역 다중화기의 버퍼 형태에 관한 분석

정 두 영
忠北大學校 情報通信工學科

A Behavioral Analysis of Demultiplexing Buffer

Doo Young Jung

Dept. of Computer and Communication Eng.

Abstract

Computer-Communication System may be classified into two kinds of systems considering connection capability among terminal and computing facility : time sharing and distributed computer systems.

According to the different input characteristic, the buffer behavior of two systems is quite different. Here we restrict our study to the analysis of queueing behavior of a demultiplexing buffer of time sharing systems. The analysis shows that M/G/1 queueing model can be used to model such behavior.

요 약

컴퓨터 통신 시스템은 단말기와 C.P.U와의 접속 방법에 따라 시분할 시스템과 분산 시스템으로 나누어지는데 이들 두 가지 시스템에서 버퍼에서의 큐잉 형태는 각기 다르다. 본고에서는 시분할 시스템의 역 다중화기의 큐잉 형태에 대해서 분석하였고 그것은 결국 M/G/1모델로 나타낼 수 있음을 보여주고 있다.

I. Introduction

Computer-communication system may be classified into two kinds of systems considering connection capability among terminals and computing facility : timesharing and distributed computer systems(Fig. 1).

For each system above, the characteristic of input traffic is different. The input traffic of a time sharing system is the characters with fixed length (a constant length message) and, on the other hand, the characteristic of input traffic of a distributed computer system is a mixture of characters with fixed length and bursts (the string of characters) which has a random length messages.

According to the different input characteristic, the buffer behavior of each system is quite different. When we come to consider about demultiplexing buffer of two system (Fig. 1), we must analyse differently the queueing model of the demultiplexing buffer.

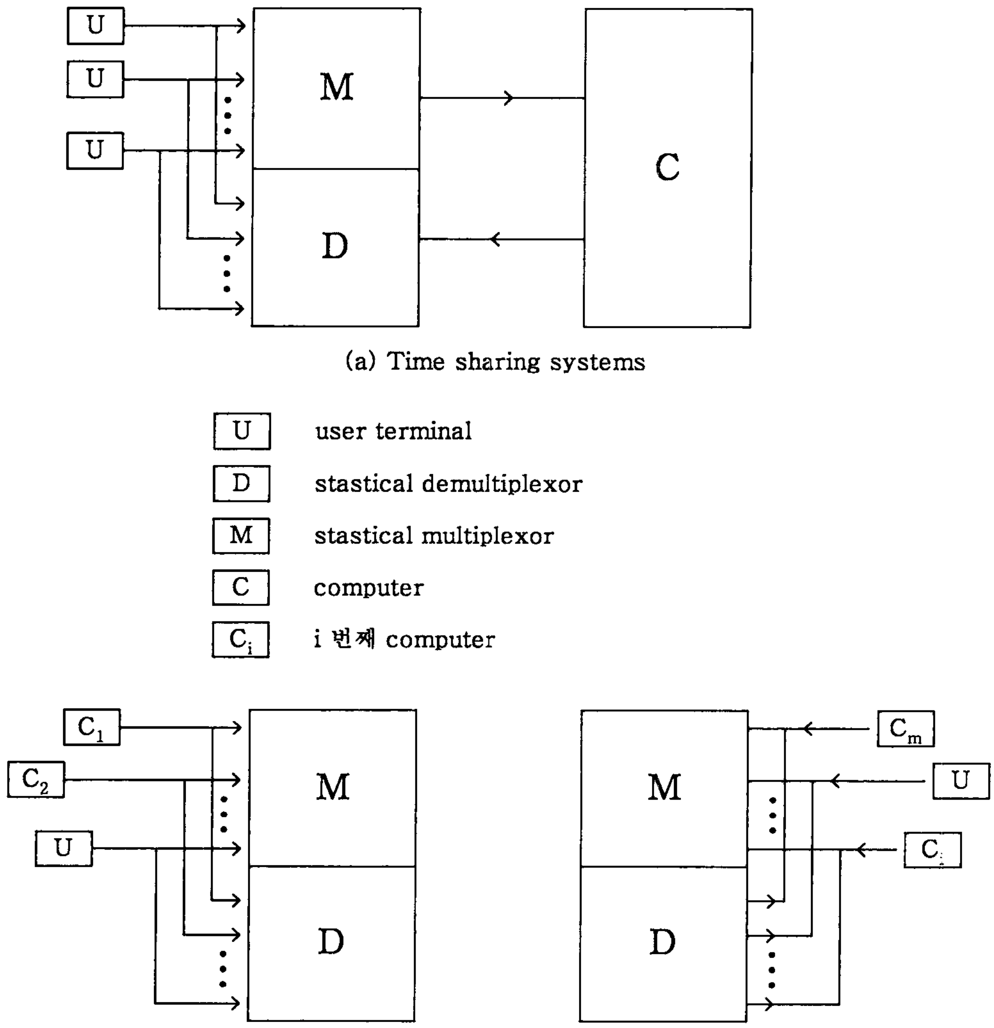

Fig. 1. Computer Communication system's Classification

In the case of the buffer behavior of demultiplexing buffer(computer-to-use buffer) of the time sharing system, the input traffic of the demultiplexing buffer (the output traffic of C.P.U.) is inherently bursts (the string of characters) which has random length message. The previous research work[1] has founded that the bursty arrivals of data from C.P.U. can be approximated as poisson arrival and the length of burst can be approximated as geometrically distributed. Here we will study that the buffer behavior with bursty input traffic is very different from that of the characters with fixed length.

Therefore we can assume that the queueing model of this study is like batch (burst) poisson arrival, a limited waiting room and a single synchronous servers (transmission channel) with constant service time (constant transmission rate).

Here we restrict our study to the analyse of queueing behavior of demultiplexing buffer of time sharing system.

II. The analysis of computer-to-use buffer time sharing system

By the fact that the multiplexing line (channel) transmits with constant speed (by above queueing model), let us define the time to transmit a character on the multiplexed line as a unit service interval devoted by μ. For a multiplexed line with a transmission rate R characters per second, μ=1/R second per character.

Let us assume that the burst length, L is geometrically distributed with mean $\bar{\ell} = \frac{1}{\theta}$ and the total number of bursts arrived during a unit service interval Y is poisson distributed with rate of λ bursts per service time.

The probability density function of L and Y are as follows :

$$f_L(l) = \theta(1-\theta)^{1-l} \quad l=1,2,\ldots\ldots\ldots\ldots(1)$$

$$f_Y(n) = \frac{e^{-\lambda}\cdot\lambda^n}{n!} \quad n=0,1,\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), characteristic functions of two p.d.f. can be founded as below ;

The characteristic function of geometric distribution is (Appendix I)

$$\Phi_L(u) = \sum_{L=1}^{n} e^{iuL}\cdot f_L(L=l)$$
$$= \theta\cdot\left(\frac{e^{iu}}{1-(1-\theta)e^{iu}}\right)\ldots\ldots\ldots\ldots(3)$$

And also, the characteristic function of poisson distribution is (Appendix II)

$$\Phi_L(u) = \sum_{n=0}^{\infty} e^{iuY}\cdot f_Y(Y=n)$$
$$= \exp[-\lambda+\lambda\exp(u)]\ldots\ldots\ldots\ldots(4)$$

The total number of characters that arrived during the time to transmit one character on the multiplexed line is random sum, S_Y and equals to

$$S_Y = \sum_{i=0}^{Y} L_i \quad \ldots\ldots\ldots\ldots(5)$$

Where L_i a random variable distributed geometrically, is the number of characters contained in the i^{th} arriving burst and Y, a random variable wit poisson distribution, is the total number of burst arriving during the unit service interval.

We let $S_Y \equiv S$ for simplication ($S_Y \equiv S = \sum_{i=1}^{Y} L_i$)

The characteristic function of S, $\Phi_S(u)$, can be devoted in terms of the characteristic function of L, $\Phi_L(u)$, and λ [3].

$$\Phi_S(u) \equiv \sum_{S=1}^{n} e^{iuS} \cdot f_S\ (S=s)$$
$$= \exp[-\lambda + \lambda\Phi_L(u)] \ldots\ldots(6)$$

Since the probability distribution function of the burst length is geometric, we can obtain the characteristic function of L as below.

$$\Phi_L(u) \equiv \sum_{L=1}^{n} e^{iuL} \cdot f_L\ (L=l)$$
$$= \theta \cdot \exp(iu)/[1-(1-\theta)\exp(iu)] (7)$$
$$\text{where } i = \sqrt{i}$$

From (4) and (5), then complete from of the characteristic function of S

$$\Phi_S(u) = \exp[-\lambda + \lambda\theta\exp(iu)/\{1-(1-\theta)\exp(iu)\}] \ldots\ldots(8)$$

From(6) we can found that probability density function of j characters arriving during a unit service interval P_r (s=j) = T_j is compound poisson distribution as shown below [3].

$$T_j = \begin{cases} \sum_{k=1}^{j}\left(\frac{e^{-\lambda}\cdot\lambda^k}{k!}\right)\binom{j-1}{k-1}\theta^k(1-\theta)^{j-k} \\ j=1,2, \\ e^{-\lambda}, j=0 \end{cases} \ldots(9)$$

From (9), we can see that the compound poisson distribution is compared with the total sum of the product of poisson distribution and a negative binomial distribution at each discrete value of k.

An overflow will be able to result when a character arrives and at the moment the waiting room(buffer) is full. And consequently, the average character departure rate from the buffer, α (carried load) is less than the average character arrival rate to the buffer, β (offered load) ($\beta = \frac{\lambda}{\theta} = \lambda\bar{l}$)

The overflow probability of the buffer (the average fraction of th total number of arriving characters rejected by the buffer) is defined as

$$\rho \equiv \frac{\text{offered load} - \text{carried load}}{\text{offered load}}$$
$$= \frac{\beta - \alpha}{\beta} = 1 - \frac{\alpha}{\beta} \ldots\ldots(10)$$

And ρ defined as the traffic intensity measures the degree of congestion. We define ρ as the ratio of offered load and unit service interval.
Thus.

$$\rho \equiv \frac{\beta}{\mu} = \frac{\lambda}{\mu\theta} = \frac{\lambda\bar{l}}{\mu} \ldots\ldots(11)$$

Finally let define another parameter, the channel utiligation (u), which measures the fraction of time that the channel is busy.
It can be expressed as

$$u = \rho(1-P_{of})$$
$$= \frac{\beta}{\mu}(1-P_{of}) - \frac{\infty}{\mu} \le 1 \ldots\ldots(12)$$

The degree of the channel utiligation is always less than 1 (unit) because it is impossible physically for the channel to be more than 100 percent busy. At P_{of}=0 which the buffer size is unlimited (no less case), the channel utiligation become same value as the traffic intensity ($u=\rho$) from equation(12).

Since most actual system allow a very low overflow probability($P_{of}=10^{-6}$), the expected queueing delay (or waiting time) due to buffering can be approximated by that of infinite waiting room, poisson arrivals and a single server with geometric service time, M/G/1 queueing model (i. e. single server system with poisson arrival and arbitrary-general service time distribution).

Then expected queueing delay (on waiting time) (D) is [3]

$$D = \frac{\lambda E[L^2]}{2(1-\rho)} = \frac{\lambda(2-\theta)}{2(\theta-\lambda)\theta} \quad \text{...........(13)}$$

Where $E[L^2]$is second moment of burst length L.

The equation (11) and (13) must be changed when we consider that the output messages(burst) from the buffer are sent to various destination. Accordingly, the traffic intensity of each destination (ρ_i) is

$$\rho_i = \frac{\lambda_i \bar{l_i}}{\mu_i} \quad \text{..................(14)}$$

where λ_i : burst arrival rate for the i^{th} destination

$\bar{l}$: average burst length for the i^{th} destination

μ_i : transmission rate for the i^{th} destination

The average traffic intensity ($\bar{\rho}$) is the average traffic of all the destinations (if we assume that there are total n destinations)

$$\bar{\rho} = \frac{1}{n}\sum_{i=1}^{n} \rho_i \quad \text{..................(15)}$$

Thus the expected queueing delay (waiting time) D_i for the t^{th} destination is [3]

$$D_i = \frac{\lambda_i E[L_i^2]}{2(1-\rho_i)} = \frac{\rho_i(2\bar{l_i}-1)}{2(1-\rho_i)} \quad \text{.......(16)}$$

Data collected from several operating time sharing systems disclose that the average number of characters sent from C.P.U. to the group of users terminals is greater than the number of characters sent from the group of users terminals to the C.P.U.

Then much large buffer is needed for the computer-to-user multiplexor to handle the large volume of data generated by C.P.U.

Since the multiplexing system and the central processor intimately interact with each other, we must jointly optimize the central processor and multiplexing system to have an optimal operating system.

Conclusion

According to the different input characteristic, the buffer behavior of two Computer-Communication Systems, time sharing system and the distributed computer system is quite different.

Our study is restricted to analysis of queueing behavior of demultiplexing buffer of the time sharing system.

The analyse shows that it can be represented to the poisson arrival, general service and one server queueing model (M/G/1 model).

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APPENDIX 1

The derivation of characteristic function of geometric distribution.

$$f_L(L=1) = \theta(1-\theta)^{l-1},\ l=1,2,\cdots,n$$

$$\Phi_L(L) = \sum_{L=1}^{n} e^{iuL} f_L(L=1)$$

$$= \sum_{l=1}^{n} e^{iul}\cdot\theta\cdot(1-\theta)^{l-1} \quad l=1,2,\cdots,n$$

$$= \theta\sum_{l=1}^{n} e^{iul}\cdot(1-\theta)^{l-1}$$

$$= \theta\ [\ e^{iul} + e^{2iu}(1-\theta) + e^{3iu}(1-\theta)^2 + \cdots]$$

$$= \theta\ [\ e^{iu}\{1 + e^{iu}(1-\theta) + e^{2iu}(1-\theta)^2 + \cdots\}]$$

$$= \theta\cdot\left(\frac{e^{iu}}{1-(1-\theta)e^{iu}}\right)$$

$$\because\ e^{iu}\{1 + e^{iu}(1-\theta) + e^{2iu}(1-\theta)^2 + \cdots\} = \frac{e^{iu}}{1-(1-\theta)e^{iu}}$$

for $-1 < e^{iu}$, $(1-\theta) < 1$

APPENDIX 2

The derivation of characteristic function of poisson distribution.

$$f_Y(Y=n) = e^{\lambda}\frac{\lambda^n}{n!} \qquad n=0,1,\cdots$$

$$\Phi_Y(u) = \sum_{n=0}^{\infty} e^{iuY}\cdot f_Y(y=n)$$

$$= \sum_{n=0}^{\infty} e^{iun}\cdot e^{-\lambda}\frac{\lambda^n}{n!}$$

$$= e^{-\lambda}\sum_{n=0}^{\infty} e^{iun}\frac{\lambda^n}{n!}$$

$$= e^{-\lambda}(1 + e^{iu}\lambda + e^{2iu}\frac{\lambda^2}{2!} + e^{3iu}\frac{\lambda^3}{3!} + \cdots)$$

$$= e^{-\lambda}\cdot(e^{e^{iu}\cdot\lambda})$$

$$= \exp\ [-\lambda + \lambda\exp(iu)]$$

$$\because\ 1 + e^{iu}\lambda + e^{2iu}\frac{\lambda^2}{2!} + e^{3iu}\frac{\lambda^3}{3!}\cdots = e^{e^{iu}\cdot\lambda}$$