poincaré section.