퍼지시스템에 대한 관측가능성

Continuously initial observability for the fuzzy system

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ABSTRACT

This paper is concerned with fuzzy number whose values are normal, convex, upper semicontinuous and compactly supported interval in E_N. We study continuously initial observability for the following fuzzy system:

$$\begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)), \\ x(0) = x_0, \quad y(t) = {}_{\alpha}\Pi(x(t)), \end{cases}$$

where $a: [0, T] \to E_N$ is fuzzy coefficient, initial value $x_0 \in E_N$ and nonlinear funtion $f: [0, T] \times E_N \to E_N$ satisfies a Lipschitz condition. Given fuzzy mapping $\Pi: C([0, T]: E_N) \to Y$ and Y is an another E_N.

I. 서론

Many authors have studied several concepts of fuzzy systems. Kloeden ([4]) investigated the fuzzy dynamic system and Kaleva ([3]) discussed the Cauchy problem for the fuzzy differential equations. Seikkala ([7]) proved the existence and uniqueness of fuzzy solution for the following equation:

$$\begin{cases} \dot{x}(t) = f(t, x(t)), \\ x(0) = x_0, \end{cases}$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number. Observability of linear and nonlinear systems represented by ordinary differential equations has been extensively studied ([1]). In particular Quinn and Carmichael ([9])

studied the continuously initial observability problems using fixed point methods, degress theory and pseudo-inverses. But the case of general fuzzy system has not been treated yet. The purpose of this note is to investigate the continuously initial observability of the fuzzy system in E_N space by utilizing the method of ([8]).

Let E_N be the set of all upper semicontinuous convex normal fuzzy numbers with bounded α-level intervals.

II. 본론

1. Continuously initial observability

We consider the continuously initial observable problem for the following nonlinear fuzzy system:

$$\text{(F.N.S.)}\quad \begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)), \\ x(0) = x_0, \\ y(t) = {}_\alpha\, \Pi(x(t)), \end{cases}$$

where $a:[0,T]\to E_N$ is fuzzy coefficient, initial value $x_0 \in E_N$ and the nonlinear funtion $f:[0,T]\times E_N \to E_N$ satisfies the Lipschitz condition.

Let $\Pi: C([0,T]:E_N)\to Y$ be a given fuzzy mapping where Y is in E_N. The (F.N.S.) is related to the following fuzzy integral system: (F.I.S.)

$$\begin{cases} x(t) = S(t)x_0 + \int_0^t S(t-s)f(s, x(s))ds, \\ x(0) = x_0 \in E_N, \\ y(t) = {}_\alpha\, \Pi(x(t)), \end{cases}$$

where $S(t)$ is a fuzzy number,

$$[S(t)]^\alpha = [S_l^\alpha(t), S_r^\alpha(t)]$$
$$= [\exp\{\int_0^t a_l^\alpha(s)ds\}, \exp\{\int_0^t a_r^\alpha(s)ds\}]$$

and $S_i^\alpha(t)$ $(i = l, r)$ is continuous, that is, there exists a constant $c>0$ such that $|S_i^\alpha(t)| \le c$ for all $t\in[0,T]$.

Now we give definitions relevant to the linear fuzzy system:

$$\text{(F.L.S.)}\quad \begin{cases} \dot{x}(t) = a(t)x(t), \\ x(0) = x_0, \\ y(t) = {}_\alpha\, \Pi(x(t)), \end{cases}$$

We define the fuzzy mapping $\widehat{H}$ from $\widehat{P}(R)$ to E_N by

$$[\widehat{H}(v(t))]^\alpha = \begin{cases} [\Pi(S(t)v(t))]^\alpha, & v(t)\subset \overline{\Gamma_{x_0}}, \\ 0, & \text{otherwise}. \end{cases}$$

where $\widehat{P(R)}$ is set of subsets in R and Γ_{x_0} is support of fuzzy initial value x_0. Then there exist $\widetilde{H}_i^\alpha$ $(i = l, r)$ such that

$$\widehat{H}_l^\alpha(v_l(t)) = \Pi_l^\alpha(S_l^\alpha(t)v_l(t)),\ v_l(t) \in [x_{0l}^\alpha, x_0^1],$$
$$\widehat{H}_r^\alpha(v_r(t)) = \Pi_r^\alpha(S_r^\alpha(t)v_r(t)),\ v_r(t) \in [x_0^1, x_{0r}^\alpha].$$

<u>Definition 1.1.</u>
The (F.L.S.) is continuously initial observable if, for any initial state x_0 there exists a fuzzy mapping $\widehat{H}$ such that the fuzzy out put $y(t)$ satisfies $\widehat{H}(x_0) = {}_\alpha\, y(t)$.

Since (F.L.S.) is continuously initially observable, $\widehat{H}_l^\alpha$, $\widehat{H}_r^\alpha$ are bijective mappings.

$$x_{0l}^\alpha = (\widehat{H}_l^\alpha)^{-1}(y_l(t)),\quad x_{0r}^\alpha = (\widehat{H}_r^\alpha)^{-1}(y_r(t)).$$

Let

$$x_0 = {}_\alpha\, \widehat{H}^{-1}(y(t) - \Pi(\int_0^t S(t-s)f(s, x(s))ds))$$

then substituting this expression

into the (F.I.S.1) yields α- level of $x(t)$ as

$$[x(t)]^{\alpha}=[S(t)\ \widetilde{H}^{-1}(y(t)-\Pi(\int_0^t S(t-s)f(s,x(s))ds))+\int_0^t S(t-s)f(s,x(s))ds]^{\alpha}.$$

Thus

$$[\Pi(x(t))]^{\alpha}=[\Pi_l^{\alpha}(x_l^{\alpha}(t)),\ \Pi_r^{\alpha}(x_r^{\alpha}(t))]$$

$$=[\Pi_l^{\alpha}(S_l^{\alpha}(t)(\widetilde{H}_l^{\alpha})^{-1}(y_l^{\alpha}(t)-\Pi_l^{\alpha}(\int_0^t S_l^{\alpha}(t-s)f_l^{\alpha}(s,x(s))ds)))+\Pi_l^{\alpha}(\int_0^t S_l^{\alpha}(t-s)f_l^{\alpha}(s,x(s))ds),$$
$$\Pi_r^{\alpha}(S_r^{\alpha}(t)(\widetilde{H}_r^{\alpha})^{-1}(y_r^{\alpha}(t)-\Pi_r^{\alpha}(\int_0^t S_r^{\alpha}(t-s)f_r^{\alpha}(s,x(s))ds)))+\Pi_r^{\alpha}(\int_0^t S_r^{\alpha}(t-s)f_r^{\alpha}(s,x(s))ds)]$$

$$=[\widetilde{H}_l^{\alpha}((\widetilde{H}_l^{\alpha})^{-1}(y_l^{\alpha}(t)-\Pi_l^{\alpha}(\int_0^t S_l^{\alpha}(t-s)f_l^{\alpha}(s,x(s))ds)))+\Pi_l^{\alpha}(\int_0^t S_l^{\alpha}(t-s)f_l^{\alpha}(s,x(s))ds),$$
$$\widetilde{H}_r^{\alpha}((\widetilde{H}_r^{\alpha})^{-1}(y_r^{\alpha}(t)-\Pi_r^{\alpha}(\int_0^t S_r^{\alpha}(t-s)f_r^{\alpha}(s,x(s))ds)))+\Pi_r^{\alpha}(\int_0^t S_r^{\alpha}(t-s)f_r^{\alpha}(s,x(s))ds)]$$

$$=[y_l^{\alpha}(t),y_r^{\alpha}(t)]=[y(t)]^{\alpha}.$$

Define

$$\Phi x(t)=_{\alpha}S(t)\ \widetilde{H}^{-1}(y(t)-\Pi(\int_0^t S(t-s)f(s,x(s))ds))+\int_0^t S(t-s)f(s,x(s))ds$$

where the fuzzy mapping $\widetilde{H}^{-1}$ satisfies the above conditions.

The supremum metric d_{∞} on E^n is defined by

$$d_{\infty}(u,v)=\sup\{d_H([u]^{\alpha},[v]^{\alpha}):\alpha\in(0,1]\}$$

for all $u,v\in E^n$ and d_H is the Hausdorff distance.

We assume the following hypotheses:

(H1) The linear equation (F.L.S.) is continuously initial observable.

(H2) There exists a positive constant $l>0$ such that $d_H([f(s,\xi_1(s))]^{\alpha},[f(s,\xi_2(s))]^{\alpha})\leq l d_H([\xi_1(s)]^{\alpha},[\xi_2(s)]^{\alpha})$ forall $\xi_1(s),\xi_2(s)\in E_N$

(H3) There exists a positive constant $k>0$ such that

$$d_H([\widetilde{H}^{-1}(\Pi(\int_0^{\cdot} S(\cdot-s)f(s,\xi_1(s))ds))]^{\alpha},[\widetilde{H}^{-1}(\Pi(\int_0^{\cdot} S(\cdot-s)f(s,\xi_2(s))ds))]^{\alpha})\leq k\int_0^{\cdot} d_H([f(s,\xi_1(s))]^{\alpha},[f(s,\xi_2(s))]^{\alpha}).$$

Theorem 1.1.

Suppose that hypotheses are hold. Then the state of the (F.I.S.) is continuously initial observabe.

Proof. Omitted.

2. Examples

Example 2.1.

Consider the following fuzzy differential equation:

$$\begin{cases}\dot{x}(t)=\tilde{2}x(t),\\ x(0)=x_0,\\ y(t)=\tilde{3}.\end{cases}$$

The α-level sets of fuzzy numbers $\tilde{2}$ and $\tilde{3}$ are

$$[\tilde{2}]^{\alpha}=[\alpha+1,3-\alpha],[\tilde{3}]^{\alpha}=[\alpha+2,4-\alpha]$$

for all $\alpha\in[0,1]$.

From the definition of fuzzy solution,

$$[x(t)]^{\alpha}=[e^{(\alpha+1)t}x_{0l}^{\alpha},\ e^{(3-\alpha)t}x_{0r}^{\alpha}]$$

for all $\alpha\in[0,1]$.

Since $y(t)=_{\alpha}\Pi(x(t))$,

$$[\alpha+2,4-\alpha]=[\Pi_l^{\alpha}(e^{(\alpha+1)t}x_{0l}^{\alpha}),\ \Pi_r^{\alpha}(e^{(3-\alpha)t}x_{0r}^{\alpha})].$$

Hence

$$\widetilde{H}_l^{\alpha}(x_{0l}^{\alpha}) = \Pi_l^{\alpha}(e^{(\alpha+1)t}x_{0l}^{\alpha}) = \alpha+2,$$

$$\widetilde{H}_r^{\alpha}(x_{0r}^{\alpha}) = \Pi_r^{\alpha}(e^{(3-\alpha)t}x_{0r}^{\alpha}) = 4-\alpha.$$

Hence

$$x_{0l}^{\alpha} = (\widetilde{H}_l^{\alpha})^{-1}(\alpha+2), \quad x_{0r}^{\alpha} = (\widetilde{H}_r^{\alpha})^{-1}(4-\alpha).$$

Example 2.2.

Consider the following fuzzy differential equation:

$$\begin{cases} \dot{x}(t) = \tilde{2}x(t) + \tilde{2}tx(t)^2, \\ x(0) = x_0, \\ y(t) = \tilde{3}. \end{cases}$$

Let $f(t, x(t)) = \tilde{2}tx(t)^2$.

Then α-level set of $f(t, x(t))$ is

$$[f(t,x(t))]^{\alpha} = [\tilde{2}tx(t)^2]^{\alpha} = t[\tilde{2}]^{\alpha} \cdot [x(t)^2]^{\alpha}$$

$$= t[\alpha+1, 3-\alpha] \cdot [(x_l^{\alpha}(t))^2, (x_r^{\alpha}(t))^2]$$

$$= t[(\alpha+1)(x_l^{\alpha}(t))^2, (3-\alpha)(x_r^{\alpha}(t))^2]$$

where $[x(t)]^{\alpha} = [x_l^{\alpha}(t), x_r^{\alpha}(t)]$ and $[\tilde{2}]^{\alpha} = [\alpha+1, 3-\alpha]$ for all $\alpha \in [0,1]$.

Hence

$$x_{0l}^{\alpha} = (\widetilde{H}_l^{\alpha})^{-1}((\alpha+2) - \Pi_l^{\alpha}(\int_0^t te^{(\alpha+1)t}(\alpha+1)(x_l^{\alpha}(t))^2 ds)),$$

$$x_{0r}^{\alpha} = (\widetilde{H}_r^{\alpha})^{-1}((4-\alpha) - \Pi_r^{\alpha}(\int_0^t te^{(3-\alpha)t}(3-\alpha)(x_r^{\alpha}(t))^2 ds)).$$

III. 참고문헌

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