VEGA를 이용한 웨이브렛 기반 퍼지 시스템 모델링

Wavelet-Based Fuzzy System Modeling Using VEGA

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Abstract : This paper addresses the wavelet fuzzy modeling using Virus-Evolutionary Genetic Algorithm (VEGA). We build a fuzzy system model which is equivalent to the wavelet transform after identifying the coefficients of wavelet transform. We can obtain an accurate system model with a small number of coefficients due to the energy compaction property of the wavelet transform. It thus means that we can construct a fuzzy system model with a small number of rules. In order to identify the wide-ranged coefficients of the wavelet transform, VEGA is adopted, which has prominent ability to avoid premature local convergence that is suitable to complex optimization problems. We demonstrate the superiority of our proposed fuzzy system modeling method over the previous results by modeling nonlinear function.

I. Introduction

When researcher wants to find the model of a system mathematically, the differential equation has been widely used. However, there are so much nonlinearity and a number of time constants in realistic system that the accurate differential equation can hardly be obtained. Though comparatively precise model is acquired, the efficiency decreases by model approximation. In order to solve this problem, the fuzzy inference system introduced by Zadeh [1].

The conventional fuzzy model is based on the knowledge of an expert, and the parameters and the structure of a fuzzy model are tuned through trial and error. It, however, is time-consuming. Subsequently, many self-tuning methods which the parameters and the structure of a fuzzy model are tuned using genetic algorithm(GA) and clustering method by have been studied [2][3]. Wang and Zeng [4][5] represented the fuzzy system with the linear combination of fuzzy basis function(FBF), and Lin [6] obtained equivalent model to discrete wavelet transform through the modification of fuzzy model. Hereby, it was solved that the conventional fuzzy model can hardly deal with abrupt change of signal.

The wavelet transform is very effective in analyzing physical status of certain signal with singularity and element of high frequency as compared with Fourier transform. The wavelet theory has been developed by Daubechies [7], and made rapid progress by Donoho [8]'s proof that wavelet can be basis function for any signal using unconditional basis. The wavelet fuzzy modeling based on Donoho's unconditional basis has the advantage of wavelet transform by constituting FBF and conclusion part to equalize the linear combination of FBF with the linear combination of wavelet function and modifying fuzzy system model to be equivalent to wavelet transform. In wavelet fuzzy model the accurate fuzzy model can be obtained because energy compaction by unconditional basis and the description of transient signal by wavelet basis functions is distinguished.

GA has been used to identify the coefficients of wavelet transform [11]. However, the identification using GA can hardly deal with the coefficients of wavelet transform because it is so wide-ranging. Consequently, in this paper, the method that the coefficients of wavelet transform and the parameters of wavelet function is simultaneously self-tuned using VEGA is proposed.

II. Discrete Wavelet Transform and Wavelet Frame

Any function or signal in $L^2(R)$ can be represented as the linear combination of basis function, in the following form

$$f(x)=\sum_k c_{j_0}(k)\varphi_{j_0,k}(x)+\sum_k\sum_{j=j_0}^{\infty}d_j(k)\psi_{j,k}(x), \quad (1)$$

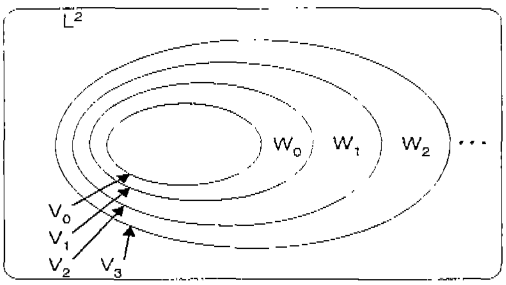

Figure 1. Scaling function and wavelet vector spaces

where $j,k,l\in Z$, $\varphi_{j_0,k}(x)$ is scale function and $\psi_{j,k}(x)$ is wavelet function. The choice of j_0 sets the coarest scale whose space is spanned by $\varphi_{j_0,k}(t)$. The rest of $L^2(R)$ is spanned by the wavelets which provide the high resolution details of the signal.

The multi-resolution equation can represent the signal with time-scale by dividing into precise elements. Therefore, $\psi_{j,k}(x)$ in Eqn. (1) can be represented in the following form:

$$\psi_{j,k}(x)=a^{-j/2}\psi(a^{-j}x-bk). \quad (2)$$

The equation (1) is elucidated by multi-resolution analysis. The various subspaces can be seen from the following expressions.

As illustrated in Fig. 1, at a given $j=-\infty$, the following equation can be considered.

$$L^2=\cdots\oplus W_{-2}\oplus W_{-1}\oplus W_0\oplus W_1\oplus W_2\oplus\cdots. \quad (3)$$

The above equation (3) means that any function can be represented by linear combination of infinite wavelet functions. Thus, Eqn. (1) becomes as follows:

$$f(x)=\sum_{j,k}d_{j,k}\psi_{j,k}(x) \quad (4)$$

It is so difficult to implement multi-scaling wavelet transform that multi-scaling wavelet frame has been studied [9][10]. Equation which represent multi-scaling function wavelet with single-scaling wavelet is as follows:

$$\psi(x)=\psi_1(x_1)\cdot\cdot\cdot\psi_n(x_n). \quad (5)$$

Single-scaling wavelet function is divided into n orthogonal directions.

$$\widehat{\psi}(\omega)=\widehat{\psi}_1(\omega_1)\cdot\cdot\widehat{\psi}_n(\omega_n). \quad (6)$$

where $\widehat{\psi}(\omega)$ is Fourier transform of $\psi(x)$. It is well proven that the following admissibility and inequality equation conditions should be satisfied to use a function as wavelet function [10].

$$\int\frac{|\widehat{\Psi}_i(\omega_i)|^2}{|\omega_i|}d\omega_i<\infty. \quad (7)$$

$$A\|f\|^2\le\sum_{j,k}|\langle f,\Psi_{j,k}\rangle|^2\le B\|f\|^2, \quad (8)$$

where $A>0$, $B<\infty$.

Therefore, $\Psi_i(x_i)$ which satisfies conditions (7) and (8) should be set as wavelet frame.

In this paper, 'Mexican Hat' is employed as a mother wavelet function which satisfies both of the conditions as follows:

$$\Psi_i(x_i)=\alpha_i(1-\alpha_i x_i^2)e^{-\frac{\alpha_i x_i^2}{2}} \quad (9)$$

Substituting Eqn. (9) for (4) and (5), the following is computed.

$$\psi_{j,k}= a^{-\frac{j_1}{2}}\alpha_1[1-\alpha_1(a^{-j_1}x_1-b_1k_1)^2]e^{-\frac{\alpha_1(a^{-j_1}x_1-b_1k_1)^2}{2}}\cdot\cdot\cdot a^{-\frac{j_n}{2}}\alpha_n[1-\alpha_n(a^{-j_n}x_n-b_nk_n)^2]e^{-\frac{\alpha_n(a^{-j_n}x_n-b_nk_n)^2}{2}} \quad (10)$$

III. Identification wavelet fuzzy model with VEGA

The fuzzy model which is used in this paper is as follows:

Rule $_i$: *If* x_i *is* A_{i1}, $\cdots$, x_n *is* A_{in},
Then y_i *is* $d_i\,\alpha_1(1-\alpha_1x_1^2)\alpha_2(1-\alpha_2x_2^2)\cdots\alpha_n(1-\alpha_nx_n^2)$, (11)

where *Rule* $_i$ is ith rule, x_j is jth input variable, y_i is ith output variable, and A_{ij} is membership function for ith rule of jth input defined as Gaussian function.

The conclusion part is constituted by the product of the remainders except Gaussian function of 'Mexican Hat' wavelet function in Eqn. (11).Thus, the model which is equivalent to wavelet transform can be obtained by modifying the conclusion part of general fuzzy model.

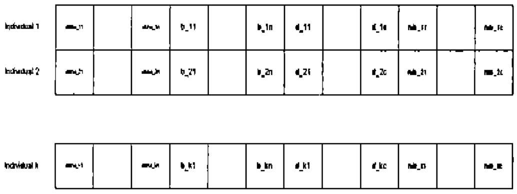

Figure 2 Genetic coding for wavelet fuzzy model

In this paper, the parameters of wavelet function and the coefficients of wavelet transform are simultaneously tuned by operators of VEGA [11].

Genetic coding for wavelet fuzzy model is shown in Fig. 2. n is the number of inputs, c is the maximum number of rules, and k is the number of individual in population. *alpha*, b, and d is real and represent *alpha*, b, and d in Eqn. (10). *rule* is defined as 0 or 1 and decides the existence of the rule. Initializing the chromosome of host individual, VEGA is implemented by the operators of VEGA.

In order to identify the parameters and structure of fuzzy model simultaneously, the fitness function which has two terms, RMSE and the number of rule has been used [12]. In this paper, Pareto optimality is adopted instead of fitness function [13]. Because it is not acceptable to represent multiobjective optimization with a single function.

The objectives can be interpreted as follows:

$$f_1 = e, \quad f_2 = 1/r, \tag{12}$$

where e is root mean square error and r is the number of rules. Two objectives are competing each other and non-commensurable. We do not have to think about the scale of f_1 and f_2. A host individual is ranked as how many number of individuals in the population is superior to it. The rule for comparison can be considered when individual I_1 is superior to I_2 if and only if

$$\forall j=1,2 \quad f_j(I_1)\le f_j(I_2), \quad \exists k=1,2 \quad f_k(I_1)<f_k(I_2). \tag{13}$$

The individual I is then ranked as

$$rank(I) = 1+s, \tag{14}$$

where I is superior to other s host individuals in the population. Consequently, all we have to design is to decide the best individual. Initializing r_1 and r_2 in the range of f_1 and f_2, r_1 and r_2 are decreased until only one individual exists in s, then r_1 and r_2 are increased.

Obtaining the individual of rank(I)=1, the individual which is the middle of individuals in the same rank is standardized.

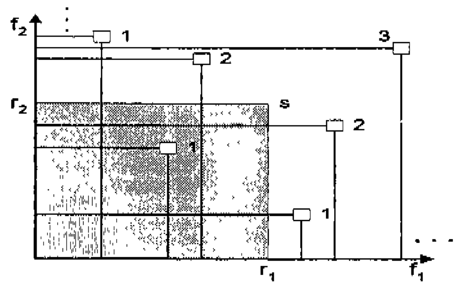

Figure 3. Pareto ranking of two objectives

Then, all of the host individuals are numbered by Eqn. (13) and (14). As illustrated in Fig. 3, the method that the middle value of rank 1 is selected as standard is proposed. Because it is natural that the individual with a rule is selected as the best in case that other method is used.

IV. Computer simulation

With time series input-output data which is presented by the chaotic Box and Jenkins [14], we obtain wavelet fuzzy model of the system using VEGA. This 256 data is constituted by the gas influx $u(t)$ as input and the density of CO_2 $y(t)$ as output. This process is so dynamic that $u(t-4)$ and $y(t-1)$ are selected as input data. Regarding that $u(t-4)$ and $y(t-1)$ are the main factors to influence the present output $y(t)$, the training data is constituted.

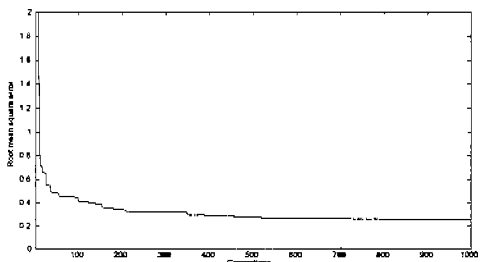

Figure 4. Changes of root mean square error

Figure 4 shows that RMSE fluctuates in the beginning of simulation and changes even in the end of simulation. It means that host individual and virus individuals coevolve.

Our model was compared with other models in Table 1.(Note that the errors of other models were transformed into RMSE's.) It can be noticed that our model is preferable to other models not only on RMSE but also on the number of rules.

Table 1. Comparisons of our model with other models

Model	Number of rules	RMSE
Liska's[15]	10	0.367
Xu's[16]	25	0.573
Ours	3	0.257

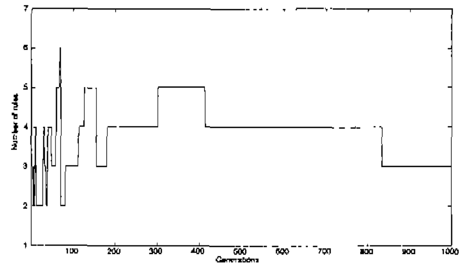

Figure 5. Changes of the number of rules

Figure 5 shows the changes of the number of rules. The number of rules widely changes from the minimum value, 2, to the maximum value, 6. The number of rules was ultimately fixed with three. It is quite few compared with other models as indicated in Table 1.

Figure 6 shows actual output and model output. It could be recognized that model output follows actual output almost exactly. Therefore, our purpose was successfully accomplished.

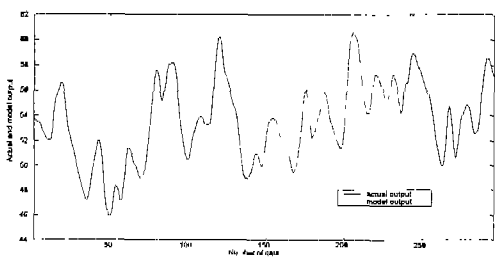

Figure 6. Comparison between actual output and model output

V. Conclusion

Wavelet fuzzy modeling using virus-evolutionary genetic algorithm(VEGA) was proposed, in this paper. Modifying conventional fuzzy model to have several fuzzy rule bases and the conclusion part to be the part of the 'Mexican Hat' wavelet function, fuzzy model was equivalent to discrete wavelet function. The wavelet fuzzy model, thus, inherited the advantage from discrete wavelet transform. The prominent fuzzy model was obtained with the small number of rules.

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