Fuzzy Modeling of a PMSM Chaotic System

Zhong Li*, Young Hoon Joo** and Jin Bae Park*

*Dept. of Electrical and Computer Eng., Yonsei Univ., Seoul 120-749, Korea

**School of Electronics and Information Eng., Kunsan Univ., Kunsan, Chunbuk , 573-701, Korea

Abstract—In this paper, a mathematical model of a permanent-magnet synchronous motor (PMSM) is derived, and the steady-state characteristics of this system, when subject to constant input voltages and constant external torque, are formulated. It is shown that the PMSM model can exhibit a variety of chaotic phenomena, under some choices of system parameters and external inputs. Based on TS fuzzy modeling methodology, the TS fuzzy model of the PMSM chaotic system is presented, so the interaction between fuzzy system and chaos can be explored, and then fuzzy-model-based control methodologies can be used to control chaos in chaotic systems. Computer simulations show that the strange attractors in the derived TS fuzzy system and original chaotic system are topologically equivalent.

Keywords—Fuzzy Modeling; Permanent-magnet Synchronous Motors (PMSMs); TS Fuzzy Model; Chaos; Strange Attractors

1. INTRODUCTION

Since the 1970s, dynamic characteristics of various motors are widely studied, to deal with starting up, speed control, and oscillations of the motors. In the study of dynamic characteristics of motors, many problems remain to be further addressed, such as their low-speed feature, known as "low-frequency oscillations," of speed-controlled motors. These problems are closely related to the studies of chaos in nonlinear systems.

Some dynamic characteristics of PMSMs were discussed in [1], and some numerical features of which, such as Poincare map, Lyapunov exponents and capacity dimension, were also studied [2]. Besides, several typical chaos control methods, such as OGY method and Entrainment and Migration method, were adopted to control chaos in PMSMs[3].

While chaos has become one of the most focusing research topics in the literature, we have witnessed rapidly growing interest in making the control systems more intelligent. Among intelligent control approaches, fuzzy control has enjoyed remarkable success in various applications.

In this paper, we first derive the chaotic model of a PMSM, which is suitable for undergoing chaos study, computer simulations show that it can exhibit different dynamical behaviors, such as limit cycles and chaos, under some choices of system parameters and external inputs. Then, a TS fuzzy model of the PMSM chaotic model is established, and it's identical to the original system. This can lay a foundation for using fuzzy-model-based control methods to control chaotic systems, which supply a new control method of chaotic systems.

2. THE PMSM SYSTEM MODEL

The dynamics of a PMSM can be modeled, based on the $d-q$ axis, as

$$\begin{cases} \dfrac{di_d}{dt} = (u_d - R_1 i_d + \omega L_q i_q)/L_d \\ \dfrac{di_q}{dt} = (u_q - R_1 i_q - \omega L_d i_d - \omega\psi_r)/L_q \\ \dfrac{d\omega}{dt} = [n_p \psi_r i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta\omega]/J \end{cases} \quad (1)$$

where i_d, i_q and ω are the state variables, $\tilde{u}_d$ and $\tilde{u}_q$ are the direct- and quadrate-axis stator voltage components, respectively, J is the polar moment of inertia, T_l is the external load torque, β is the viscous damping coefficient, R_1 is the stator winding resistance, L_d and L_q are the direct- and quadrature-axis stator inductors, respectively, i_d and i_q are currents, ω is the motor angular frequency,

ψ_r is the permanent-magnet flux, and n_p represents the number of pole-pairs.

By applying an affine transformation of the form

$$x = \lambda \tilde{x}, \tag{2}$$

and a time-scaling transformation

$$t = \tau \tilde{t}, \tag{3}$$

where $x = [i_d \quad i_q \quad \omega]^T$, $\tilde{x} = [\tilde{i}_d \quad \tilde{i}_q \quad \tilde{\omega}]^T$,

$\lambda = \begin{bmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_q & 0 \\ 0 & 0 & \lambda_\omega \end{bmatrix} = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & \frac{1}{\tau} \end{bmatrix}$, $b = \frac{L_q}{L_d}$,

$k = \frac{\beta}{n_p \tau \psi_r}$, and $\tau = \frac{L_q}{R_1}$,

we obtain a system of equations in the dimensionless form:

$$\begin{cases} \frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q + \tilde{u}_d \\ \frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q \\ \frac{d\tilde{\omega}}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) + \varepsilon\tilde{i}_d\tilde{i}_q - \tilde{T}_L \end{cases} \tag{4}$$

where $\gamma = -\frac{\psi_r}{kL_q}$, $\sigma = \frac{\beta\tau}{J}$, $\tilde{u}_q = \frac{1}{R_1 k}u_q$,

$\tilde{u}_d = \frac{1}{R_1 k}u_d$, $\varepsilon = \frac{n_p b \tau^2 k^2 (L_d - L_q)}{J}$, and

$\tilde{T}_L = \frac{\tau^2}{J}T_L$.

Next, we study the dynamic characteristics of a smooth-air-gap PMSM, with $L_d = L_q = L$ in the model. Thus, system (4) becomes

$$\begin{cases} \frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q + \tilde{u}_d \\ \frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q \, . \\ \frac{d\tilde{\omega}}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) - \tilde{T}_L \end{cases} \tag{5}$$

3. STRANGE ATTRACTORS IN THE PMSM

In [1], a method was proposed to adjust system parameters so that the system exhibits steady states, limit cycles, or chaos. Here, we just show the simulation results in the case of $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$ for brevity's sake.

We supposed σ is fixed, and $\sigma = 5.46$, if $\gamma = 14.1$, $\gamma = 14.93$, and $\gamma = 20$, respectively. For a given initial value of $(0.01, 0.01, 0.01)$, the simulation results are as follows:

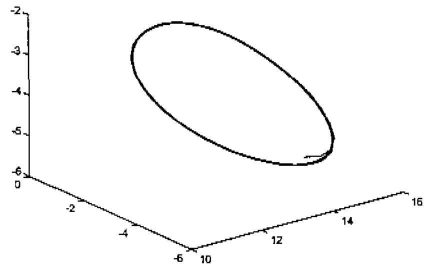

Fig. 1 A limit cycle generated by $\gamma = 14.1$

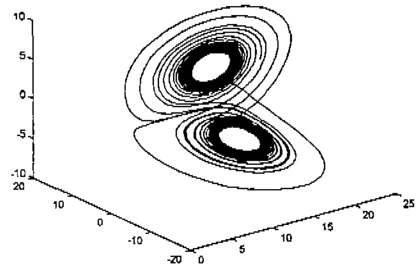

Fig. 2 A chaotic attractor generated by $\gamma = 14.93$

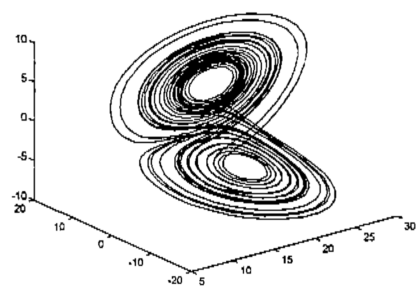

Fig. 3 A chaotic attractor generated by $\gamma = 20$

4. FUZZY MODELING OF THE PMSM CHAOTIC MODEL

The particular fuzzy modeling framework employed here is the so-called Takagi-Sugeno model [4], and this fuzzy modeling method is control-oriented.

The system dynamics is captured by a set of fuzzy implications, which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models.

Specifically, the Takagi-Sugeno fuzzy system is of the following form:

Rule i: IF $x_1(t)$ is M_{i1} ...and $x_n(t)$ is M_{in}

$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t),$$

where

$$\mathbf{x}^T(t) = [x_1(t), x_2(t), \ldots\ x_n(t)],$$
$$\mathbf{u}^T(t) = [u_1(t), u_2(t), \ldots\ u_m(t)],$$

$i = 1,2,\ldots,r$ and r is the number of IF-THEN rules.

M_{ij} are fuzzy sets, and $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ is the output from the i-th IF-THEN rule. Given a pair of $(\mathbf{x}(t), \mathbf{u}(t))$, the final output of the fuzzy system is inferred as follows

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\{\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\}}{\sum_{i=1}^{r} w_i(t)}, \qquad (6)$$

where

$$w_i(t) = \prod_{j=1}^{n} M_{ij}(x_j(t)).$$

$M_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in M_{ij}. The open-loop system of (6) is

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\mathbf{A}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(t)}, \qquad (7)$$

where it is assumed that

$$\sum_{i=1}^{r} w_i(t) > 0;$$
$$w_i(t) \geq 0, \quad i = 1,2,\ldots,r.$$

Next, we show the application of the fuzzy model (7) to the PMSM chaotic model.

The system (5) has two nonlinear quadratic terms, $\widetilde{\omega}\widetilde{i}_q$ and $-\widetilde{\omega}\widetilde{i}_d$. Therefore, we can rewrite it to a linear system with a nonlinear term as follows:

$$\frac{d}{dt}\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q \\ \tilde{\omega}\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma\end{bmatrix}\begin{bmatrix}\tilde{i}_d \\ \tilde{i}_q \\ \tilde{\omega}\end{bmatrix} + \begin{bmatrix}\widetilde{\omega}\widetilde{i}_q \\ -\widetilde{\omega}\widetilde{i}_d \\ 0\end{bmatrix}. \qquad (8)$$

In order to construct a TS fuzzy model, the nonlinear terms $\widetilde{\omega}\widetilde{i}_q$ and $-\widetilde{\omega}\widetilde{i}_d$ must be expressed as a weighted linear sum of some linear functions. For this purpose, we have the following corollary from [5].

Assume $x \in [M_1, M_2]$. The nonlinear term

$$f(x,y) = xy$$

can be represented by a linear weighted sum of linear functions of the form

$$f(x,y) = \left(\sum_{i_2=1}^{2} \mu_{i_2} g_{i_2}(x,y)\right) y,$$

where

$$g_1(x,y) = M_1, \quad g_2(x,y) = M_2$$

and

$$\mu_1 = \Gamma_1^2, \quad \mu_2 = \Gamma_2^2$$

$$\Gamma_1^2 = \frac{-x + M_2}{M_2 - M_1}, \quad \Gamma_2^2 = \frac{x - M_1}{M_2 - M_1}.$$

Using this corollary, system (5) can be expressed as follows:

Rule 1: IF $\tilde{\omega}$ is $M_{\omega_{\min}}$, THEN $\dot{\mathbf{x}} = A_1\mathbf{x}$

Rule 1: IF $\tilde{\omega}$ is $M_{\omega_{\max}}$, THEN $\dot{\mathbf{x}} = A_2\mathbf{x}$ (9)

where

$$A_1 = \begin{bmatrix} -1 & \omega_{\min} & 0 \\ -\omega_{\min} & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & \omega_{\max} & 0 \\ -\omega_{\max} & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$$

and the membership function are

$$M_{\omega_{\min}} = \frac{-\omega + \omega_{\max}}{\omega_{\max} - \omega_{\min}}, \quad M_{\omega_{\max}} = \frac{\omega - \omega_{\min}}{\omega_{\max} - \omega_{\min}} \tag{10}$$

and shown in Fig.4. The defuzzified output of the TS fuzzy model of the PMSM chaotic system with membership function (10) can be obtained by (7).

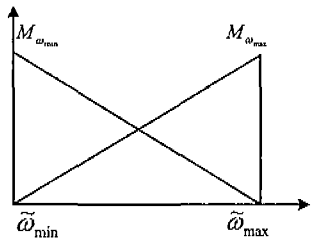

Fig.4 Membership functions

In order to construct a TS fuzzy model, the range of state variables is set to be $(\omega_{\min}, \omega_{\max}) = (-10, 10)$, which is determined from the above numerical simulation of the strange attractor, using the parameter values, $\sigma = 5.46$ and $\gamma = 20$, the initial value $(0.01, 0.01, 0.01)$. The trajectory of the TS fuzzy model of the PMSM model is shown in Fig.5, which is identical to that of the original system, because both have exactly the same outputs.

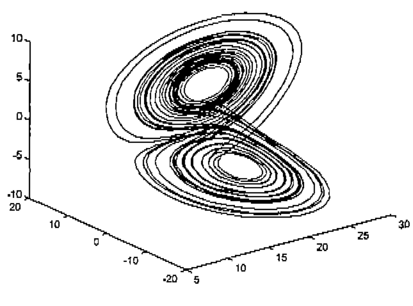

Fig.5 Trajectory of the TS fuzzy model of the chaotic Lorenz system

5. CONCLUSION

A chaotic model of the PMSM was derived, and it can exhibit different characteristic behavior, under different choices of system parameters and external inputs. Then the TS fuzzy model of the PMSM model was established, which is identical to the original system. So fuzzy modeling techniques can be used to model chaotic dynamical systems, which also implies that fuzzy system can be chaotic. Then we can explore the interaction between fuzzy control systems and chaos, can also use fuzzy-model-based control methodologies to control chaotic systems.

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